CBSE NCERT Solutions for Class 11 Mathematics Chapter 03

Back of Chapter Questions

1. Find the radian measures corresponding to the following degree measures:
(i)25^{o}

(ii)$-47^{o}30'$

(iii)240^{o}

(iv)520^{o}

Solution:

(i) Step1:
Given degree is 25^{o}

It is known that $180^{o} = \pi$radian

Therefore, $25^{o} = \frac{\pi}{180} \times 25$radian

$= \frac{5\pi}{36}$ radian

Overall Hint: Convert the degree measures to radians by using the formula Angle in degree = $\frac{\pi}{180} \times Measure\ of\ Angle\ in\ degrees$ radian

(ii) Step1:
Given degree is $-47^{o}30'$

$-47^{o}30' = -47\frac{1}{2}$degree $[\because\ 1^{o} = 60']$

$= -\frac{95^{o}}{2}$

As$180^{o} = \pi$radian

$\Rightarrow -\frac{95^{o}}{2} = \frac{\pi}{180} \times \left(\frac{-95}{2}\right)$radian$= \left(\frac{-19}{72}\right)\pi$radian

Therefore, $-47^{o}30' = \left(\frac{-19}{72}\right)\pi$radian

Overall Hint: Convert the degree measures to radians by using the formula Angle in degree = $\frac{\pi}{180} \times Measure\ of\ Angle\ in\ degrees$ radian

(iii) Step1:
Given degree is240^{o}

It is known that $180^{o} = \pi$radian

$\therefore 240^{o} = \frac{\pi}{180} \times 240$radian

$= \frac{4\pi}{3}$ radian

Overall Hint: Convert the degree measures to radians by using the formula Angle in degree = $\frac{\pi}{180} \times Measure\ of\ Angle\ in\ degrees$ radian

a. Step1:
Given degree is520^o

As $180^o = \pi$radian

Therefore, $520^o = \frac{\pi}{180} \times 520$radian= $\frac{26\pi}{9}$radian

Overall Hint: Convert the degree measures to radians by using the formula Angle in degree = $\frac{\pi}{180} \times Measure\ of\ Angle\ in\ degrees$ radian

2. Find the degree measures corresponding to the following radian measures(Use$\pi = \frac{22}{7}$)

(i)$\frac{11}{16}$

(ii)-4

(iii)$\frac{5\pi}{3}$

(iv)$\frac{7\pi}{6}$

Solution:

(i) Step1:
Given radian is$\frac{11}{16}$

As πradian= 180^o

$\Rightarrow \frac{11}{16}$radian= $\frac{180}{\pi} \times \frac{11}{16}$degree= $\frac{45\times11}{\pi\times4}$degree

$= \frac{45\times11\times7}{22\times4}$degree

$= \frac{315}{8}$degree

$= 39\frac{3}{8}$degree

Step2:

$= 39^o + \frac{3\times60}{8}$Minutes $[1^o = 60']$

$= 39^o + 22' + \frac{1}{2}$minutes

$=39^o22'30''$ $[1' = 60'']$

Overall Hint: Convert the given radian angle into degrees by using the formula R**ad** × 180/π = **Deg**

(ii) Step1:
Given radian is -4

As πradian= 180^o

-4radian= $\frac{180}{\pi} \times (-4)$radian= $\frac{180 \times 7 \times (-4)}{22}$degree

$= \frac{-2520}{11}$degree= $-229\frac{1}{11}$degree

$= -229^o + \frac{1 \times 60}{11}$minutes $[1^o = 60']$

Step2:

$= -229^o + 5' + \frac{5}{11}$minutes

$= -229^o + 5' + \frac{5 \times 60}{11}$ minutes

$= -229^o5'27''$ $[1' = 60'']$

Overall Hint: Convert the given radian angle into degrees by using the formula R**ad** × 180/π = **Deg**

(iii) Step1:
Given radian is $\frac{5\pi}{3}$

As πradian= 180^o

$\frac{5\pi}{3}$radian= $\frac{180}{\pi} \times \frac{5\pi}{3}$radian

$= 300^o$

Overall Hint: Convert the given radian angle into degrees by using the formula R**ad** × 180/π = **Deg**

(iv) Step1:
Given radian is$\frac{7\pi}{6}$

As πradian= 180^o

$\frac{7\pi}{6}$radian$= \frac{180}{\pi} \times \frac{7\pi}{6}$radian$= 210^{0}$

Overall Hint: Convert the given radian angle into degrees by using the formula R**ad** × 180/π = **Deg**

3. A wheel makes 360revolutions in one minute. Through how many radians does it turn in one second?

Solution:

Step1:

Given,

Numberofrevolutionsmadebythewheelin1minute$= 360$

Therefore, Numberofrevolutionmadebythewheelin1second$= \frac{360}{60} = 6$

For one completerevolution,thewheel willturnanangleof2πradian.

Therefore,in6completerevolutions,itwillturnanangleof$= 6 \times 2\pi = 12\pi$ radian

Hence,inonesecond,thewheelturnsanangleof12πradian.

Overall Hint: Convert the revolutions per minute to radians per second by using the formula 1 **rpm** = 2(pi)/60 = pi/30 **radians per second**

4. Find the degree measure of the angle subtended at the Centre of a circle of radius100 cm by an arc of length22 cm (Use$\pi = \frac{22}{7}$)

Solution:

Step1:

Given,

radiusofcircle$= 100$ cm,

lengthofarc$= 22$ cm.

Consider theanglesubtendedbythearcattheCentreofthecircleasθ.

Weknow, $\theta = \frac{\text{arc}}{\text{radius}}$

Step2:

$\theta = \frac{22}{100}$radian$= \frac{180}{\pi} \times \frac{22}{100}$degree$= \frac{180\times7\times22}{22\times100}$degree

$= \frac{126}{10}$degree

$= 12\frac{3}{5}$degree

$= 12\frac{3\times60}{5}$ degree $\quad [1^o = 60']$

$= 12^o36'$

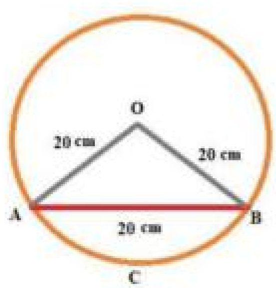
Therefore,therequiredangleis$12^o36'$.
Overall DL: M

Overall Hint: Find the angle subtended at the center of the circle by using the formula $\theta = \frac{\text{arc}}{\text{radius}}$ then find the angle measure in degree by using $\theta = \frac{180\times7\times22}{22\times100}$degree

5. In a circle of diameter40 cm,the length of a chord is20 cm.Find the length of minor arc of the chord.

Solution:

Step1:



Step1:

Step2:

Given, Diameter of the circle$= 40$ cm

So, Radius of the circle$= \frac{40}{2}\text{cm} = 20$ cm

Let AB be a chord of the circle.

InΔOAB, $OA = OB =$radiusofcircle$= 20$ cm

Also, $AB = 20$ cm

Therefore ΔOAB is equilateral triangle.

$\therefore \theta = 60^o = \frac{\pi}{3}$radian

Step3:

We know that $\theta = \frac{\text{arc}}{\text{radius}}$

Arc= $\theta \times$Radius

$= \frac{\pi}{3} \times 20$ cm

$= \frac{20\pi}{3}$ cm

Therefore, the length of the minor arc of the chord is $\frac{20\pi}{3}$ cm.

Overall Hint: Draw a rough figure making use of the given data then find the measure of the angle in radians. Also find the arc length using the formula Arc= $\theta \times$Radius

6. If in two circles,arcs of the same length subtend angles60^oand75^oat the Centre,find the ratio of their radii.

Solution:

Step1:

Let the angle formed by the arc of first circle $\theta_1 = 60^o = 60 \times \left(\frac{\pi}{180}\right)$radian$= \frac{60\pi}{180}$

Let the angle formed by the arc of second circle $\theta_2 = 75^o = 75 \times \left(\frac{\pi}{180}\right) = \frac{75\pi}{180}$

Let the radius of first circle be r_1 and radius of second circle be r_2

It is known that, $\theta = \frac{\text{arc}}{\text{radius}}$

Step2:

Radius$= \frac{\text{arc}}{\theta}$

$$\frac{r_1}{r_2} = \frac{\frac{arc}{\theta_1}}{\frac{arc}{\theta_2}} = \frac{\theta_2}{\theta_1}$$

$$\frac{r_1}{r_2} = \frac{\frac{75\pi}{180}}{\frac{60\pi}{180}} = \frac{75}{60} = \frac{5}{4}$$

Therefore, the ratio of their radii is 5: 4

Overall Hint:First find the angles formed by the arcs of the two circles in radians then make use of the formula $\frac{r_1}{r_2} = \frac{\frac{arc}{\theta_1}}{\frac{arc}{\theta_2}} = \frac{\theta_2}{\theta_1}$ to find ratio of their radii.

7. Find the angle in radian through which a pendulum swings if its length is75 cm and the tip describes an arc of length.

(i)10 cm

(ii)15 cm

(iii)21 cm

Solution:

We know that in a circle of radius r, if an arc of length l unit subtends an angle θ radian at the Centre, then $\theta = \frac{l}{r}$

Given that $r = 75$ cm

(i) Step1:

Here,$l = 10$ cm

$$\theta = \frac{l}{r}$$

$$\theta = \frac{10}{75}\text{radian} = \frac{2}{15}\text{radian}$$

Overall Hint: Find the angle using formula

$\theta = \frac{l}{r}$ radian

(ii) Step1:

Here,$l = 15$ cm

$$\theta = \frac{l}{r}$$

$$\Rightarrow \theta = \frac{15}{75}\text{radian} = \frac{1}{5}\text{radian}$$

Overall Hint: Find the angle using formula
$\theta = \frac{l}{r}$ radian

(iii) Step1:
Here, $l = 21$ cm

$\theta = \frac{l}{r}$

$\theta = \frac{21}{75}$ radian

$= \frac{7}{25}$ radian

Overall Hint: Find the angle using formula
$\theta = \frac{l}{r}$ radian

Exercise3.2

Find the values of other five trigonometric functions in Exercise 1 to 5.

1. $\cos x = -\frac{1}{2}$, x lies in third quadrant.

Solution:

Step1:

Given, $\cos x = \frac{1}{2}$

Therefore, $\sec x = \frac{1}{\cos x} = \frac{1}{-\frac{1}{2}} = -2$

We know that $\sin^2 x + \cos^2 x = 1$

Step2:

$\Rightarrow \sin^2 x = 1 - \cos^2 x$

$\Rightarrow \sin^2 x = 1 - \left(-\frac{1}{2}\right)^2$

$\Rightarrow \sin^2 x = 1 - \frac{1}{4}$

$\Rightarrow \sin^2 x = \frac{3}{4}$

Therefore, $\sin x = \pm \frac{\sqrt{3}}{2}$

The value of $\sin x$ is negative as x lies in third quadrant. So, $\sin x = -\frac{\sqrt{3}}{2}$

Step3:

We know that $\operatorname{cosec} x = \frac{1}{\sin x}$

$\Rightarrow \operatorname{cosec} x = -\frac{2}{\sqrt{3}}$

We know that $\tan x = \frac{\sin x}{\cos x} = \frac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}}$

Therefore, $\tan x = \sqrt{3}$

We know that $\cot x = \frac{1}{\tan x}$

Therefore, $\cot x = \frac{1}{\sqrt{3}}$

Overall Hint: As cos x is given we can find sec x as its reciprocal then we will find sin x using 1 – cos^2(x) and then cosec x as its reciprocal the cot x and tan x can be found using the ratio of cos x and sin x

2. $\sin x = \frac{3}{5}$, xLiesinsecondquadrant.

Solution:

Step1:

Given, $\sin x = \frac{3}{5}$

$\operatorname{cosec} x = \frac{1}{\sin x}$

$\Rightarrow \operatorname{cosec} x = \frac{5}{3}$

We know that, $\sin^2 x + \cos^2 x = 1$

Step2:

$\Rightarrow \cos^2 x = 1 - \sin^2 x$

$\Rightarrow cos^2 x = 1 - \left(\frac{3}{5}\right)^2$

$\Rightarrow \cos^2 x = 1 - \frac{9}{25}$

$\Rightarrow \cos^2 x = \frac{16}{25}$

$\Rightarrow \cos x = \pm\frac{4}{5}$

The value of cos is negative as x lies in second quadrant. So, $\cos x = -\frac{4}{5}$

Step3:

We know that, $\sec x = \frac{1}{\cos x}$

$\Rightarrow \sec x = -\frac{5}{4}$

We know that, $\tan x = \frac{\sin x}{\cos x}$

$= \frac{\frac{3}{5}}{-\frac{4}{5}} = -\frac{3}{4}$

We know that, $\cot x = \frac{1}{\tan x} = -\frac{4}{3}$

Overall Hint: As sin x is given we can find cosec x as its reciprocal then we will find cos x using 1 – sin^2(x) and then sec x as its reciprocal the cot x and tan x can be found using the ratio of cos x and sin x

3. $\cot x = \frac{3}{4}, x$ lies in third quadrant.

Solution:

Step1:

Given, $\cot x = \frac{3}{4}$

We know that, $\tan x = \frac{1}{\cot x}$

$\Rightarrow \tan x = \frac{4}{3}$

We know that $\operatorname{cosec}^2 x = 1 + \cot^2 x$

$\Rightarrow \operatorname{cosec}^2 x = 1 + \left(\frac{3}{4}\right)^2$

$\Rightarrow \operatorname{cosec}^2 x = 1 + \frac{9}{16} = \frac{25}{16}$

$\Rightarrow \operatorname{cosec} x = \pm\sqrt{\frac{25}{16}}$

Therefore, $\operatorname{cosec} x = \pm\frac{5}{4}$

Step2:

But x lies in third quadrant so $\operatorname{cosec} x$ will be negative.

$\Rightarrow \operatorname{cosec} x = -\frac{5}{4}$

We know that $\sin x = \frac{1}{\operatorname{cosec} x}$

$\sin x = -\frac{4}{5}$

Also, $\sin^2 x + \cos^2 x = 1$

$\Rightarrow \cos^2 x = 1 - \sin^2 x$

$\Rightarrow \cos^2 x = 1 - \left(-\frac{4}{5}\right)^2$

$\Rightarrow \cos^2 x = 1 - \frac{16}{25}$

$\Rightarrow \cos^2 x = \frac{9}{25}$

$\Rightarrow \cos x = \pm\frac{3}{5}$

Step3:

But x lies in third quadrant,so value of $\cos x$ is negative.

Therefore, $\cos x = -\frac{3}{5}$

$\sec x = \frac{1}{\cos x} = -\frac{5}{3}$

Overall Hint: As cot x is given we can find tan x as its reciprocal then we will find cosec x using 1 + cot^2(x) and then sin x as its reciprocal the cos x and sec x can be found using the 1- sin^(2)x and the reciprocal of cos x will be sec x

4. $\sec x = \frac{13}{5}$, x lies in fourth quadrant.

Solution:

Step1:

Given, $\sec x = \frac{13}{5}$

$$\Rightarrow \cos x = \frac{1}{\sec x} = \frac{5}{13}$$

We know that $\sin^2 x + \cos^2 x = 1$

$$\Rightarrow \sin^2 x = 1 - \cos^2 x$$

$$\Rightarrow \sin^2 x = 1 - \left(\frac{5}{13}\right)^2$$

$$\Rightarrow \sin^2 x = \frac{144}{169}$$

$$\Rightarrow \sin x = \pm\frac{12}{13}$$

Step2:

But x lie in a fourth quadrant, so value of $\sin x$ is negative.

Therefore, $\sin x = -\frac{12}{13}$

Also, $\operatorname{cosec} x = \frac{1}{\sin x} = -\frac{13}{12}$

$$\tan x = \frac{\sin x}{\cos x} = \frac{-\frac{12}{13}}{\frac{5}{13}} = -\frac{12}{5}$$

$$\cot x = \frac{1}{\tan x}$$

Therefore, $\cot x = -\frac{5}{12}$

Overall Hint: As sec x is given we can find cos x as its reciprocal then we will find sin x using 1 – cos^2(x) and then cosec x as its reciprocal the cot x and tan x can be found using the ratio of cos x and sin x and vice versa

5. $\tan x = -\frac{5}{12}$, x in second quadrant.

Solution:

Step1:

Given, $\tan x = -\frac{5}{12}$

$\cot x = \frac{1}{\tan x}$

$\Rightarrow \cot x = -\frac{12}{5}$

We know that $\operatorname{cosec}^2 x = 1 + \cot^2 x$

$\Rightarrow \operatorname{cosec}^2 x = 1 + \left(-\frac{12}{5}\right)^2$

$\Rightarrow \operatorname{cosec}^2 x = 1 + \frac{144}{25} = \frac{169}{25}$

$\Rightarrow \operatorname{cosec} x = \pm\sqrt{\frac{169}{25}}$

$\Rightarrow \operatorname{cosec} x = \pm\frac{13}{5}$

Step2:

But x lies in second quadrant, so $\operatorname{cosec} x$ is positive.

$\therefore \operatorname{cosec} x = \frac{13}{5}$

We know that $\sin x = \frac{1}{\operatorname{cosec} x}$

$\sin x = \frac{5}{13}$

Also, $\sin^2 x + \cos^2 x = 1$

$\Rightarrow \cos^2 x = 1 - \sin^2 x$

$\Rightarrow \cos^2 x = 1 - \left(\frac{5}{13}\right)^2$

$\Rightarrow \cos^2 x = 1 - \frac{25}{169}$

$\Rightarrow \cos^2 x = \frac{144}{169}$

$\Rightarrow \cos x = \pm\frac{12}{13}$

Step3:

But x lies in these con quadrant, so value of $\cos x$ is negative.

Therefore, $\cos x = -\frac{12}{13}$

We know that, $\sec x = \frac{1}{\cos x}$

Therefore, $\sec x = -\frac{13}{12}$

Overall Hint: As tan x is given we can find cot x as its reciprocal then we will find cosec x using 1 + cot^2(x) and then sin x as its reciprocal the cos x can be found using 1 – sin^2(x) and sec x as its reciprocal

Find the values of the trigonometric functions in Exercises 6 to 10

6. $\sin 765^o$

Solution:

Step1:

$\sin 765^o$ can be written as $\sin(2 \times 360^o + 45^o)$

$= \sin 45^o [\because \sin$ is positive in first quadrant]

$= \frac{1}{\sqrt{2}}$

Therefore, $\sin 765^o = \frac{1}{\sqrt{2}}$

Overall Hint: Write sin 765^o as sin (2*360 + 45) using Sin(2π+ x) = sin x we get sin 765 as $\frac{1}{\sqrt{2}}$

7. $\text{cosec}\,(-1410^o)$

Solution:

Step1:

Given, $\text{cosec}(-1410^o)$

$= -\text{cosec}(1410^o)$

$= -\text{cosec}(4 \times 360^o - 30^o)$

$= -\text{cosec}(-30^o)$

$= \text{cosec}(30^o)$

$= 2$

Overall Hint: Write cosec (-1410^o) as cosec $(4 \times 360^o - 30^o)$ using cosec (2nπ+x) = Cosec we get cosec (-1410^o) as 2

8. $\tan\frac{19\pi}{3}$

Solution:

Step1:

Given, $\tan\frac{19\pi}{3}$

$\tan\frac{19\pi}{3} = \tan\left(6\pi + \frac{\pi}{3}\right)$

$= \tan\frac{\pi}{3}$ [We know that value of $\tan x$ repeats after an interval of π or 180^o]

$= \sqrt{3}$

Therefore, $\tan\frac{19\pi}{3} = \sqrt{3}$

Overall Hint: Write $\tan\frac{19\pi}{3}$ as $\tan(6\pi + \frac{\pi}{3})$ and using $\tan(2n\pi + x) = \tan x$ we get $\tan\frac{19\pi}{3} = \sqrt{3}$

9. $\sin\left(-\frac{11\pi}{3}\right)$

Solution:

Step1:

Given, $\sin\left(-\frac{11\pi}{3}\right)$

$\sin\left(-\frac{11\pi}{3}\right) = -\sin\left(\frac{11\pi}{3}\right)$ [∵sin is negative in fourth quadrant]

$= -\sin\left(4\pi - \frac{\pi}{3}\right)$

$= -\left[-\sin\left(\frac{\pi}{3}\right)\right]$ [∵sin is negative in fourth quadrant]

$=\sin\left(\frac{\pi}{3}\right)$

$=\frac{\sqrt{3}}{2}$

Overall Hint: Write $\sin\left(-\frac{11\pi}{3}\right)$ as - sin $(\frac{11\pi}{3})$ = - $\sin\left(4\pi-\frac{\pi}{3}\right)$ then using sin $(2n\pi - x)$ = -sin x we get $\sin\left(-\frac{11\pi}{3}\right)$ as $\frac{\sqrt{3}}{2}$

10. $\cot\left(-\frac{15\pi}{4}\right)$

Solution:

Step1:

Given, $\cot\left(-\frac{15\pi}{4}\right)$

$\cot\left(-\frac{15\pi}{4}\right)=-\cot\left(\frac{15\pi}{4}\right)$ [∵cot is negative in fourth quadrant]

$=-\cot\left(4\pi-\frac{\pi}{4}\right)$

$=-\left[-\cot\left(\frac{\pi}{4}\right)\right]$ [∵cot is negative in fourth quadrant]

$=\cot\left(\frac{\pi}{4}\right)$

$=1$

Overall Hint: Write $\cot\left(-\frac{15\pi}{4}\right)$ as - cot $(\frac{15\pi}{4})$ then as -- $\cot\left(4\pi-\frac{\pi}{4}\right)$ using cot $(2n\pi$-x) = cot x we get $\cot\left(-\frac{15\pi}{4}\right)$ as 1

Exercise:3.3

1. $\sin^2\frac{\pi}{6}+\cos^2\frac{\pi}{3}-\tan^2\frac{\pi}{4}=-\frac{1}{2}$

Solution:

L.H.S$\sin^2\frac{\pi}{6}+\cos^2\frac{\pi}{3}-\tan^2\frac{\pi}{4}$

$=\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2-(1)^2$

$=\frac{1}{4}+\frac{1}{4}-1$

$= \frac{1}{2} - 1$

$= -\frac{1}{2}$

=R.H.S

Hence LHS = RHS.

Overall Hint: Substitute the values of $\sin\frac{\pi}{6}\cos\frac{\pi}{3}, \tan\frac{\pi}{4}$ in the LHS . This will be equal to what's given in the RHS hence proved

2. Prove that : $2\sin^2\frac{\pi}{6} + \mathrm{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3} = \frac{3}{2}$

Solution:

L.H.S= $2\sin^2\frac{\pi}{6} + \mathrm{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}$

$= 2\left(\frac{1}{2}\right)^2 + \mathrm{cosec}^2\left(\pi + \frac{\pi}{6}\right)\left(\frac{1}{2}\right)^2$

$= 2 \times \frac{1}{4} + \left(-\mathrm{cosec}\frac{\pi}{6}\right)^2\left(\frac{1}{4}\right)$

$= \frac{1}{2} + (-2)^2\left(\frac{1}{4}\right)$

$= \frac{1}{2} + 1$

$= \frac{3}{2}$

=R.H.S

Hence LHS = RHS.

Overall Hint: Substitute the values of $\sin\frac{\pi}{6}, \mathrm{cosec}\ \frac{7\pi}{6}, \cos\frac{\pi}{3}$ in the LHS . This will be equal to what's given in the RHS

3. Prove that $\cot^2\frac{\pi}{6} + \mathrm{cosec}^5\frac{\pi}{6} + 3\tan^2\frac{\pi}{6} = 6$

Solution:

L.H.S= $\cot^2\frac{\pi}{6} + \mathrm{cosec}\frac{5\pi}{6} + 3\tan^2\frac{\pi}{6}$

$= (\sqrt{3})^2 + \operatorname{cosec}\left(\pi - \frac{\pi}{6}\right) + 3\left(\frac{1}{\sqrt{3}}\right)^2$

$= 3 + \operatorname{cosec}\left(\frac{\pi}{6}\right) + 3 \times \frac{1}{3}$

$= 3 + 2 + 1$

$= 6$

=R.H.S

Hence, LHS = RHS.

Overall Hint: Substitute the values of $\cot\frac{\pi}{6}, \operatorname{cosec}\frac{\pi}{6}, \tan\frac{\pi}{6}$ in the LHS . This will be equal to what's given in the RHS

4. Prove that $2\sin^2\frac{3\pi}{4} + 2\cos^2\frac{\pi}{4} + 2\sec^2\frac{\pi}{3} = 10$

Solution:

GIVEN, L.H.S= $2\sin^2\frac{3\pi}{4} + 2\cos^2\frac{\pi}{4} + 2\sec^2\frac{\pi}{3}$

$2\sin^2\left(\pi - \frac{\pi}{4}\right) + 2\left(\frac{1}{\sqrt{2}}\right)^2 + 2(2)^2$

$= 2\sin^2\left(\frac{\pi}{4}\right) + 2 \times \frac{1}{2} + 2 \times 4$

$= 2\left(\frac{1}{\sqrt{2}}\right)^2 + 1 + 8$

$= 1 + 1 + 8$

$= 10$

=R.H.S

Hence LHS = RHS.

Overall Hint: Substitute the values of $\sin\frac{3\pi}{4}, \cos\frac{\pi}{4}, \sec\frac{\pi}{3}$ in the LHS . This will be equal to what's given in the RHS

5. Find the value of:

(i) $\sin 75^o$

(ii) $\tan 15^o$

Solution:

(i) Step1: $\sin 75^o = \sin(45^o + 30^o)$

$= \sin 45^o \cos 30^o + \cos 45^o \sin 30^o$ $[\because \sin(X+Y) = \sin X \cos Y + \cos X \sin Y]$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

Overall Hint: The value of sin 75 can be found using sin (60+15) with the help of formula $\sin(X+Y) = \sin X \cos Y + \cos X \sin Y$ by taking 60 and 15 as X and Y respectively

(ii) Step1:

$\tan 15^o = \tan(45^o - 30^o)$

$= \frac{\tan 45^o - \tan 30^o}{1 + \tan 45^o \tan 30^o}$ $\left[\because \tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}\right]$

$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1 \times \frac{1}{\sqrt{3}}}$

$= \frac{\sqrt{3}-1}{\sqrt{3}+1}$

$= \frac{(\sqrt{3}-1)^2}{(\sqrt{3}-1)\ (\sqrt{3}+1)} = \frac{3+1-2\sqrt{3}}{(\sqrt{3})^2 - 1^2} = \frac{4-2\sqrt{3}}{3-1}$

$= 2 - \sqrt{3}$

Overall Hint: The value of tan 15 can be found using tan(45-30) with the help of formula $\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$ by taking 45 and 30 as x and y respectively

6. Prove the following : $\cos\left(\frac{\pi}{4} - x\right)\cos\left(\frac{\pi}{4} - y\right) - \sin\left(\frac{\pi}{4} - x\right)\sin\left(\frac{\pi}{4} - y\right) = \sin(x+y)$

Solution:

Step1:

L.H.S= $\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)-\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)$

$=\frac{1}{2}\left[2\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)\right]+\frac{1}{2}\left[-2\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)\right]$

$=\frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4}-x\right)+\left(\frac{\pi}{4}-y\right)\right\}\right]+\frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4}-x\right)-\left(\frac{\pi}{4}-y\right)\right\}\right]$
$+\frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4}-x\right)+\left(\frac{\pi}{4}-y\right)\right\}\right]-\frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4}-x\right)-\left(\frac{\pi}{4}-y\right)\right\}\right]$

[$\because 2\cos A\cos B=\cos(A+B)+\cos(A-B)$ and $-2\sin A\sin B=\cos(A+B)-\cos(A-B)$]

Step2:

$=\frac{1}{2}\times 2\cos\left(\frac{\pi}{2}-(x+y)\right)$

$=\cos\left[\frac{\pi}{2}-(x+y)\right]$

$=\sin(x+y)$

=R.H.S

Hence RHS = LHS

Overall Hint: By multiplying the LHS throughout with 2 and dividing it with ½ and using the identity $2\cos A\cos B=\cos(A+B)+\cos(A-B)$ and $-2\sin A\sin B=\cos(A+B)-\cos(A-B)$ We can prove the LHS and RHS as equal

7. Prove that: $\frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}=\left(\frac{1+\tan x}{1-\tan x}\right)^2$

Solution:

Step1:

We know that,

$\tan(x+y)=\frac{\tan x+\tan y}{1-\tan x.\tan y}$ and $\tan(x-y)=\frac{\tan x-\tan y}{1+\tan x.\tan y}$

Given, L.H.S= $\frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}$

$$=\frac{\frac{\tan\frac{\pi}{4}+\tan x}{1-\tan\frac{\pi}{4}\tan x}}{\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}}$$

$$=\frac{\frac{1+\tan x}{1-\tan x}}{\frac{1-\tan x}{1+\tan x}}$$

$$=\left(\frac{1+\tan x}{1-\tan x}\right)^2$$

=R.H.S

Hence, LHS = RHS

Overall Hint:Use the identities $\tan(x+y)=\frac{\tan x+\tan y}{1-\tan x.\tan y}$ and$\tan(x-y)=\frac{\tan x-\tan y}{1+\tan x.\tan y}$ In the numerator and denominator of The LHS to prove the RHS

8. Prove that $\frac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\frac{\pi}{2}+x\right)}=\cot^2 x$

Solution:

Step1:

Given, L.H.S= $\frac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\frac{\pi}{2}+x\right)}$

$$=\frac{(-\cos x)(\cos x)}{(\sin x)(-\sin x)}$$

$$=\frac{\cos^2 x}{\sin^2 x}$$

$$=\cot^2 x$$

=R.H.S

Hence LHS = RHS

Overall Hint: Cos(π+x) = - cos x and cos (-x) = cos x and sin$(\pi-x)$ = sinx, cos$(\frac{\pi}{2}+x)$ – sin x in the numerator and denominator of the LHS to prove the RHS

9. $\cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi+x)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi+x)\right]=1$

Solution:

Step1:

Given, L.H.S.= $\cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi+x)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi+x)\right]$

$=\sin x\cos x\,[\tan x+\cot x]$

$=\sin x\cos x\left[\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}\right]$

$=\sin x\cos x\left[\frac{\sin^2 x+\cos^2 x}{\cos x\sin x}\right]$

$=\sin^2 x+\cos^2 x$

$=1$

=R.H.S.

Hence LHS = RHS.

Overall Hint: Use Cos($\frac{2n+1\pi}{2}+x$) = sin x , cos($2n\pi+x$) = cos x and $\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi+x)\right]$ = $\tan x+\cot x$ in the LHS to prove the RHS

10. Prove that $\sin(\mathrm{n}+1)\,x\sin(\mathrm{n}+2)\,x+\cos(\mathrm{n}+1)\,x\cos(\mathrm{n}+2)\,x=\cos x$

Solution:

Step1:

Given, L.H.S.= $\sin(n+1)\,x\sin(n+2)x+\cos(n+1)\,x\cos(n+2)x$

$=\frac{1}{2}(2\sin(n+1)\,x\sin(n+2)x+2\cos(n+1)\,x\cos(n+2)x)$

$=\frac{1}{2}[\cos\{(n+1)x-(n+2)x\}-\cos\{(n+1)x+(n+2)x\}]$

$+\frac{1}{2}[\cos\{(n+1)x+(n+2)x\}+\cos\{(n+1)x-(n+2)x\}]$

$=\frac{1}{2}\times 2\cos\{(n+1)x-(n+2)x\}$

$=\cos(-x)$

$= \cos x$

=R.H.S

Hence LHS = RHS

Overall Hint: Multiply the LHS of the equation throughout by 2 and divide with ½ then use the formula cos(A − B) = cos A cos B + sin A sin B to prove the RHS

11. Prove that $\cos\left(\frac{3\pi}{4}+x\right)-\cos\left(\frac{3\pi}{4}-x\right)=-\sqrt{2}\sin x$

Solution:

Step1:

L.H.S= $\cos\left(\frac{3\pi}{4}+x\right)-\cos\left(\frac{3\pi}{4}-x\right)$

We know that, $\cos A-\cos B=-2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

So, L.H.S.= $-2\sin\left\{\frac{\left(\frac{3\pi}{4}+x\right)+\left(\frac{3\pi}{4}-x\right)}{2}\right\}\sin\left\{\frac{\left(\frac{3\pi}{4}+x\right)-\left(\frac{3\pi}{4}-x\right)}{2}\right\}$

$= -2\sin\left(\frac{3\pi}{4}\right)\sin x$

$= -2\sin\left(\pi-\frac{\pi}{4}\right)\sin x$

$= -2\sin\left(\frac{\pi}{4}\right)\sin x$

$= -2\times\frac{1}{\sqrt{2}}\sin x$

$= -\sqrt{2}\sin x$

=R.H.S

Hence Proved

Overall Hint: Use the formula $\cos A-\cos B=-2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$ in the LHS to prove the RHS. Also use $\sin\left(\pi-\frac{\pi}{4}\right)=\sin\left(\frac{\pi}{4}\right)$

12. Prove that $\sin^2 6x - \sin^2 4x = \sin 2x \sin 10x$

Solution:

Step1:

Given, L.H.S.= $\sin^2 6x - \sin^2 4x$

$= (\sin 6x - \sin 4x)(\sin 6x + \sin 4x)$

$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$and $\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

L.H.S.= $\left[2\cos\left(\frac{6x+4x}{2}\right)\sin\left(\frac{6x-4x}{2}\right)\right]\left[2\sin\left(\frac{6x+4x}{2}\right)\cos\left(\frac{6x-4x}{2}\right)\right]$

$= [2\cos 5x \sin x][2\sin 5x \cos x]$

$= [2\sin 5x \cos 5x][2\sin x \cos x]$

$= \sin 10x \sin 2x$

=R.H.S.

Hence RHS = LHS

Overall Hint: Write the LHS using a^2 – b^2 = (a+b)*(a-b) then make use of the formulae $\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$and $\sin A - \sin B =$ $2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$
to prove the RHS

13. Prove that $\cos^2 2x - \cos^2 6x = \sin 4x \sin 8x$

Solution:

Step1:

L.H.S.= $\cos^2 2x - \cos^2 6x$

$= (\cos 2x + \cos 6x)(\cos 2x - \cos 6x)$

We know that,

$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)and\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

So,L.H.S.= $\left[-2\sin\left(\frac{2x+6x}{2}\right)\sin\left(\frac{2x-6x}{2}\right)\right]\left[2\cos\left(\frac{2x+6x}{2}\right)\cos\left(\frac{2x-6x}{2}\right)\right]$

$= [-2\sin(4x)\sin(-2x)][2\cos(4x)\cos(-2x)]$

$= [2\sin 4x \sin 2x][2\cos 4x \cos 2x]$

$= [2\sin 4x \cos 4x][2\sin 2x \cos 2x]$

$= \sin 8x \sin 4x$

=R.H.S.

Hence LHS = RHS.

Overall Hint: Write the LHS in the form of a^2 – b^2 = (a+b)*(a-b) and make use of the formulae $\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$ and $\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$
To prove the RHS

14. Prove that $\sin 2x + 2\sin 4x + \sin 6x = 4\cos^2 x \sin 4x$

Solution:

Step1:

Given, L.H.S.= $\sin 2x + 2\sin 4x + \sin 6x$

$= (\sin 2x + \sin 6x) + 2\sin 4x$

We know that $\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

So,L.H.S.= $2\sin\left(\frac{2x+6x}{2}\right)\cos\left(\frac{2x-6x}{2}\right) + 2\sin 4x$

Step2:

$= 2\sin 4x \cos(-2x) + 2\sin 4x$

$= 2\sin 4x\,[\cos 2x + 1]$

$= 2\sin 4x\,[2\cos^2 x - 1 + 1]$

$= 2\sin 4x(2\cos^2 x)$

$= 4\cos^2 x.\sin 4x$

=R.H.S

Hence LHS = RHS.

Overall Hint: Write $\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$ to prove the RHS

15. Prove that $\cot 4x\,(\sin 5x + \sin 3x) = \cot x\,(\sin 5x - \sin 3x)$

Solution:

Step1:

Given, L.H.S.= $\cot 4x\,(\sin 5x + \sin 3x)$

We know that $\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

So,L.H.S.= $\frac{\cos 4x}{\sin 4x}\left[2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)\right]$

$= \frac{\cos 4x}{\sin 4x}(2\sin 4x\cos x)$

$= 2\cos 4x\cos x$

R.H.S.= $\cot x(\sin 5x - \sin 3x)$

Step2:

We know that $\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

So,R.H.S.= $\frac{\cos x}{\sin x}\left[2\cos\left(\frac{5x+3x}{2}\right)\sin\left(\frac{5x-3x}{2}\right)\right]$

$= \frac{\cos x}{\sin x}[2\cos 4x\sin x]$

$= 2\cos 4x\cos x$

L.H.S.=R.H.S.

Hence Proved

Overall Hint: Write $\cot 4x\,(\sin 5x + \sin 3x)$ as $\frac{\cos 4x}{\sin 4x}\left[2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)\right]$ using formula $\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$ in the LHS and $\cot x\,(\sin 5x - \sin 3x)$ as $\frac{\cos x}{\sin x}\left[2\cos\left(\frac{5x+3x}{2}\right)\sin\left(\frac{5x-3x}{2}\right)\right]$ using the formula $\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

In the RHS

16. Prove that $\frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} = -\frac{\sin 2x}{\cos 10x}$

Solution:

Step1:

Given, L.H.S= $\frac{\cos 9x-\cos 5x}{\sin 17x-\sin 3x}$

We know that,

$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$ and $\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

L.H.S = $\frac{-2\sin\left(\frac{9x+5x}{2}\right)\sin\left(\frac{9x-5x}{2}\right)}{2\cos\left(\frac{17x+3x}{2}\right)\sin\left(\frac{17x-3x}{2}\right)}$

$= -\frac{2\sin 7x.\sin 2x}{2\cos 10x.\sin 7x}$

$= -\frac{\sin 2x}{\cos 10x}$

=R.H.S.

Hence LHS = RHS.

Overall Hint: Change the numerator and denominator in the LHS as $\frac{\cos 9x-\cos 5x}{\sin 17x-\sin 3x} = \frac{-2\sin\left(\frac{9x+5x}{2}\right)\sin\left(\frac{9x-5x}{2}\right)}{2\cos\left(\frac{17x+3x}{2}\right)\sin\left(\frac{17x-3x}{2}\right)}$ with the help of formula $\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$ and $\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$
For the numerator and denominator respectively then simplify it to show that its equal to RHS

17. Prove that : $\frac{\sin 5x+\sin 3x}{\cos 5x+\cos 3x} = \tan 4x$

Solution:

Step1:

L.H.S= $\frac{\sin 5x+\sin 3x}{\cos 5x+\cos 3x}$

We know that,

$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$ and $\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

Given, L.H.S = $\frac{2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)}{2\cos\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)}$

$= \frac{2\sin(4x)\cos(x)}{2\cos(4x)\cos(x)}$

$= \frac{\sin 4x}{\cos 4x}$

$= \tan 4x$

=R.H.S

Hence LHS = RHS

Overall Hint: Change the numerator and denominator of the LHS ($\frac{\sin 5x+\sin 3x}{\cos 5x+\cos 3x}$) as $\frac{2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)}{2\cos\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)}$ with the help of the formulae $\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$ and $\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$ then simplify it to show that its equal to RHS

18. Prove that $\frac{\sin x-\sin y}{\cos x+\cos y} = \tan\frac{x-y}{2}$

Solution:

Step1:

Given, L.H.S= $\frac{\sin x-\sin y}{\cos x+\cos y}$

We know that $\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$ and $\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

L.H.S $= \frac{2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)}$

$= \frac{\sin\left(\frac{x-y}{2}\right)}{\cos\left(\frac{x-y}{2}\right)}$

$= \tan\left(\frac{x-y}{2}\right)$

=R.H.S.

Hence LHS = RHS

Overall Hint: Change the numerator and denominator of LHS ($\frac{\sin x-\sin y}{\cos x+\cos y}$) to $\frac{2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)}$ using the formulae $\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$ and $\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$ and then simplify it to show that its equal to RHS

19. Prove that $\frac{\sin x+\sin 3x}{\cos x+\cos 3x}=\tan 2x$

Solution:

Step 1:

Given, L.H.S.$=\frac{\sin x+\sin 3x}{\cos x+\cos 3x}$

$=\frac{2\sin\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}{2\cos\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}$ $\left[\because \sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$

$=\frac{\sin 2x}{\cos 2x}$ $\left[\because \cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$

$=\tan 2x$

=R.H.S.

Hence LHS = RHS.

Overall Hint: Change the numerator and denominator of the LHS ($\frac{\sin x+\sin 3x}{\cos x+\cos 3x}$) to $\frac{2\sin\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}{2\cos\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}$ using the formulae $\left[\sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$ and $\left[\cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$ to prove the LHS equal to RHS

20. Prove that $\frac{\sin x-\sin 3x}{\sin^2 x-\cos^2 x}=2\sin x$

Solution:

Step1:

Given, L.H.S.$=\frac{\sin x-\sin 3x}{\sin^2 x-\cos^2 x}$

$=\frac{2\cos\left(\frac{x+3x}{2}\right)\sin\left(\frac{x-3x}{2}\right)}{-\cos 2x}$ $\left[\because \sin A-\sin B=\ 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$

$=\frac{2\cos 2x\sin(-x)}{-\cos 2x}$ $[\because \cos^2 x-\sin^2 x=\cos 2x]$

$=\frac{-2\cos 2x\ \sin x}{-\cos 2x}$

$=2\sin x$

=R.H.S.

Hence LHS = RHS.

Overall Hint: Change the numerator and denominator of LHS ($\frac{\sin x-\sin 3x}{\sin^2 x-\cos^2 x}$) to $\frac{2\cos\left(\frac{x+3x}{2}\right)\sin\left(\frac{x-3x}{2}\right)}{-\cos 2x}$ using the formulae $\left[\sin A-\sin B= 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$ and $[\cos^2 x-\sin^2 x=\cos 2x]$ *and simplify it* to prove the LHS equal to RHS

21. Prove that $\frac{\cos4x+\cos3x+\cos2x}{\sin4x+\sin3x+\sin2x}=\cot 3x$

Solution:

Step1:

Given, L.H.S.$=\frac{\cos4x+\cos3x+\cos2x}{\sin4x+\sin3x+\sin2x}$

$$=\frac{(\cos4x+\cos2x)+\cos3x}{(\sin4x+\sin2x)+\sin3x}$$

We know that,

$\sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$ and $\cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$$=\frac{2\cos\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right)+\cos3x}{2\sin\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right)+\sin3x}$$

Step2:

$$=\frac{2\cos 3x\cos x+\cos 3x}{2\sin 3x\cos x+\sin 3x}$$

$$=\frac{\cos 3x\,(\cos x+1)}{\sin 3x\,(\cos x+1)}$$

$$=\frac{\cos 3x}{\sin 3x}$$

$=\cot 3x$

=R.H.S.

Hence LHS = RHS

Overall Hint: Change the numerator and denominator of LHS($\frac{\cos4x+\cos3x+\cos2x}{\sin4x+\sin3x+\sin2x}$) to $\frac{2\cos\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right)+\cos3x}{2\sin\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right)+\sin3x}$ using the formulae $\sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$ and $\cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$ and simplify it to prove the RHS

22. Prove that $\cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x = 1$

Solution:

Step1:

L.H.S.= $\cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x$

$= \cot x \cot 2x - \cot 3x\,(\cot 2x + \cot x)$

$= \cot x \cot 2x - \cot(2x + x)(\cot 2x + \cot x)$

$= \cot x \cot 2x - \left[\frac{\cot 2x \cot x - 1}{\cot 2x + \cot x}\right](\cot 2x + \cot x)$ $\left[\because \cot(A + B) = \left[\frac{\cot A \cot B - 1}{\cot A + \cot B}\right]\right.$

$= \cot x \cot 2x - \cot 2x \cot x + 1$

$= 1$

=R.H.S.

Hence LHS = RHS.

Overall Hint: take cot3x in the 2nd and 3rd term and Use the formula $\cot(A + B) = \left[\frac{\cot A \cot B - 1}{\cot A + \cot B}\right]$ to show that LHS is equal to RHS

23. Prove that $\tan 4x = \frac{4\tan x(1-\tan^2 x)}{1-6\tan^2 x+\tan^4 x}$

Solution:

Step1:

We know that, $\tan 2A = \frac{2\tan A}{1-\tan^2 A}$

L.H.S.= $\tan 4x = \tan 2(2x)$

$$= \frac{2\tan 2x}{1-\tan^2(2x)}$$

$$= \frac{2\left(\frac{2\tan x}{1-\tan^2 x}\right)}{1-\left(\frac{2\tan x}{1-\tan^2 x}\right)^2}$$

Step2:

$=\dfrac{\frac{4\tan x}{1-\tan^2 x}}{\frac{(1-\tan^2 x)^2-4\tan^2 x}{(1-\tan^2 x)^2}}$

$=\dfrac{4\tan x(1-\tan^2 x)}{(1-\tan^2 x)^2-4\tan^2 x}$

$=\dfrac{4\tan x(1-\tan^2 x)}{1+\tan^4 x-2\tan^2 x-4\tan^2 x}$

$=\dfrac{4\tan x(1-\tan^2 x)}{1-6\tan^2 x+\tan^4 x}$

=R.H.S.

Hence LHS = RHS

Overall Hint: Change the LHS tan4x as $\dfrac{2\left(\frac{2\tan x}{1-\tan^2 x}\right)}{1-\left(\frac{2\tan x}{1-\tan^2 x}\right)^2}$ using the formula tan 2A = $\dfrac{2\tan A}{1-\tan^2 A}$ and then expand it to prove its equal to RHS

24. Prove thatcos $4x = 1 - 8\sin^2 x\cos^2 x$

Solution:

Step1:

Given, L.H.S= $\cos 4x = \cos 2(2x)$

$= 1 - 2\sin^2(2x)$ $[\because \cos 2A = 1 - 2\sin^2 A]$

$= 1 - 2(2\sin x\cos x)^2]$ $[\because \sin 2A = 2\sin A\cos A$

$= 1 - 8\sin^2 x\cos^2 x$

= R.H.S.

Hence LHS = RHS

Overall Hint: Write cos(4x)= cos2(2x) and then apply formulae $\cos 2A = 1 - 2\sin^2 A$

$\sin 2A = 2\sin A\cos A$ to show it equal to RHS

25. Prove that $\cos 6x = 32\cos^6 x - 48\cos^4 x + 18\cos^2$

SteP1:

Given, L.H.S. $= \cos 6x$

$= \cos 3(2x)$

$= 4cos^3 2x - 3\cos 2x$

$[\because \cos 3A = 4\cos^3 A - 3\cos A]$

$= 4[(2\cos^2 x - 1)^3 - 3(2\cos^2 x - 1)]$

$[\because cos\ 2x = 2\cos^2 x - 1]$

Step2:

$= 4[(2\cos^2 x)^3 - (1)^3 - 3(2\cos^2 x)^2 + 3(2\cos^2 x)] - 6\cos^2 x + 3$

$= 4[8\cos^6 x - 1 - 12\cos^4 x + 6\cos^2 x] - 6\cos^2 x + 3$

$= 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$

=R.H.S.

Hence LHS = RHS.

Overall Hint: Change the terms in LHS by using formulae $\cos 3A = 4\cos^3 A - 3\cos A$

$cos\ 2x = 2\cos^2 x - 1$ and simplify it to prove that its equal to RHS

Exercise: 3.4

Find the principal and general solutions of the following equations:

1. $\tan x = \sqrt{3}$

Solution:

Step1:

We know that $\tan\frac{\pi}{3} = \sqrt{3}$and $\tan\frac{4\pi}{3} = \tan\left(\pi + \frac{\pi}{3}\right) = \tan\frac{\pi}{3} = \sqrt{3}$

Thus ,the principal solutions are $x = \frac{\pi}{3}$and$x = \frac{4\pi}{3}$

And,tan$x = \tan\frac{\pi}{3}$

$\Rightarrow x = n\pi + \frac{\pi}{3}$,where $n \in Z$

Thus, the general solution is= $n\pi + \frac{\pi}{3}$, where $n \in Z$

Overall Hint : The principal solution is π and $\frac{4\pi}{3}$ and the general solution is $n\pi + \frac{\pi}{3}, n \in Z$

2. Find the principal and general solutions of the following equations:

$\sec x = 2$

Solution:

Step1:

Given, $\sec x = 2$

We know that $\sec\frac{\pi}{3} = 2$ and $\sec\frac{5\pi}{3} = \sec\left(2\pi - \frac{\pi}{3}\right) = \sec\frac{\pi}{3} = 2$

Thus, the principal solutions are $x = \frac{\pi}{3}$ and $x = \frac{5\pi}{3}$

Step2:

Now, $\sec x = \sec\frac{\pi}{3}$

$\Rightarrow \cos x = \cos\frac{\pi}{3}$ $\left[\because \sec x = \frac{1}{\cos x}\right]$

$\Rightarrow x = 2n\pi \pm \frac{\pi}{3}$, where $n \in Z$

Thus, the general solution is $= 2n\pi \pm \frac{\pi}{3}$, where $n \in Z$

Overall Hint: The principal solutions are $x = \frac{\pi}{3}$ and $x = \frac{5\pi}{3}$ and the general solutions are $2n\pi \pm \frac{\pi}{3}$, where $n \in Z$

3. Find the principal and general solutions of the following equations:

$\cot x = -\sqrt{3}$

Solution:

Step1:

We know that $\cot\frac{\pi}{6} = \sqrt{3}$

So, $\cot\left(\pi - \frac{\pi}{6}\right) = -\cot\frac{\pi}{6} = -\sqrt{3}$ and $\cot\left(2\pi - \frac{\pi}{6}\right) = -\cot\frac{\pi}{6} = -\sqrt{3}$

i.e., $\cot\left(\frac{5\pi}{6}\right) = -\sqrt{3}$ and $\cot\left(\frac{11\pi}{6}\right) = -\sqrt{3}$

Thus, the principal solutions are $x = \frac{5\pi}{6}$and$x = \frac{11\pi}{6}$

Step2:

Now,$\cot x = \cot\left(\frac{5\pi}{6}\right)$

$\Rightarrow \tan x = \tan\frac{5\pi}{6}\left[\tan x = \frac{1}{\cot x}\right]$

$\Rightarrow x = n\pi + \frac{5\pi}{6}$,where $n \in Z$

Thus, the general solution is= $n\pi + \frac{5\pi}{6}$,where $n \in Z$

Overall Hint: The principal solution is $x = \frac{5\pi}{6}$and$x = \frac{11\pi}{6}$ and the general solution is $n\pi + \frac{5\pi}{6}$,where $n \in Z$

4. Find the principal and general solution

$\operatorname{cosec} x = -2$

Solution:

Step1:

$\operatorname{cosec} x = -2$

We know that $\operatorname{cosec}\frac{\pi}{6} = 2$

Also, $\operatorname{cosec}\left(\pi + \frac{\pi}{6}\right) = -\operatorname{cosec}\frac{\pi}{6} = -2and\operatorname{cosec}\left(2\pi - \frac{\pi}{6}\right) = -\operatorname{cosec}\frac{\pi}{6} = -2$

i.e., $\operatorname{cosec}\left(\frac{7\pi}{6}\right) = -2and\operatorname{cosec}\left(\frac{11\pi}{6}\right) = -2$

Thus, the principal solutions are $x = \frac{7\pi}{6}$and$x = \frac{11\pi}{6}$

Step2:

Now,$\operatorname{cosec} x = \operatorname{cosec}\left(\frac{7\pi}{6}\right)$

$\Rightarrow \sin x = \sin\frac{7\pi}{6}\left[\sin x = \frac{1}{\operatorname{cosec} x}\right]$

$\Rightarrow x = n\pi + (-1)^n\frac{7\pi}{6}$,where$n \in Z$

Thus, the general solution is= $n\pi + (-1)^n\frac{7\pi}{6}$, where $n \in Z$

Overall Hint: The principal solution is $x = \frac{7\pi}{6}$ and $x = \frac{11\pi}{6}$ and the general solution is $n\pi + (-1)^n \frac{7\pi}{6}$, where $n \in Z$

5. Find the general solution for each of the following equations:

$\cos 4x = \cos 2x$

Solution:

Step1:

Given, $\cos 4x = \cos 2x$

$\Rightarrow \cos 4x - \cos 2x = 0$

$\cos 4x - \cos 2x = -2\sin\left(\frac{4x+2x}{2}\right)\sin\left(\frac{4x-2x}{2}\right)$

$\left[\because \cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$

Step2:

$\Rightarrow -2\sin 3x \sin x = 0$

$\Rightarrow \sin 3x = 0$ or $\sin x = 0$

$\Rightarrow 3x = n\pi$ or $x = n\pi$, where $n \in Z$

Thus, $x = \frac{n\pi}{3}$ or $x = n\pi$, where $n \in Z$

Overall Hint: The general solution is $x = \frac{n\pi}{3}$ or $x = n\pi$, where $n \in Z$

Use $\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

6. Find the general solution for each of the following equations:

$\cos 3x + \cos x - \cos 2x = 0$

Solution:

Step1:

Given, $\cos 3x + \cos x - \cos 2x = 0$

$\Rightarrow 2\cos\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right) - \cos 2x = 0$

$\left[\because \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$

Step2:

$\Rightarrow 2\cos 2x \cos x - \cos 2x = 0$

$\Rightarrow \cos 2x\,(2\cos x - 1) = 0$

$\Rightarrow \cos 2x = 0$ or $2\cos x - 1 = 0$

$\Rightarrow \cos 2x = 0$ or $\cos x = \frac{1}{2}$

Therefore, $2x = (2n+1)\frac{\pi}{2}$ or $\cos x = \cos\frac{\pi}{3}$, where $n \in Z$

Thus, $x = (2n+1)\frac{\pi}{4}$ or $x = 2n\pi \pm \frac{\pi}{3}$, where $n \in Z$
Overall Hint: The general solution is $x = (2n+1)\frac{\pi}{4}$ or $x = 2n\pi \pm \frac{\pi}{3}$, where $n \in Z$

Use $\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

7. Find the general solution for each of the following equations:

$\sin 2x + \cos x = 0$

Solution:

Step1:

Given, $\sin 2x + \cos x = 0$

$\Rightarrow 2\sin x \cos x + \cos x = 0$

$[\because \sin Ax = 2\sin A \cos A]$

$\Rightarrow \cos x\,(2\sin x + 1) = 0$

$\Rightarrow \cos x = 0$ or $2\sin x + 1 = 0$

Step2:

$\Rightarrow \cos x = 0 \Rightarrow x = (2n+1)\frac{\pi}{2}$, where $n \in Z$

Also, $2\sin x + 1 = 0$

$\Rightarrow \sin x = -\frac{1}{2}$

$\Rightarrow \sin x = -\sin\frac{\pi}{6} \Rightarrow \sin x = \sin\left(\pi + \frac{\pi}{6}\right) = \sin\frac{7\pi}{6}$

$\Rightarrow x = n\pi + (-1)^n \frac{7\pi}{6}$,where $n \in Z$

Thus, the general solution is $x = (2n+1)\frac{\pi}{2}$or$x = n\pi + (-1)^n \frac{7\pi}{6}$,where $n \in Z$
Overall Hint: The general solution is $x = (2n+1)\frac{\pi}{2}$or$x = n\pi + (-1)^n \frac{7\pi}{6}$,where $n \in Z$,Use $\sin Ax = 2\sin A \cos A$

8. Find the general solution for each of the following equations

$\sec^2 2x = 1 - \tan 2x$

Solution:

Step1:

Given, $\sec^2 2x = 1 - \tan 2x$

$1 + \tan^2 2x = 1 - \tan 2x$

$\Rightarrow \tan^2 2x = -\tan 2x$

$\Rightarrow \tan^2 2x + \tan 2x = 0$

$\Rightarrow \tan 2x\,(\tan 2x + 1) = 0$

$\Rightarrow \tan 2x = 0$ Or $\tan 2x + 1 = 0$

Now, $\tan 2x = \tan 0$

$\Rightarrow 2x = n\pi + 0$, Where $n \in Z$

Step2:

$\Rightarrow x = \frac{n\pi}{2}$, where $n \in Z$

$\tan 2x + 1 = 0$

$\Rightarrow \tan 2x = -1$

$\Rightarrow \tan 2x = -\tan\frac{\pi}{4} \Rightarrow \tan 2x = \tan\left(\pi - \frac{\pi}{4}\right) = \tan\frac{3\pi}{4}$

So,$2x = n\pi + \frac{3\pi}{4}$, where $n \in Z$

$\Rightarrow x = \frac{n\pi}{2} + \frac{3\pi}{8}$, where $n \in Z$

Thus, the general solution is $x = \frac{n\pi}{2}$ or $x = \frac{n\pi}{2} + \frac{3\pi}{8}$, where $n \in Z$

Overall Hint: The general solution is $x = \frac{n\pi}{2}$ or $x = \frac{n\pi}{2} + \frac{3\pi}{8}$, where $n \in Z$, Use $\sec^2 2x$ =

$1 + \tan^2 2x$. Take tan2x common.

9. Find the general solution for each of the following equation

$\sin x + \sin 3x + \sin 5x = 0$

Solution:

Step1:

Given, $\sin x + \sin 3x + \sin 5x = 0$

$(\sin x + \sin 5x) + \sin 3x = 0$

$\Rightarrow 2\sin\left(\frac{x+5x}{2}\right)\cos\left(\frac{x-5x}{2}\right) + \sin 3x = 0$

$\left[\because \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$

$\Rightarrow 2\sin 3x\cos(-2x) + \sin 3x = 0$

$\Rightarrow \sin 3x\,(2\cos 2x + 1) = 0$

$\Rightarrow \sin 3x = 0$ or $2\cos 2x + 1 = 0$

$\Rightarrow \sin 3x = 0$ or $\cos 2x = -\frac{1}{2}$

$\sin 3x = 0$

Step2:

$3x = n\pi$, where $n \in Z$

$\Rightarrow x = \frac{n\pi}{3}$, where $n \in Z$

$\cos 2x = -\frac{1}{2}$

$\cos 2x = -\cos\frac{\pi}{3} \Rightarrow \cos 2x = \cos\left(\pi - \frac{\pi}{3}\right) = \cos\frac{2\pi}{3}$

So, $2x = 2n\pi \pm \frac{2\pi}{3}$, where $n \in Z$

$\Rightarrow x = n\pi \pm \frac{\pi}{3}$, where $n \in Z$

Thus, the general solution is $x = \frac{n\pi}{3}$ or $x = n\pi \pm \frac{\pi}{3}$, where $n \in Z$

Overall Hint: The general solution is $x = \frac{n\pi}{3}$ or $x = n\pi \pm \frac{\pi}{3}$, where $n \in Z$, Use $\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

Miscellaneous Exercise

1. Prove that: $2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13} = 0$

Solution:

Step1:

Given, L.H.S.= $2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13}$

$2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + 2\cos\left(\frac{\frac{3\pi}{13}+\frac{5\pi}{13}}{2}\right)\cos\left(\frac{\frac{3\pi}{13}-\frac{5\pi}{13}}{2}\right)$

$[\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos(\frac{x-y}{2})]$

$\Rightarrow 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + 2\cos\left(\frac{4\pi}{13}\right)\cos\left(-\frac{\pi}{13}\right)$

Step2:

$\Rightarrow 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + 2\cos\left(\frac{4\pi}{13}\right)\cos\left(\frac{\pi}{13}\right)$

$\Rightarrow 2\cos\frac{\pi}{13}\left(\cos\frac{9\pi}{13} + \cos\left(\frac{4\pi}{13}\right)\right)$

$= 2\cos\frac{\pi}{13}\left(2\cos\left(\frac{\frac{9\pi}{13}+\frac{4\pi}{13}}{2}\right)\cos\left(\frac{\frac{9\pi}{13}-\frac{4\pi}{13}}{2}\right)\right)$

$= 4\cos\frac{\pi}{13}\left(\cos\left(\frac{\pi}{2}\right)\cos\left(\frac{5\pi}{26}\right)\right)$

$= 4\cos\frac{\pi}{13}\left(0 \times \cos\left(\frac{5\pi}{26}\right)\right)$

$= 0$

=R.H.S.

Hence LHS = RHS.

Overall Hint: Use $\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos(\frac{x-y}{2})$ in the :LHS to prove it is equal to RHS

2. Prove that:$(\sin 3x + \sin x)\sin x + \left(\cos 3x - \cos x\right)\cos x = 0$

Solution:

Step1:

Given, L.H.S.= $(\sin 3x + \sin x)\sin x + (\cos 3x - \cos x)\cos x$

$= \sin 3x \sin x + \sin^2 x + \cos 3x \cos x - \cos^2 x$

$= (\sin 3x \sin x + \cos 3x \cos x) + (\sin^2 x - \cos^2 x)$

$= \cos(3x - x) - \cos 2x)$ $[\because \cos(A - B) = \cos A \cos B + \sin A \sin B \ \&\cos 2A = \cos^2 A - \sin^2 A]$

$= \cos 2x - \cos 2x$

$= 0$

=R.H.S.

Hence LHS = RHS..

Overall Hint: Use $\cos(A - B) = \cos A \cos B + \sin A \sin B \ \&\cos 2A = \cos^2 A - \sin^2 A$ to prove LHS is equal to RHS

3. Prove that:$(\cos x + \cos y)^2 + (\sin x - \sin y)^2 = 4\cos^2\frac{x+y}{2}$

Solution:

Step1:

Given, L.H.S.= $(\cos x + \cos y)^2 + (\sin x - \sin y)^2$

$= \cos^2 x + \cos^2 y + 2\cos x \cos y + \sin^2 x + \sin^2 y - 2\sin x \sin y$

$= (\sin^2 x + \cos^2 x) + (\sin^2 y + \cos^2 y) + 2(\cos x \cos y - \sin x \sin y)$

$= (1) + (1) + 2\cos(x+y)$ $[\because \cos(A+B) = \cos A \cos B - \sin A \sin B]$

Step2:

$= 2[1 + \cos(x+y)]$

$= 2\left[2\cos^2\left(\frac{x+y}{2}\right)\right]$ $[\because 1 + \cos 2A = 2\cos^2 A]$

$= 4\cos^2\left(\frac{x+y}{2}\right)$

=R.H.S.

Hence LHS = RHS.

Overall Hint: Use $\cos(A+B) = \cos A \cos B - \sin A \sin B$, $1 + \cos 2A = 2\cos^2 A$, (a+b)^2 = (a^2+b^2+2ab) , (a-b)^2 = (a^2+b^2-2ab) to prove LHS is equal to RHS

4. Prove that: $(\cos x - \cos y)^2 + (\sin x - \sin y)^2 = 4\sin^2\frac{x-y}{2}$

Solution:

Step1:

Given, L.H.S.= $(\cos x - \cos y)^2 + (\sin x - \sin y)^2$

$= \cos^2 x + \cos^2 y - 2\cos x \cos y + \sin^2 x + \sin^2 y - 2\sin x \sin y$

$= (\sin^2 x + \cos^2 x) + (\sin^2 y + \cos^2 y) - 2(\cos x \cos y + \sin x \sin y)$

$= (1) + (1) - 2\cos(x-y)$

Step2:

$[\because \cos(A-B) = \cos A \cos B + \sin A \sin B]$

$= 2[1 - \cos(x-y)]$

$= 2\left[1 - \left(1 - 2\sin^2\left(\frac{x-y}{2}\right)\right)\right]$ $[\because 1 - \cos 2A = 2\sin^2 A]$

$= 4\sin^2\left(\frac{x-y}{2}\right)$

=R.H.S.

Hence LHS = RHS.

Overall Hint: Use (a-b)^2 = (a^2+b^2-2ab) , $1 - \cos 2A = 2\sin^2 A$,

$\cos(A-B) = \cos A \cos B + \sin A \sin B$ to prove LHS equal to RHS

5. Prove that :$\sin x + \sin 3x + \sin 5x + \sin 7x = 4\cos x \cos 2x \sin 4x$

Solution:

Step1:

Given, L.H.S.= $\sin x + \sin 3x + \sin 5x + \sin 7x$

$\Rightarrow (\sin x + \sin 5x) + (\sin 3x + \sin 7x)$

$\Rightarrow 2\sin\left(\frac{x+5x}{2}\right)\cos\left(\frac{x-5x}{2}\right) + 2\sin\left(\frac{3x+7x}{2}\right)\cos\left(\frac{3x-7x}{2}\right)$

$\left[\because \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$

$\Rightarrow 2\sin 3x \cos(-2x) + 2\sin 5x \cos(-2x)$

Step2:

$= 2\sin 3x \cos 2x + 2\sin 5x \cos 2x$

$= 2\cos 2x\,(\sin 3x + \sin 5x)$

$= 2\cos 2x\left[2\sin\left(\frac{3x+5x}{2}\right)\cos\left(\frac{3\,x-5x}{2}\right)\right]$

$= 4\cos 2x \sin 4x \cos(-x)$

$= 4\cos 2x \sin 4x \cos x$

=R.H.S.

Hence RHS = LHS

Overall Hint: Use $\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$ formula to show that the LHS is equal to RHS

6. Prove that $\frac{(\sin 7x+\sin 5x)+(\sin 9x+\sin 3x)}{(\cos 7x+\cos 5x)+(\cos 9x+\cos 3x)} = \tan 6x$

Solution:

Given, L.H.S.= $\frac{(\sin 7x+\sin 5x)+(\sin 9x+\sin 3x)}{(\cos 7x+\cos 5x)+(\cos 9x+\cos 3x)}$

We know that,

$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$ & $\cos A + \cos B$
$= 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

L.H.S. $= \frac{\left\{2\sin\left(\frac{7x+5x}{2}\right)\cos\left(\frac{7x-5x}{2}\right)\right\}+\left\{2\sin\left(\frac{9x+3x}{2}\right)\cos\left(\frac{9x-3x}{2}\right)\right\}}{\left\{2\cos\left(\frac{7x+5x}{2}\right)\cos\left(\frac{7x-5x}{2}\right)\right\}+\left\{2\cos\left(\frac{9x+3x}{2}\right)\cos\left(\frac{9x-3x}{2}\right)\right\}}$

Step2:

$= \frac{2\sin 6x \cos x + 2\sin 6x \cos 3x}{2\cos x \cos x + 2\cos 6x \cos 3x}$

$= \frac{2\sin 6x\,(\cos x + \cos 3x)}{2\cos 6x\,(\cos x + \cos 3x)}$

$= \frac{\sin 6x}{\cos 6x}$

$= \tan 6x$

=R.H.S.

Hence LHS = RHS.

Overall Hint: Change the LHS($\frac{(\sin 7x+\sin 5x)+(\sin 9x+\sin 3x)}{(\cos 7x+\cos 5x)+(\cos 9x+\cos 3x)}$ to

$\frac{\left\{2\sin\left(\frac{7x+5x}{2}\right)\cos\left(\frac{7x-5x}{2}\right)\right\}+\left\{2\sin\left(\frac{9x+3x}{2}\right)\cos\left(\frac{9x-3x}{2}\right)\right\}}{\left\{2\cos\left(\frac{7x+5x}{2}\right)\cos\left(\frac{7x-5x}{2}\right)\right\}+\left\{2\cos\left(\frac{9x+3x}{2}\right)\cos\left(\frac{9x-3x}{2}\right)\right\}}$ using formula

$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$ & $\cos A + \cos B$
$= 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

And show it is equal to RHS

7. Prove that: $\sin 3x + \sin 2x - \sin x = 4\sin x \cos\frac{x}{2}\cos\frac{3x}{2}$

Solution:

Step1:

Given, L.H.S.$\sin 3x + \sin 2x - \sin x = \sin 3x + \sin 2x - \sin x$

$= \sin 3x + 2\cos\left(\frac{2x+x}{2}\right)\sin\left(\frac{2x-x}{2}\right)$

$\left[\because \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$

$= \sin 3x + 2\cos\frac{3x}{2}\sin\frac{x}{2}$

$= 2\sin\frac{3x}{2}\cos\frac{3x}{2} + 2\cos\frac{3x}{2}\sin\frac{x}{2}$

$= 2\cos\frac{3x}{2}\left(\sin\frac{3x}{2} + \sin\frac{x}{2}\right)$

$= 2\cos\frac{3x}{2}\left(2\sin\left(\frac{\frac{3x}{2}+\frac{x}{2}}{2}\right)\cos\left(\frac{\frac{3x}{2}-\frac{x}{2}}{2}\right)\right)$

Step2:

$\left[\because \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$

$= 2\cos\frac{3x}{2}\left(2\sin x \cos\left(\frac{x}{2}\right)\right)$

$= 4\sin x \cos\frac{x}{2}\cos\frac{3x}{2}$

=R.H.S.

Hence LHS = RHS.

Overall Hint: Change LHS $\sin 3x + \sin 2x - \sin x$ to $\sin 3x + 2\cos\left(\frac{2x+x}{2}\right)\sin\left(\frac{2x-x}{2}\right)$ by using the formula

$\text{Sin}A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$ use $\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$ for similarly changing the RHS and showing they both are equal

8. Find $\sin\frac{x}{2}, \cos\frac{x}{2}$ and $\tan\frac{x}{2}$ in $\tan x = -\frac{4}{3}$, x in quadrant II

Solution:

Step1:

Here, x lies in second quadrant.

So, $\frac{\pi}{2} < x < \pi$

$\Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$

Therefore, $\sin\frac{x}{2}, \cos\frac{x}{2}, \tan\frac{x}{2}$liesinfirstquadrant so they all are positive.

Given,$\tan x = -\frac{4}{3}$

We know that ,$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$

So,$\tan x = \frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}}$

$$\Rightarrow -\frac{4}{3} = \frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}}$$

$$\Rightarrow -4\left(1-\tan^2\frac{x}{2}\right) = 6\tan\frac{x}{2}$$

Step2:

$$\Rightarrow 4\tan^2\frac{x}{2} - 6\tan\frac{x}{2} - 4 = 0$$

$$\Rightarrow 2\tan^2\frac{x}{2} - 3\tan\frac{x}{2} - 2 = 0$$

$$\Rightarrow 2\tan^2\frac{x}{2} - 4\tan\frac{x}{2} + \tan\frac{x}{2} - 2 = 0$$

$$\Rightarrow 2\tan\frac{x}{2}\left(\tan\frac{x}{2} - 2\right) + 1\left(\tan\frac{x}{2} - 2\right) = 0$$

$$\Rightarrow \left(\tan\frac{x}{2} - 2\right)\left(2\tan\frac{x}{2} + 1\right) = 0$$

$\tan\frac{x}{2} = 2$or$\tan\frac{x}{2} = -\frac{1}{2}$

$\tan\frac{x}{2} = 2$ [$\tan\frac{x}{2}$ Cannot be negative as $\frac{x}{2}$ is in first quadrant]

Step3:

$$\sec^2\frac{x}{2} = 1 + \tan^2\frac{x}{2}$$

$$\sec^2\frac{x}{2} = 1 + (2)^2$$

$$\Rightarrow \sec^2\frac{x}{2} = 5$$

$\Rightarrow \sec\frac{x}{2} = \sqrt{5}$ [$\sec\frac{x}{2}$ Cannot be negative as $\frac{x}{2}$ is in first quadrant]

$$\Rightarrow \cos\frac{x}{2} = \frac{1}{\sqrt{5}}$$

Therefore, $\tan\frac{x}{2} = 2$

$$\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = 2$$

$$\Rightarrow \sin\frac{x}{2} = 2\cos\frac{x}{2}$$

Therefore, $\sin\frac{x}{2} = \frac{2}{\sqrt{5}}$

Overall Hint: Find $\tan\frac{x}{2}$ using $\tan 2x = \frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}}$ then find similarly values of $\sin\frac{x}{2}, \cos\frac{x}{2}$ using $\sin\frac{x}{2} = \tan\frac{x}{2}\cos\frac{x}{2}$ and $\cos\frac{x}{2} = \sin\frac{x}{2} / \tan\frac{x}{2}$

9. Find $\sin\frac{x}{2}, \cos\frac{x}{2}$ and $tan\frac{x}{2}$ for $\cos x = -\frac{1}{3}, x$ in quadrant III

Solution:

Step1:

Here,$x$liesinthirdquadrant.

So,$\pi < x < \frac{3\pi}{2}$

$$\Rightarrow \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$$

It is known that $\sin\frac{x}{2}$is positive whereas, $\cos\frac{x}{2}$ and$\tan\frac{x}{2}$are negative, lies in third quadrant.

Given,$\cos x = -\frac{1}{3}$

We know that,$\cos 2A = 2\cos^2 A - 1$

So,$\cos x = 2\cos^2\frac{x}{2} - 1$

$$\Rightarrow -\frac{1}{3} = 2\cos^2\frac{x}{2} - 1$$

$$\Rightarrow \cos^2\frac{x}{2} = \frac{1+\cos x}{2} = \frac{1+\left(-\frac{1}{3}\right)}{2} = \frac{\left(\frac{2}{3}\right)}{2} = \frac{1}{3}$$

$$\Rightarrow \cos\frac{x}{2} = \pm\frac{1}{\sqrt{3}}$$

Step2:

Whereas, $\cos\frac{x}{2}$ is negative as $\frac{x}{2}$ lies in second quadrant.

So, $\cos\frac{x}{2} = -\frac{1}{\sqrt{3}}$

We know that, $\sin^2 A + \cos^2 A = 1$

$$\sin^2\frac{x}{2} + \left(\frac{1}{\sqrt{3}}\right)^2 = 1$$

$$\sin^2\frac{x}{2} = 1 - \frac{1}{3}$$

$$\Rightarrow \sin^2\frac{x}{2} = \frac{2}{3}$$

$$\sin\frac{x}{2} = \pm\sqrt{\frac{2}{3}}$$

Step3:

But $\sin\frac{x}{2}$ is positive as $\frac{x}{2}$ lies in second quadrant.

So, $\sin\frac{x}{2} = \sqrt{\frac{2}{3}}$

Also, $\tan\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}$

$$\tan\frac{x}{2} = \frac{\sqrt{\frac{2}{3}}}{-\frac{1}{\sqrt{3}}}$$

Therefore, $\tan\frac{x}{2} = -\sqrt{2}$

Overall Hint: Find

$\cos\frac{x}{2}$ *using*

cos 2A$= 2\cos^2 A - 1$ and then the values of

$\sin\frac{x}{2} = \sqrt{1-\cos^2\frac{x}{2}}$

$tan\frac{x}{2}$ using $\tan x = \frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}}$

10. Find $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$ for $\sin x = \frac{1}{4}$, x in quadrant II.

Solution:

Step1:

Here, x lies in second quadrant.

So, $\frac{\pi}{2} < x < \pi$

$$\Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$$

That means $\sin\frac{x}{2}, \cos\frac{x}{2}, \tan\frac{x}{2}$ lies in first quadrant.

Given, $\sin x = \frac{1}{4}$

We know that, $\sin^2 A + \cos^2 A = 1$

$\sin^2 x + \cos^2 x = 1$

$$= \cos^2 x = 1 - \left(\frac{1}{4}\right)^2$$

$$\Rightarrow \cos^2 x = 1 - \frac{1}{16} = \frac{15}{16}$$

$$\cos x = \pm\frac{\sqrt{15}}{4}$$

Step2:

But $\cos x$ is negative as x lies in second quadrant.

So, $\cos x = -\frac{\sqrt{15}}{4}$

$$\sin^2\frac{x}{2} = \frac{1 - \cos x}{2} \quad [\because \cos 2A = 1 - 2\sin^2 A]$$

$$\sin^2\frac{x}{2} = \frac{1 + \frac{\sqrt{15}}{4}}{2}$$

$$\sin^2\frac{x}{2} = \frac{4 + \sqrt{15}}{8}$$

$\sin\frac{x}{2} = \sqrt{\left(\frac{4+\sqrt{15}}{8}\right)}$ [$\sin\frac{x}{2}$ is positive as $\frac{x}{2}$ lies in first quadrant]

$\cos^2\frac{x}{2} = \frac{1+\cos x}{2}$ [$\because \cos 2A = 1 - 2\sin^2 A$]

$$\cos^2\frac{x}{2} = \frac{1-\frac{\sqrt{15}}{4}}{2}$$

$$\cos^2\frac{x}{2} = \frac{4-\sqrt{15}}{8}$$

$\cos\frac{x}{2} = \sqrt{\left(\frac{4-\sqrt{15}}{8}\right)}$ [$\cos\frac{x}{2}$ Is positive as $\frac{x}{2}$ lies in first quadrant]

Step3:

Also, $\tan\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}$

$$\tan\frac{x}{2} = \frac{\sqrt{\left(\frac{4+\sqrt{15}}{8}\right)}}{\sqrt{\left(\frac{4-\sqrt{15}}{8}\right)}}$$

$$\Rightarrow \tan\frac{x}{2} = \frac{\sqrt{4+\sqrt{15}}}{\sqrt{4-\sqrt{15}}}$$

$$\Rightarrow \sqrt{\frac{4+\sqrt{15}}{4-\sqrt{15}} \times \frac{4+\sqrt{15}}{4+\sqrt{15}}}$$

$$\Rightarrow \sqrt{\frac{\left(4+\sqrt{15}\right)^2}{16-15}}$$

$$\Rightarrow 4+\sqrt{15}$$

Therefore, the values of $\sin\frac{x}{2}$, $\cos\frac{x}{2}$, $\tan\frac{x}{2}$ are $\sqrt{\left(\frac{4+\sqrt{15}}{8}\right)}$, $\sqrt{\left(\frac{4-\sqrt{15}}{8}\right)}$ and $4+\sqrt{15}$.

Overall Hint: First find $\cos x$ using the formula $\sin^2 A + \cos^2 A = 1$ then proceed accordingly