NCERT solutions for class 11 Maths Chapter 2 Relations and Functions

Question:1 If $\left(\frac{x}{3}+1, y-\frac{2}{3}\right)=\left(\frac{5}{3}, \frac{1}{3}\right)$, find the values of x and y.

Answer:

It is given that

$$\left(\frac{x}{3}+1, y-\frac{2}{3}\right)=\left(\frac{5}{3}, \frac{1}{3}\right)$$

Since the ordered pairs are equal, the corresponding elements will also be equal

Therefore,

$$\frac{x}{3}+1=\frac{5}{3} \quad and \quad y-\frac{2}{3}=\frac{1}{3}$$
$$\frac{x}{3}=\frac{5}{3}-1 \quad and \quad y=\frac{1}{3}+\frac{2}{3}$$
$$\frac{x}{3}=\frac{5-3}{3} \quad and \quad y=\frac{1+2}{3}$$
$$\frac{x}{3}=\frac{2}{3} \quad and \quad y=\frac{3}{3}$$
$$x=2 \quad and \quad y=1$$

Therefore, values of x and y are **2 and 1** respectively

Question:2 If the set A has 3 elements and the set B = {3, 4, 5}, then find the number of elements in $(A \times B)$.

Answer:

It is given that set A has 3 elements and the elements in set B are 3 , 4 , and 5

Therefore, the number of elements in set B is 3

Now,

Number of elements in $(A \times B)$

= (Number of elements in set A) $\times$ (Number of elements in set B)

= 3×3

= 9

Therefore, number of elements in $(A \times B)$ is **9**

Question:3 If G = {7, 8} and H = {5, 4, 2}, find $G \times H \ and \ H \times G$

Answer:

It is given that

G = {7, 8} and H = {5, 4, 2}

We know that the cartesian product of two non-empty sets P and Q is defined as

$P \times Q$ = {(p,q) , where p ϵ P , q ϵ Q }

Therefore,

$G \times H$ = {(7,5),(7,4),(7,2),(8,5),(8,4),(8,2)}

And

$H \times G$ = {(5,7),(5,8),(4,7),(4,8),(2,7),(2,8)}

Question:4 (i) State whether each of the following statements are true or false. If the statement is false, rewrite the given statement correctly. If P = {m, n} and Q = { n, m}, then $P \times Q$ = {(m, n),(n, m)}.

Answer:

FALSE

If P = {m, n} and Q = { n, m}

Then,

$P \times Q = \{(m,m),(m,n),(n,m),(n,n)\}$

Question:4(ii) State whether each of the following statements are true or false. If the statement is false, rewrite the given statement correctly. If A and B are non-empty sets, then A × B is a non-empty set of ordered pairs (x, y) such that $x \epsilon A$ and $y \epsilon B$

Answer:

It is a **TRUE** statement

∵ If A and B are non-empty sets, then A × B is a non-empty set of ordered pairs (x, y) such that $x \epsilon A$and $y \epsilon B$

Question:4 (iii) State whether each of the following statements are true or false. If the statement is false, rewrite the given statement correctly. If A = {1, 2}, B = {3, 4}, then $A \times (B \cap \phi) = \phi$

Answer:

This statement is **TRUE**

∵ If A = {1, 2}, B = {3, 4}, then

$B \cap \phi = \phi$

There for

$A \times (B \cap \phi) = \phi$

Question:5 If A = {–1, 1}, find $A \times A \times A$

Answer:

It is given that

A = {–1, 1}

A is an non-empty set

Therefore,

Lets first find $A \times A$

Now,

$$= \{(-1,-1,-1),(-1,-1,1),(-1,1,-1),(-1,1,1),(1,-1,-1),(1,-1,1),(1,1,-1),(1,1,1)\}$$

Question:6 If $A \times B$ = {(a, x),(a , y), (b, x), (b, y)}. Find A and B.

Answer:

It is given that

$A \times B$ = {(a, x),(a , y), (b, x), (b, y)}

We know that the cartesian product of two non-empty set P and Q is defined as

$$P \times Q = \{(p, q) : p \in P, q \in Q\}$$

Now, we know that A is the set of all first elements and B is the set of all second elements

Therefore,

$$A = \{a, b\} \quad and \quad B = \{x, y\}$$

Question:7 (i) Let A = {1, 2}, B = {1, 2, 3, 4}, C = {5, 6} and D = {5, 6, 7, 8}. Verify that $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

Answer:

It is given that

A = {1, 2}, B = {1, 2, 3, 4}, C = {5, 6} and D = {5, 6, 7, 8}

Now,

$B \cap C = \{1, 2, 3, 4\} \cap \{5, 6\} = \Phi$

Now,

$A \times (B \cap C) = A \times \phi = \phi \qquad -(i)$

$A \times B = \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4)\}$

And

$A \times C = \{(1,5), (1,6), (2,5), (2,6)\}$

Now,

$(A \times B) \cap (A \times C) = \phi \qquad -(ii)$

From equation (i) and (ii) it is clear that

$L.H.S. = R.H.S.$

Hence,

$A \times (B \cap C) = (A \times B) \cap (A \times C)$

Question:7 (ii) Let A = {1, 2}, B = {1, 2, 3, 4}, C = {5, 6} and D = {5, 6, 7, 8}. Verify that $A \times C$ is a subset of $B \times D$

Answer:

It is given that

A = {1, 2}, B = {1, 2, 3, 4}, C = {5, 6} and D = {5, 6, 7, 8}

Now,

$A \times C = \{(1,5), (1,6), (2,5), (2,6)\}$

And

We can clearly observe that all the elements of the set $A \times C$ are the elements of the set $B \times D$

Therefore, $A \times C$ is a subset of $B \times D$

Question:8 Let A = {1, 2} and B = {3, 4}. Write $A \times B$. How many subsets will $A \times B$ have?

List them.

Answer:

It is given that

A = {1, 2} and B = {3, 4}

Then,

$A \times B = \{(1,3),(1,4),(2,3),(2,4)\}$

$\Rightarrow n(A \times B) = 4$

Now, we know that if C is a set with $n(C) = m$

Then,

$n[P(C)] = 2^m$

Therefore,

The set $A \times B$ has $2^4 = 16$ subsets.

Question:9 Let A and B be two sets such that n(A) = 3 and n(B) = 2. If (x, 1), (y, 2), (z, 1) are in $A \times B$, find A and B, where x, y and z are distinct elements.

Answer:

It is given that

n(A) = 3 and n(B) = 2 and If (x, 1), (y, 2), (z, 1) are in A × B.

By definition of Cartesian product of two non-empty Set P and Q:

$P \times Q = \{(p, q) : p \epsilon P, q \epsilon Q\}$

Now, we can see that

P = set of all first elements.

And

Q = set of all second elements.

Now,

$\Rightarrow$ (x, y, z) are elements of A and (1,2) are elements of B

As n(A) = 3 and n(B) = 2

Therefore,

A = {x, y, z} and B = {1, 2}

Question:10 The Cartesian product $A \times A$ has 9 elements among which are found (–1, 0) and (0,1). Find the set A and the remaining elements of $A \times A$

Answer:

It is given that Cartesian product A × A having 9 elements among which are found (–1, 0) and (0,1).

Now,

Number of elements in (A× B) = (Number of elements in set A) × (Number of elements in B)

$n(A \times A) = n(A) \times n(A)$

It is given that $n(A \times A) = 9$

Therefore,

$n(A) \times n(A) = 9$

$\Rightarrow n(A) = 3$

Now,

By definition A × A = {(a, a): a ? A}

Therefore,

-1, 0 and 1 are the elements of set A

Now, because, n(A) = 3 therefore, A = {-1, 0, 1}

Therefore,

the remaining elements of set (A × A) are

(-1,-1), (-1,1), (0,0), (0, -1), (1,1), (1, -1) and (1, 0)

NCERT solutions for class 11 maths chapter 2 relations and funcions- Exercise: 2.2

Question:1 Let A = {1, 2, 3,...,14}. Define a relation R from A to A by $R=\left\{(x,y):3x-y=0, where\ \ x,y\epsilon A\right\}$.Write down its domain, codomain and range.

Answer:

It is given that

Now, the relation R from A to A is given as
$R=\left\{(x,y):3x-y=0, where\ \ x,y\epsilon A\right\}$

Therefore,

the relation in roaster form is , $R=\left\{(1,3),(2,6),(3,9),(4,12)\right\}$

Now,

We know that Domain of R = set of all first elements of the order pairs in the relation

Therefore,

Domain of $R=\left\{1,2,3,4\right\}$

And

Codomain of R = the whole set A

i.e. Codomain of $R = \{1, 2, 3, ..., 14\}$

Now,

Range of R = set of all second elements of the order pairs in the relation.

Therefore,

range of $R = \{3, 6, 9, 12\}$

Question:2 Define a relation R on the set N of natural numbers by $R = \{(x, y) : y = x + 5, x$ is a natural number less than $4; x, y \epsilon N\}$. Depict this relationship using roster form. Write down the domain and the range.

Answer:

It is given that

$R = \{(x, y) : y = x + 5, x$ is a natural number less than $4; x, y \epsilon N\}$

As x is a natural number which is less than 4.

Therefore,

the relation in roaster form is, $R = \{(1, 6), (2, 7), (3, 8)\}$

As Domain of R = set of all first elements of the order pairs in the relation.

Therefore,

Domain of $R = \{1, 2, 3\}$

Now,

Range of R = set of all second elements of the order pairs in the relation.

Therefore,

the range of $R = \{6, 7, 8\}$

Therefore, domain and the range are $\{1,2,3\}\quad and\quad \{6,7,8\}$ respectively

Question:3 A = {1, 2, 3, 5} and B = {4, 6, 9}. Define a relation R from A to B by . Write R in roster form.

Answer:

It is given that

A = {1, 2, 3, 5} and B = {4, 6, 9}

And

Now, it is given that the difference should be odd. Let us take all possible differences.

(1 - 4) = - 3, (1 - 6) = - 5, (1 - 9) = - 8(2 - 4) = - 2, (2 - 6) = - 4, (2 - 9) = - 7(3 - 4) = - 1, (3 - 6) = - 3, (3 - 9) = - 6(5 - 4) = 1, (5 - 6) = - 1, (5 - 9) = - 4

Taking the difference which are odd we get,

Therefore,

the relation in roaster form, $R=\{(1,4),(1,6),(2,9),(3,4),(3,6),(5,4),(5,6)\}$

Question:4 (i) The Fig2.7 shows a relationship between the sets P and Q. Write this relation in set-builder form

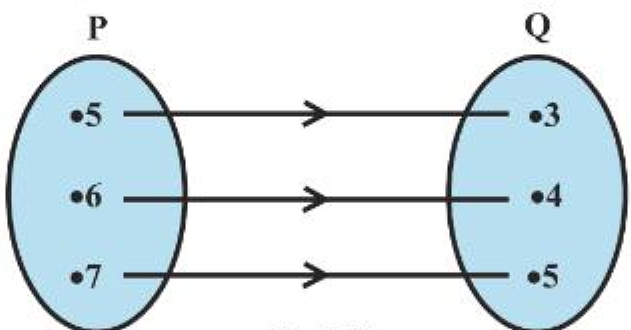

Fig 2.7

Answer:

It is given in the figure that

P = {5,6,7}, Q = {3,4,5}

Therefore,

the relation in set builder form is ,

$R=\{(x,y):y=x-2;x\,\epsilon\,P\}$

OR

$R=\{(x,y):y=x-2;\ for\ x=5,6,7\}$

Question:4 (ii) The Fig2.7 shows a relationship between the sets P and Q. Write this relation roster form. What is it domain and range?

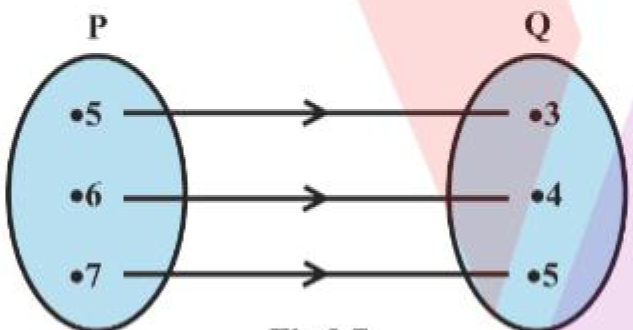

Fig 2.7

Answer:

From the given figure. we observe that

P = {5,6,7}, Q = {3,4,5}

And the relation in roaster form is , $R=\{(5,3),(6,4),(7,5)\}$

As Domain of R = set of all first elements of the order pairs in the relation.

Therefore,

Domain of $R = \{5, 6, 7\}$

Now,

Range of R = set of all second elements of the order pairs in the relation.

Therefore,

the range of $R = \{3, 4, 5\}$

Question:5 (i) Let A = {1, 2, 3, 4, 6}. Let R be the relation on A defined by Write R in roster form

Answer:

It is given that

A = {1, 2, 3, 4, 6}

And

Therefore,

the relation in roaster form is ,

Question:5 (ii) Let A = {1, 2, 3, 4, 6}. Let R be the relation on A defined by Find the domain of R

Answer:

It is given that

A = {1, 2, 3, 4, 6}

And

Now,

As Domain of R = set of all first elements of the order pairs in the relation.

Therefore,

Domain of $R = \{1, 2, 3, 4, 6\}$

Question:5 (iii) Let A = {1, 2, 3, 4, 6}. Let R be the relation on A defined by Find the range of R.

Answer:

It is given that

A = {1, 2, 3, 4, 6}

And

Now,

As the range of R = set of all second elements of the order pairs in the relation.

Therefore,

Range of $R = \{1, 2, 3, 4, 6\}$

Question:6 Determine the domain and range of the relation R defined by $R = \{(x, x+5) : x\epsilon \{0, 1, 2, 3, , 4, 5\}\}$

Answer:

It is given that

$R = \{(x, x+5) : x\epsilon \{0, 1, 2, 3, , 4, 5\}\}$

Therefore,

the relation in roaster form is , $R=\{(0,5),(1,6),(2,7),(3,8),(4,9),(5,10)\}$

Now,

As Domain of R = set of all first elements of the order pairs in the relation.

Therefore,

Domain of $R=\{0,1,2,3,4,5\}$

Now,

As Range of R = set of all second elements of the order pairs in the relation.

Range of $R=\{5,6,7,8,9,10\}$

Therefore, the domain and range of the relation R is $\{0,1,2,3,4,5\} \quad and \quad \{5,6,7,8,9,10\}$ respectively

Question:7 Write the relation in roster form.

Answer:

It is given that

Now,

As we know the prime number less than 10 are 2, 3, 5 and 7.

Therefore,

the relation in roaster form is , $R=\{(2,8),(3,27),(5,125),(7,343)\}$

Question:8 Let A = {x, y, z} and B = {1, 2}. Find the number of relations from A to B.

Answer:

It is given that

A = {x, y, z} and B = {1, 2}

Now,

$A \times B = \{(x,1),(x,2),(y,1),(y,2),(z,1),(z,2)\}$

Therefore,

$n(A \times B) = 6$

Then, the number of subsets of the set $(A \times B) = 2^n = 2^6$

Therefore, the number of relations from A to B is 2^6

Question:9 Let R be the relation on Z defined by $R = \{(a,b) : a, b \epsilon Z, a - b \ is \ an \ integer\}$

Find the domain and range of R.

Answer:

It is given that

$R = \{(a,b) : a, b \epsilon Z, a - b \ is \ an \ integer\}$

Now, as we know that the difference between any two integers is always an integer.

And

As Domain of R = set of all first elements of the order pairs in the relation.

Therefore,

The domain of R = Z

Now,

Range of R = set of all second elements of the order pairs in the relation.

Therefore,

range of R = Z

Therefore, the domain and range of R is **Z** and **Z** respectively

NCERT solutions for class 11 maths chapter 2 relations and functions-Exercise: 2.3

Question:1 (i) Which of the following relations are functions? Give reasons. If it is a function,determine its domain and range. {(2,1), (5,1), (8,1), (11,1), (14,1), (17,1)}

Answer:

Since, 2, 5, 8, 11, 14 and 17 are the elements of domain R having their unique images.
Hence, this relation R is a function.
Now,
As Domain of R = set of all first elements of the order pairs in the relation.
Therefore,
Domain of $R = \{2, 5, 8, 11, 14, 17\}$

Now,
As Range of R = set of all second elements of the order pairs in the relation.
Therefore,
Range of $R = \{1\}$

Therefore, domain and range of R are $\{2, 5, 8, 11, 14, 17\} \ \ and \ \ \{1\}$ respectively

Question:1 (ii) Which of the following relations are functions? Give reasons. If it is a function, determine its domain and range.{(2,1), (4,2), (6,3), (8,4), (10,5), (12,6), (14,7)}

Answer:

Since, 2, 4, 6, 8, 10,12 and 14 are the elements of domain R having their unique images. Hence, this relation R is a function.

Now,

As Domain of R = set of all first elements of the order pairs in the relation.

Therefore,

Domain of $R = \{2, 4, 6, 8, 10, 12, 14\}$

Now,

As Range of R = set of all second elements of the order pairs in the relation.

Therefore,

Range of $R = \{1, 2, 3, 4, 5, 6, 7\}$

Therefore, domain and range of R are $\{2, 4, 6, 8, 10, 12, 14\} \ and \ \{1, 2, 3, 4, 5, 6, 7\}$ respectively

Question:1 (iii) Which of the following relations are functions? Give reasons. If it is a function, determine its domain and range. {(1,3), (1,5), (2,5)}.

Answer:

Since the same first element 1 corresponds to two different images 3 and 5. Hence, this relation is not a function.

Question:2 (i) Find the domain and range of the following real functions:

$f(x) = -|x|$

Answer:

Given function is

$f(x) = -|x|$

Now, we know that

$$|x| \begin{cases} x & if\ x > 0 \\ -x & if\ x < 0 \end{cases}$$

$$\Rightarrow f(x) = -|x| \begin{cases} -x & if\ x > 0 \\ x & if\ x < 0 \end{cases}$$

Now, for a function f(x),

Domain: The values that can be put in the function to obtain real value. For example f(x) = x, now we can put any value in place of x and we will get a real value. Hence, the domain of this function will be Real Numbers.

Range: The values that we obtain of the function after putting the value from domain. For Example: f(x) = x + 1, now if we put x = 0, f(x) = 1. This 1 is a value of Range that we obtained.

Since f(x) is defined for $x \epsilon R$, **the domain of f is R.**

It can be observed that the range of f(x) = -|x| is all real numbers except positive real numbers. Because will always get a negative number when we put a value from the domain.

Therefore, the range of f is $(-\infty, 0]$

Question:2 (ii) Find the domain and range of the following real functions:

$f(x)=\sqrt{9-x^2}$

Answer:

Given function is

$f(x)=\sqrt{9-x^2}$

Now,

Domain: These are the values of x for which f(x) is defined.

for the given f(x) we can say that, f(x) should be real and for that, $9 - x^2 \geq 0$ [Since a value less than 0 will give an imaginary value]

$\Rightarrow 3^2-x^2=(3-x)(3+x)\geq 0$

$\Rightarrow -3\leq x\leq 3$

Therefore,

The domain of f(x) is $[-3,3]$

Now,

If we put the value of x from $[-3,3]$ we will observe that the value of function $f(x)=\sqrt{9-x^2}$ varies from **0 to 3**

Therefore,

Range of f(x) is $[0,3]$

Question:3 (i) A function f is defined by f(x) = 2x –5. Write down the values of f (0),

Answer:

Given function is

$f(x)=2x-5$

Now,

$f(0)=2(0)-5=0-5=-5$

Therefore,

Value of **f(0)** is **-5**

Question:3 (ii) A function f is defined by f(x) = 2x –5. Write down the values of f (7)

Answer:

Given function is

$f(x) = 2x - 5$

Now,

$f(7) = 2(7) - 5 = 14 - 5 = 9$

Therefore,

Value of **f(7)** is **9**

Question:3 (iii) A function f is defined by f(x) = 2x –5. Write down the values of f (-3)

Answer:

Given function is

$f(x) = 2x - 5$

Now,

$f(-3) = 2(-3) - 5 = -6 - 5 = -11$

Therefore,

Value of **f(-3)** is **-11**

Question:4(i) The function 't' which maps temperature in degree Celsius into temperature in degree Fahrenheit is defined by $t(C) = \frac{9C}{5} + 32$ t (0)

Answer:

Given function is
$t(C) = \frac{9C}{5} + 32$

Now,
$t(0) = \frac{9(0)}{5} + 32 = 0 + 32 = 32$

Therefore,

Value of **t(0)** is **32**

Question:4(ii) The function 't' which maps temperature in degree Celsius into temperature in degree Fahrenheit is defined by $t(C) = \frac{9C}{5} + 32$ t (28)

Answer:

Given function is
$t(C) = \frac{9C}{5} + 32$

Now,
$t(28) = \frac{9(28)}{5} + 32 = \frac{252}{5} + 32 = \frac{252 + 160}{5} = \frac{412}{5}$

Therefore,
Value of **t(28)** is $\frac{412}{5}$

Question:4(iii) The function 't' which maps temperature in degree Celsius into temperature in degree Fahrenheit is defined by $t(C) = \frac{9C}{5} + 32$ t (-10)

Answer:

Given function is
$t(C) = \frac{9C}{5} + 32$

Now,

$$t(-10) = \frac{9(-10)}{5} + 32 = \frac{-90}{5} + 32 = -18 + 32 = 14$$

Therefore,

Value of **t(-10)** is **14**

Question:4(iv) The function 't' which maps temperature in degree Celsius into temperature in

degree Fahrenheit is defined by $t(C) = \frac{9C}{5} + 32$ The value of C, when t(C) = 212.

Answer:

Given function is

$$t(C) = \frac{9C}{5} + 32$$

Now,

$$212 = \frac{9(C)}{5} + 32$$

$$212 \times 5 = 9(C) + 160$$

$$9(C) = 1060 - 160$$

$$C = \frac{900}{9} = 100$$

Therefore,

When t(C) = 212 , value of C is **100**

Question:5 (i) Find the range of each of the following functions.

$$f(x) = 2 - 3x, x \epsilon R, x > 0.$$

Answer:

Given function is

$f(x) = 2 - 3x, x \epsilon R, x > 0.$

It is given that $x > 0$

Now,

$\Rightarrow 3x > 0$

$\Rightarrow -3x < 0$

Add 2 on both the sides

$\Rightarrow -3x + 2 < 0 + 2$

$\Rightarrow 2 - 3x < 2$

$\Rightarrow f(x) < 2 \qquad (\because f(x) = 2 - 3x)$

Therefore,

Range of function $f(x) = 2 - 3x$ is $(-\infty, 2)$

Question:5 (ii) Find the range of each of the following functions

$f(x) = x^2 + 2$, x is a real number.

Answer:

Given function is

$f(x) = x^2 + 2$

It is given that **x** is a real number

Now,

$\Rightarrow x^2 \geq 0$

Add 2 on both the sides

$\Rightarrow x^2 + 2 \geq 0 + 2$

$\Rightarrow f(x) \geq 2 \qquad (\because f(x) = x^2 + 2)$

Therefore,

Range of function $f(x) = x^2 + 2$ is $[2, \infty)$

Question:5 (iii) Find the range of each of the following functions.

f (x) = x, x is a real number

Answer:

Given function is

$f(x) = x$

It is given that **x** is a real number

Therefore,

Range of function $f(x) = x$ is **R**

NCERT solutions for class 11 maths chapter 2 relations and functions- Miscellaneous Exercise

Question:1 The relation f is defined by $f(x) = \begin{cases} x^2 & 0 \leq x \leq 3 \\ 3x & 3 \leq x \leq 10 \end{cases}$ The relation g is defined by $g(x) = \begin{cases} x^2 & 0 \leq x \leq 2 \\ 3x & 2 \leq x \leq 10 \end{cases}$ Show that f is a function and g is not a function.

Answer:

It is given that

$$f(x) = \begin{cases} x^2 & 0 \leq x \leq 3 \\ 3x & 3 \leq x \leq 10 \end{cases}$$

Now,

$$f(x) = x^2 \ for \ 0 \leq x \leq 3$$

And

$$f(x) = 3x \ for \ 3 \leq x \leq 10$$

At x = 3, $f(x) = x^2 = 3^2 = 9$

Also, at x = 3, $f(x) = 3x = 3 \times 3 = 9$

We can see that for $0 \leq x \leq 10$, f(x) has unique images.

Therefore, By definition of a function, the given relation is function.

Now,

It is given that

$$g(x) = \begin{cases} x^2 & 0 \leq x \leq 2 \\ 3x & 2 \leq x \leq 10 \end{cases}$$

Now,

$g(x) = x^2 \ for \ 0 \leq x \leq 2$

And

$g(x) = 3x \ for \ 2 \leq x \leq 10$

At x = 2, $g(x) = x^2 = 2^2 = 4$

Also, at x = 2, $g(x) = 3x = 3 \times 2 = 6$

We can clearly see that element 2 of the domain of relation g(x) corresponds to two different images i.e. 4 and 6. Thus, f(x) does not have unique images

Therefore, by definition of a function, the given relation is not a function

Hence proved

Question:2 If $f(x) = x^2$ find $\frac{f(1.1) - f(1)}{(1.1 - 1)}$

Answer:

Given function is

$f(x) = x^2$

Now,

$$\frac{f(1.1) - f(1)}{(1.1 - 1)} = \frac{(1.1)^2 - 1^2}{(1.1 - 1)} = \frac{1.21 - 1}{0.1} = \frac{0.21}{0.1} = 2.1$$

Therefore, value of $\frac{f(1.1) - f(1)}{(1.1 - 1)}$ is **2.1**

Question:3 Find the domain of the function $f(x) = \frac{x^2 + 2x + 1}{x^2 - 8x + 12}$

Answer:

Given function is

$$f(x) = \frac{x^2 + 2x + 1}{x^2 - 8x + 12}$$

Now, we will simplify it into

$$f(x) = \frac{x^2 + 2x + 1}{x^2 - 8x + 12}$$

$$= \frac{x^2 + 2x + 1}{x^2 - 6x - 2x + 12}$$

$$= \frac{x^2 + 2x + 1}{x(x - 6) - 2(x - 6)}$$

$$= \frac{x^2 + 2x + 1}{(x - 2)(x - 6)}$$

Now, we can clearly see that $x \neq 2, 6$

Therefore, the Domain of **f(x)** is $(R - \{2, 6\})$

Question:4 Find the domain and the range of the real function f defined by $f(x) = \sqrt{(x - 1)}$

Answer:

Given function is

$f(x) = \sqrt{(x-1)}$

We can clearly see that **f(x) is only defined** for the values of x , $x \geq 1$

Therefore,

The domain of the function $f(x) = \sqrt{(x-1)}$ is $[1, \infty)$

Now, as

$\Rightarrow x \geq 1$

$\Rightarrow x - 1 \geq 1 - 1$

$\Rightarrow x - 1 \geq 0$

take square root on both sides

$\Rightarrow \sqrt{x-1} \geq 0$

$\Rightarrow f(x) \geq 0 \qquad (\because f(x) = \sqrt{x-1})$

Therefore,

Range of function $f(x) = \sqrt{(x-1)}$ is $[0, \infty)$

Question:5 Find the domain and the range of the real function f defined by $f(x) = |x-1|$

Answer:

Given function is

$f(x) = |x-1|$

As the given function is defined of all real number

The domain of the function $f(x) = |x-1|$ is **R**

Now, as we know that the mod function always gives only positive values

Therefore,

Range of function $f(x) = |x-1|$ is all non-negative real numbers i.e. $[0, \infty)$

Question:6 Let $f = \left\{\left(x, \frac{x^2}{1+x^2}\right) : x \epsilon R\right\}$ R be a function from R into R. Determine the range of f.

Answer:

Given function is

$$f = \left\{\left(x, \frac{x^2}{1+x^2}\right) : x \epsilon R\right\}$$

Range of any function is the set of values obtained after the mapping is done in the domain of the function. So every value of the codomain that is being mapped is Range of the function.

Let's take

$$y = \frac{x^2}{1+x^2}$$

$$\Rightarrow y(1+x^2) = x^2$$

$$\Rightarrow y + yx^2 = x^2$$

$$\Rightarrow y = x^2(1-y)$$

$$\Rightarrow x^2 = \frac{y}{1-y}$$

$$\Rightarrow x = \pm\sqrt{\frac{y}{1-y}}$$

Now, 1 - y should be greater than zero and y should be greater than and equal to zero for x to exist because other than those values the x will be imaginary

Thus, $1 - y > 0, y < 1 \ and \ y \geq 0$

Therefore,

Range of given function is $[0, 1)$

Question:7 Let f, g : R → R be defined, respectively by f(x) = x + 1, g(x) = 2x – 3. Find f + g, f – g and f/g

Answer:

It is given that

$f, g : R \to R$

$f(x) = x + 1 \ \ and \ \ g(x) = 2x - 3$

Now,

$(f + g)x = f(x) + g(x)$

$= (x + 1) + (2x - 3)$

$= 3x - 2$

Therefore,

$(f + g)x = 3x - 2$

Now,

$(f - g)x = f(x) - g(x)$

$= (x + 1) - (2x - 3)$

$= x + 1 - 2x + 3$

$= -x + 4$

Therefore,

$(f - g)x = -x + 4$

Now,

$\left(\frac{f}{g}\right)x = \frac{f(x)}{g(x)}, g(x) \neq 0$

$= \frac{x+1}{2x-3} \ , x \neq \frac{3}{2}$

Therefore, values

of $(f + g)x, (f - g)x \ and \ \left(\frac{f}{g}\right)x$ are $(3x - 2), (-x + 4) \ and \ \frac{x+1}{2x-3}$ respectively

Question:8 Let f = {(1,1), (2,3), (0,–1), (–1, –3)} be a function from Z to Z defined by f(x) = ax + b, for some integers a, b. Determine a, b.

Answer:

It is given that

$f = \{(1,1),(2,3),(0,-1),(-1,-3)\}$

And

$f(x) = ax + b$

Now,

At **x = 1** , $f(x) = 1$

$\Rightarrow f(1) = a(1) + b$

$\Rightarrow a + b = 1 \qquad - (i)$

Similarly,

At $x = 0$, $f(x) = -1$

$\Rightarrow f(0) = a(0) + b$

$\Rightarrow b = -1$

Now, put this value of **b** in equation (i)

we will get,

$a = 2$

Therefore, values of **a** and **b** are **2** and **-1** respectively

Question:9 (i) Let R be a relation from N to N defined by $R = \left\{(a,b) : a, b\epsilon N \ \ and \ \ a = b^2\right\}$. Are the following true?

$(a,a)\epsilon R,$ for all $a\epsilon N$

Answer:

It is given that
$R=\left\{(a,b):a,b\epsilon N\ \ and\ \ a=b^2\right\}$

And

$(a,a)\epsilon R,$ for all $a\epsilon N$

Now, it can be seen that $2\ \epsilon\ N$ But, $2\neq 2^2=4$

Therefore, this statement is **FALSE**

Question:9 (ii) Let R be a relation from N to N defined by $R=\left\{(a,b):a,b\epsilon N\ \ and\ \ a=b^2\right\}$. Are the following true?

$(a,a)\epsilon R,$ implies (b,a) ϵ R

Answer:

It is given that
$R=\left\{(a,b):a,b\epsilon N\ \ and\ \ a=b^2\right\}$

And

$(a,b)\ \epsilon\ R,$ implies $(b,a)\ \epsilon\ R$

Now , it can be seen that $(2,4)\ \epsilon\ R,$ and $4=2^2=4$, But $2\neq 4^2=16$

Therefore, $(2,4)\ \ \notin\ \ N$

Therefore, given statement is **FALSE**

Question:9 (iii) Let R be a relation from N to N defined by $R=\left\{(a,b):a,b\epsilon N\ \ and\ \ a=b^2\right\}$. Are the following true?

(a,b) ϵ R, (b,c) ϵ R implies (a,c) ϵ R.

Answer:

It is given that
$R = \left\{(a,b) : a, b \epsilon N \ \ and \ \ a = b^2\right\}$

And
$(a,b) \ \epsilon \ R, (b,c) \ \epsilon \ R$ implies $(a,c) \ \epsilon \ R$

Now, it can be seen
that $(16,4) \ \epsilon \ R, (4,2) \ \epsilon \ R$ because $16 = 4^2 = 16$ and $4 = 2^2 = 4$, But $16 \neq 2^2 = 4$
Therefore, $(16,2) \ \notin \ N$

Therefore, the given statement is **FALSE**

Question:10 (i) Let A ={1,2,3,4}, B = {1,5,9,11,15,16} and f = {(1,5), (2,9), (3,1), (4,5), (2,11)} Are the following true? f is a relation from A to B Justify your answer

Answer:

It is given that
$A = \{1,2,3,4\}$
$B = \{1,5,9,11,15,16\}$
and $f = \{(1,5),(2,9),(3,1),(4,5),(2,11)\}$
Now,
Now, a relation from a non-empty set A to a non-empty set B is a subset of the Cartesian product A × B
And we can see that f is a subset of $A \times B$
Hence f is a relation from A to B
Therefore, given statement is **TRUE**

Question:10 (ii) Let A ={1,2,3,4}, B = {1,5,9,11,15,16} and f = {(1,5), (2,9), (3,1), (4,5), (2,11)} Are the following true? f is a function from A to B justify your answer

Answer:

It is given that

$A = \{1, 2, 3, 4\}$

$B = \{1, 5, 9, 11, 15, 16\}$

and $f = \{(1,5), (2,9), (3,1), (4,5), (2,11)\}$

Now,

As we can observe that same first element i.e. 2 corresponds to two different images that is 9 and 11.

Hence f is not a function from A to B

Therefore, given statement is **FALSE**

Question:11 Let f be the subset of $Z \times Z$ defined by $f = \{(ab, a+b) : a, b \epsilon Z\}$. Is f a function from Z to Z? Justify your answer.

Answer:

It is given that

$f = \{(ab, a+b) : a, b \epsilon Z\}$

Now, we know that relation f from a set A to a set B is said to be a function only if every element of set A has a unique image in set B

Now, for value 2, 6, -2, -6 ϵ Z

$\Rightarrow f = \{(12, 8), (12, -8), (-12, -4), (-12, 4)\}$

Now, we can observe that same first element i.e. 12 corresponds to two different images that are 8 and -8.

Thus, **f** is not a function

Question:12 Let A = {9,10,11,12,13} and let f : A→ N be defined by f (n) = the highest prime factor of n. Find the range of f.

Answer:

It is given that

A = {9,10,11,12,13}

And

$f : A \rightarrow N$ be defined by f(n) = the highest prime factor of n.

Now,

Prime factor of 9 = 3

Prime factor of 10 = 2,5

Prime factor of 11 = 11

Prime factor of 12 = 2,3

Prime factor of 13 = 13

f(n) = the highest prime factor of n.

Hence,

f(9) = the highest prime factor of 9 = 3

f(10) = the highest prime factor of 10 = 5

f(11) = the highest prime factor of 11 = 11

f(12) = the highest prime factor of 12 = 3 f(13) = the highest prime factor of 13 = 13

As the range of f is the set of all f(n), where $n \epsilon A$

Therefore, the range of f is: {3, 5, 11, 13}.