

CBSE NCERT Solutions for Class 11 Mathematics Chapter 01***Back of Chapter Questions***

I. Which of the following are sets? Justify your answer.

- (i) The collection of all months of a year beginning with the letter J .
- (ii) The collection of ten most talented writers of India.
- (iii) A team of eleven best-cricket batsmen of the world.
- (iv) The collection of all boys in your class.
- (v) The collection of all natural numbers less than 100.
- (vi) A collection of novels written by the writer Munshi Prem Chand.
- (vii) The collection of all even integers.
- (viii) The collection of questions in this Chapter.
- (ix) A collection of most dangerous animals of the world.

Solution:

(i) Step 1:

Anyone can identify a month that belongs to the collection of all the months of a year beginning with the letter J . so, it is a well-defined collection of objects. Therefore, this collection is a set.

Hint: Well-defined collection of objects is known as a set.

(ii) Step 1:

The criteria of determining a writer's talent vary from one person to other. Thus, the collection of ten most talented writer of India is not a well-defined collection Therefore; this collection is not a set.

(iii) Step 1:

The criteria of determining a batsman's talent vary from one person to other. Thus, a team of 11 best cricket batsmen of the world is not a well-defined collection Therefore; this collection is not a set.

Hint: Well defined collection of objects is known as a set.

(iv) Step 1:

A boy can easily identify who belongs to his class therefore, the collection of all boys in your class is a well-defined collection Therefore, and this collection is a set.

Hint: Well defined collection of objects is known as a set.

(v) Step 1:

One can easily identify whether a number belongs to the set of numbers less than 100, thus the collection of all natural numbers less than 100 is a well-defined collection Therefore, and this collection is a set.

Hint: Well defined collection of objects is known as a set.

(vi) Step 1:

A book written by writer Munshi Prem Chand can be easily identified therefore, the collection of novels written by the writer Munshi Prem Chand is a well-defined collection Therefore, this collection is a set.

Hint: Well defined collection of objects is known as a set.

(vii) Step 1:

The even integers can be easily identified thus, the collection of all even integers is a well-defined collection Therefore, and this collection is a set.

Hint: Well defined collection of objects is known as a set.

(viii) Step 1:

Any question belonging to the particular chapter can be easily identified thus, the collection of question in this chapter is a well-defined collection therefore, and this collection is a set.

Hint: Well defined collection of objects is known as a set.

(ix) Step 1:

Determining the dangerous animals of the world is not easy to identify thus the collection of most dangerous animals of the world is not a well-defined collection therefore; this collection is not a set.

Hint: Well defined collection of objects is known as a set.

2. Let $A = \{1, 2, 3, 4, 5, 6\}$. Insert the appropriate symbol \in or \notin in the blank spaces:

(i) $5 \dots A$

(ii) $8 \dots A$

(iii) $0 \dots A$

(iv) $4 \dots A$

(v) $2 \dots A$

(vi) $10 \dots A$

Solution:

- (i) Step 1:
 $5 \in A$ [5 belongs to A]
Hint: See whether the given number is present in $A = \{1, 2, 3, 4, 5, 6\}$ accordingly. and put \in or \notin
- (ii) Step 1:
 $8 \notin A$ [8 doesnot belong to A]
Hint: See whether the given number is present in $A = \{1, 2, 3, 4, 5, 6\}$ and put \in or \notin accordingly.
- (iii) Step 1:
 $0 \notin A$ [0 doesnot belong to A]
Hint: See whether the given number is present in $A = \{1, 2, 3, 4, 5, 6\}$ and put \in or \notin accordingly.
- (iv) Step 1:
 $4 \in A$ [4 belongs to A]
Hint: See whether the given number is present in $A = \{1, 2, 3, 4, 5, 6\}$ and put \in or \notin accordingly.
- (v) Step 1:
 $2 \in A$ [2 belongs to A]
Hint: See whether the given number is present in $A = \{1, 2, 3, 4, 5, 6\}$ and put \in or \notin accordingly.
- (vi) Step 1:
 $10 \notin A$ [10 doesnot belong to A]
Hint: See whether the given number is present in $A = \{1, 2, 3, 4, 5, 6\}$ and put \in or \notin accordingly.

3. Write the following sets in roster form:

(i) $A = \{x: x \text{ is an integer and } -3 < x < 7\}$

(ii) $B = \{x: x \text{ is a natural number less than } 6\}$

(iii) $C = \{x: x \text{ is a two - digit natural number such that the sum of its digits is } 8\}$

(iv) $D = \{x: x \text{ is a prime number which is divisor of } 60\}$.

(v) $E =$ The set of all letters in the word TRIGONOMETRY.

(vi) $F =$ The set of all letters in the word BETTER.

Solution:

(i). Step 1:

$$A = \{x: x \text{ is an integer and } -3 < x < 7\}$$

The elements belong to this set are $-2, -1, 0, 1, 2, 3, 4, 5$ and 6 .

Thus, the given set can be written in roster form as $A = \{-2, -1, 0, 1, 2, 3, 4, 5, 6\}$

Hint: In roster form, all the elements of a set are listed, the elements are being separated by commas and enclosed within braces $\{ \}$.

(ii) Step 1:

$$B = \{x: x \text{ is a natural number less than } 6\}$$

The elements belong to this set are $1, 2, 3, 4,$ and 5 .

Thus, the given set can be written in roster form as $B = \{1, 2, 3, 4, \text{ and } 5\}$

Hint: In roster form, all the elements of a set are listed, the elements are being separated by commas and enclosed within braces $\{ \}$.

(iii) Step 1:

$$C = \{x: x \text{ is a two digit number such that sum of its digits is } 8\}$$

The elements belong to this set are $17, 26, 35, 44, 53, 62, 71$ and 80

Thus, the given set can be written in roster form as $C = \{17, 26, 35, 44, 53, 62, 71, \text{ and } 80\}$

Hint: In roster form, all the elements of a set are listed, the elements are being separated by commas and enclosed within braces $\{ \}$.

(iv) Step 1:

$D = \{x: x \text{ is a prime number which is a divisor of } 60\}$

$$60 = 2 \times 2 \times 3 \times 5.$$

The elements belong to this set are 2, 3 and 5.

Thus, the given set can be written in roster form as $D = \{2, 3, 5\}$

Hint: In roster form, all the elements of a set are listed, the elements are being separated by commas and enclosed within braces { }.

(v) Step 1:

$E =$ The set of all letters in the word TRIGONOMETRY.

There are 12 letters in the word TRIGONOMETRY, out of which letters T, R and O are repeated.

Thus, the given set can be written in roster form as $E = \{T, R, I, G, O, N, M, E, Y\}$

Hint: In roster form, all the elements of a set are listed, the elements are being separated by commas and enclosed within braces { }.

(vi). Step 1:

$F =$ The set of all letters in the word BETTER.

There are 6 letters in the word BETTER, out of which letter E and T are repeated.

Thus, the given set can be written in roster form as $F = \{B, E, T, R\}$

Hint: In roster form, all the elements of a set are listed, the elements are being separated by commas and enclosed within braces { }.

4. Write the following sets in the set-builder form:

(i) $\{3, 6, 9, 12\}$

(ii) $\{2, 4, 8, 16, 32\}$

(iii) $\{5, 25, 125, 625\}$

(iv) $\{2, 4, 6, \dots\}$

(v) $\{1, 4, 9, \dots, 100\}$

Solution:

(i) Step 1:

$$\{3, 6, 9, 12\} = \{x: x = 3n, n \in N \text{ and } 1 \leq n \leq 4\}$$

Hint: In set builder form, all the elements of a set possess a single common property, which is not possessed by any element outside the set.

(ii) Step 1:

$$\{2, 4, 8, 16, 32\}$$

It can be written as, $2 = 2^1, 4 = 2^2, 8 = 2^3, 16 = 2^4, 32 = 2^5$

$$\therefore \{2, 4, 8, 16, 32\} = \{x: x = 2^n, n \in N \text{ and } 1 \leq n \leq 5\}$$

Hint: In set builder form, all the elements of a set possess a single common property, which is not possessed by any element outside the set.

(iii) Step 1:

$$\{5, 25, 125, 625\}$$

Here, $5 = 5^1, 25 = 5^2, 125 = 5^3, 625 = 5^4$

$$\therefore \{5, 25, 125, 625\} = \{x: x = 5^n, n \in N \text{ and } 1 \leq n \leq 4\}$$

Hint: In set builder form, all the elements of a set possess a single common property, which is not possessed by any element outside the set.

(iv) Step 1:

$$\{2, 4, 6 \dots \dots\}$$

It is a set of all even natural numbers.

$$\therefore \{2, 4, 6 \dots \dots\} = \{x: x \text{ is an even natural number}\}$$

Hint: In set builder form, all the elements of a set possess a single common property, which is not possessed by any element outside the set.

(v) Step 1:

$$\{1, 4, 9, \dots, 100\}$$

$$\text{Here, } 1 = 1^2, 4 = 2^2, 9 = 3^2 \dots \dots \dots 100 = 10^2$$

$$\therefore \{1, 4, 9, \dots \dots 100\} = \{x: x = n^2, n \in N \text{ and } 1 \leq n \leq 10\}$$

Hint: In set builder form, all the elements of a set possess a single common property, which is not possessed by any element outside the set.

5. List all the elements of the following sets:

(i) $A = \{x: x \text{ is an odd natural number}\}$

(ii) $B = \left\{x: x \text{ is an } -\frac{1}{2} < x < \frac{9}{2} \text{ integer,}\right\}$

(iii) $C = \{x: x \text{ is an integer } x^2 \leq 4\}$

(iv) $D = \{x: x \text{ is a letter in the word "LOYAL"}\}$

(v) $E = \{x: x \text{ is a month of a year not having 31 days}\}$

(vi) $F = \{x: x \text{ is a consonant in the English alphabet which proceeds } k\}$

Solution:

(i) Step 1:

$$A = \{x: x \text{ is an odd natural number}\}$$

$$A = \{1, 3, 5, 7, 9 \dots\}$$

Hint: Every element possessing the given property of the set should be mentioned within the braces.

(ii) Step 1:

$$B = \{x: x \text{ is an integer, } -1/2 < n < 9/2\}$$

It is seen that $-1/2 = -0.5$ and $9/2 = 4.5$

$$B = \{0, 1, 2, 3, 4\}$$

Hint: Every element possessing the given property of the set should be mentioned within the braces.

(iii) Step 1:

$$C = \{x: x \text{ is an integer, } x^2 \leq 4\}$$

It is seen that $(-1)^2 \leq 4$, $(-2)^2 \leq 4$, $0^2 \leq 4$, $(1)^2 \leq 4$, $(2)^2 \leq 4$

$$\therefore C = \{-2, -1, 0, 1, 2\}$$

Hint: Every element possessing the given property of the set should be mentioned within the braces.

(iv) Step 1:

$$D = \{x: x \text{ is a letter in the word "LOYAL"}\}$$

$$D = \{L, O, Y, A\}$$

Hint: Every element possessing the given property of the set should be mentioned within the braces.

(v) Step 1:

$$E = \{x: x \text{ is a month of a year not having 31 days}\}$$

$$E = \{\text{February, April, June, September, November}\}$$

Hint: Every element possessing the given property of the set should be mentioned within the braces.

(vi) Step 1:

$$F = \{x: x \text{ is a consonant in the English alphabet which precedes } k\}$$

$$F = \{b, c, d, f, g, h, j\}$$

Hint: Every element possessing the given property of the set should be mentioned within the braces.

6. Match each of the set on the left in the roster form with the same set on the right described in setbuilder form:

(i) $\{1, 2, 3, 6\}$ (a) $\{x: x \text{ is a prime number and a divisor of } 6\}$

(ii) $\{2, 3\}$ (b) $\{x: x \text{ is an odd natural number less than } 10\}$

(iii) $\{M, A, T, H, E, I, C, S\}$ (c) $\{x: x \text{ is natural number and divisor of } 6\}$ (iv) $\{1, 3, 5, 7, 9\}$ (d) $\{x: x \text{ is a letter of the word MATHEMATICS}\}$

Solution:

(i) Step 1:

We can see that all the elements of this set are natural numbers as well as the divisors of 6. therefore, (i) matches with (c).

Hint: Every element possessing the given property of the set should be mentioned within the braces.

(ii) Step 1:

We can see that 2 and 3 are prime numbers and are also divisors of 6. therefore, (ii) matches with (a).

Hint: Every element possessing the given property of the set should be mentioned within the braces.

(iii) Step 1:

We can see that all the elements of this set are letters of the word MATHEMATICS.

Therefore, (iii) matches with (d).

Hint: Every element possessing the given property of the set should be mentioned within the braces.

(iv) Step 1:

We can see that all the elements of this set are odd natural numbers less than 10. therefore, (iv) matches with (b).

Hint: Every element possessing the given property of the set should be mentioned within the braces.

Exercise 11.2

1. Which of the following are examples of the null set

(i) Set of odd natural numbers divisible by 2

- (ii) Set of even prime numbers
- (iii) $\{x: x \text{ is a natural number, } x < 5 \text{ and } x > 7\}$
- (v) $\{y: y \text{ is a point common to any two parallel lines}\}$

Solution:

- (i) Step 1:

We know that no odd number is divisible by 2 therefore, it is a null set.

Hint: A set, which does not contain any element is called the empty set or the null set.

- (ii) Step 1: 2 is an even prime number therefore, a set of even prime numbers is not a null set.

Hint: A set, which does not contain any element is called the empty set or the null set.

- (iii) Step 1:

$\{x: x \text{ is a natural number, } x < 5 \text{ and } x > 7\}$ is a null set because a number cannot be simultaneously less than 5 and greater than 7.

Hint: A set, which does not contain any element is called the empty set or the null set.

- (iv) Step 1:

As we know that parallel lines do not intersect. therefore, they have no common point. So, $\{y: y \text{ is a point common to any two parallel lines}\}$ is a null set.

Hint: A set, which does not contain any element is called the empty set or the null set.

2. Which of the following sets are finite or infinite

- (i) The set of months of a year
- (ii) $\{1, 2, 3 \dots\}$
- (iii) $\{1, 2, 3 \dots 99, 100\}$
- (iv) The set of positive integers greater than 100
- (v) The set of prime numbers less than 99

Solution:

(i) Step 1:

We know that there are 12 months in a year so the set of months of a year is a finite set with 12 elements in it.

Hint: A set, which is empty or consists of a definite number of elements is called finite otherwise, the set is called infinite.

(ii) Step 1:

{1,2,3} is an infinite set as it has infinite number of natural numbers.

Hint: A set, which is empty or consists of a definite number of elements is called finite otherwise, the set is called infinite.

(iii) Step 1:

{1,2,399, 100} is a finite set as the numbers from 1 to 100 are finite in numbers.

Hint: A set, which is empty or consists of a definite number of elements is called finite otherwise, the set is called infinite.

(iv) Step 1:

Positive integers greater than 100 are infinite in numbers. So, the set of positive integers greater than 100 is an infinite set.

Hint: A set, which is empty or consists of a definite number of elements is called finite otherwise, the set is called infinite.

(vi) Step 1:

Prime numbers less than 99 are finite in number. therefore, the set of prime numbers less than 99 is a finite set.

Hint: A set, which is empty or consists of a definite number of elements is called finite otherwise, the set is called infinite.

3. State whether each of the following set is finite or infinite:

- (i) The set of lines which are parallel to the x -axis
- (ii) The set of letters in the English alphabet
- (iii) The set of numbers which are multiple of 5
- (iv) The set of animals living on the earth
- (v) The set of circles passing through the origin $(0, 0)$

Solution:

- (i) Step 1:

Lines parallel to x -axis are infinite in number. So, the set of lines which are parallel to the x -axis is an infinite set.

Hint: A set, which is empty or consists of a definite number of elements is called finite otherwise, the set is called infinite.

- (ii) Step 1:

The set of letters in the English alphabet is a finite set as it has 26 elements.

Hint: A set, which is empty or consists of a definite number of elements is called finite otherwise, the set is called infinite.

- (iii) Step 1:

Multiple of 5 are infinite in number. therefore, the set of numbers which are multiple of 5 is an infinite set.

Hint: A set, which is empty or consists of a definite number of elements is called finite otherwise, the set is called infinite.

- (iv) Step 1:

The number of animals living on the earth is finite although the count is big. Therefore, the set of animals living on the earth is a finite set.

Hint: A set, which is empty or consists of a definite number of elements is called finite otherwise, the set is called infinite.

- (v) Step 1:

Infinite number of circles can pass through the origin. So, the set of circles passing through the origin $(0, 0)$ is an infinite set.

Hint: A set, which is empty or consists of a definite number of elements is called finite otherwise, the set is called infinite.

4. In the following, state whether $A = B$ or not:

(i) $A = \{a, b, c, d\}; B = \{d, c, b, a\}$

(ii) $A = \{4, 8, 12, 16\}; B = \{8, 4, 16, 18\}$

(iii) $A = \{2, 4, 6, 8, 10\}; B = \{x: x \text{ is positive even integer and } x \leq 10\}$

(iv) $A = \{x: x \text{ is a multiple of } 10\}; B = \{10, 15, 20, 25, 30 \dots\}$

Solution:

(i) Step 1:

$$A = \{a, b, c, d\}$$

$$B = \{d, c, b, a\}$$

The order in which the elements of a set are listed is not important.

Thus, $A = B$

Hint: For the sets A and B to be equal, the elements from both the sets should be same and the order in which the elements are listed is not significant.

(ii) Step 1:

$$A = \{4, 8, 12, 16\}$$

$$B = \{8, 4, 16, 18\}$$

We can see that $12 \in A$ but $12 \notin B$.

Hence, A is not equal to B.

Hint: For the sets A and B to be equal, the elements from both the sets should be same and the order in which the elements are listed is not significant.

(iii) Step 1:

$$A = \{2, 4, 6, 8, 10\}$$

$$B = \{x: x \text{ is a positive even integer and } x \leq 10\}$$

$$B = \{2, 4, 6, 8, 10\}$$

Hence, $A = B$

Hint: For the sets A and B to be equal, the elements from both the sets should be same and the order in which the elements are listed is not significant.

(iv) Step 1:

$$A = \{x: x \text{ is a multiple of } 10\}$$

$$B = \{10, 15, 20, 25, 30, \dots\}$$

It is seen that $15 \in B$ but $15 \notin A$.

Therefore, A is not equal to B.

Hint: For the sets A and B to be equal, the elements from both the sets should be same and the order in which the elements are listed is not significant.

5. Are the following pair of sets equal? Give reasons.

(i) $A = \{2, 3\}$; $B = \{x: x \text{ is solution of } x^2 + 5x + 6 = 0\}$

(ii) $A = \{x: x \text{ is a letter in the word FOLLOW}\}$; $B = \{y: y \text{ is a letter in the word WOLF}\}$

Solution:

(i) Step 1:

$$A = \{2, 3\}$$

$$B = \{x: x \text{ is solution of } x^2 + 5x + 6 = 0\}$$

The equation $x^2 + 5x + 6 = 0$ can be solved as:

$$x^2 + 3x + 2x + 6 = 0$$

$$X(x + 3) + 2(x + 3) = 0$$

$$(x + 2)(x + 3) = 0$$

$$x = -2 \text{ or } x = -3$$

$$\Rightarrow B = \{-2, -3\}$$

$$A = \{2, 3\}$$

$$B = \{-2, -3\}$$

Thus, A is not equal to B .

Hint: For the sets A and B to be equal, the elements from both the sets should be same and the order in which the elements are listed is not significant.

(ii) Step 1:

$$A = \{x: x \text{ is a letter in the word FOLLOW}\}$$

$$A = \{F, O, L, W\}$$

$$B = \{y: y \text{ is a letter in the word WOLF}\}$$

$$B = \{W, O, L, F\}$$

The order in which the elements of a set are listed is not important.

Therefore, $A = B$

Hint: For the sets A and B to be equal, the elements from both the sets should be same and the order in which the elements are listed is not significant.

6. From the sets given below, select equal sets:

$$A = \{2, 4, 8, 12\}, B = \{1, 2, 3, 4\}, C = \{4, 8, 12, 14\}, D = \{3, 1, 4, 2\}$$

$$E = \{-1, 1\}, F = \{0, a\}, G = \{1, -1\}, H = \{0, 1\}$$

Solution:

Given, Step

1:

$$A = \{2, 4, 8, 12\}, B = \{1, 2, 3, 4\}, C = \{4, 8, 12, 14\}, D = \{3, 1, 4, 2\},$$

$E = \{-1, 1\}, F = \{0, a\}, G = \{1, -1\}, H = \{0, 1\}$ We can see that

$8 \in A, 8 \notin B, 8 \notin D, 8 \notin E, 8 \notin F, 8 \notin G, 8 \notin H,$
Therefore, $A \neq B, A \neq D, A \neq E, A \neq F, A \neq G, A \neq H$

Also, $2 \in A, 2 \notin C$

Therefore, $A \neq C$ Step

2:

$3 \in B, 3 \notin C, 3 \notin E, 3 \notin F, 3 \notin G, 3 \notin H$

$\therefore B \neq C, B \neq E, B \neq F, B \neq G, B \neq H$

$12 \in C, 12 \notin D, 12 \notin E, 12 \notin F, 12 \notin G, 12 \notin H,$

$\therefore C \neq D, C \neq E, C \neq F, C \neq G, C \neq H$

$4 \in D, 4 \notin E, 4 \notin F, 4 \notin G, 4 \notin H,$

$\therefore D \neq E, D \neq F, D \neq G, D \neq H$

Similarly, $E \neq F, E \neq G, E \neq H, F \neq G, F \neq H, G \neq H$

The order in which elements of a set are listed is not important.

Therefore, $B = D$ and $E = G$

Thus, from all given sets, $B = D$ and $E = G$

Hint: For the sets A and B to be equal, the elements from both the sets should be same and the order in which the elements are listed is not significant.

Exercise 11.3

I. Make correct statements by filling in the symbols \subset or $\not\subset$ in the blank spaces:

(i) $\{2, 3, 4\} \dots \{1, 2, 3, 4, 5\}$

(ii) $\{a, b, c\} \dots \{b, c, d\}$

(iii) $\{x: x \text{ is a student of Class XI of your school}\} \dots \{x: x \text{ student of your school}\}$

- (iv) $\{x: x \text{ is a circle in a plane}\} \dots \{x: x \text{ is a circle in the same plane with radius 1 unit}\}$
- (v) $\{x: x \text{ is a triangle in a plane}\} \dots \{x: x \text{ is a rectangle in the plane}\}$
- (vi) $\{x: x \text{ is an equilateral triangle in a plane}\} \dots \{x: x \text{ is a triangle in the same plane}\}$
- (vi) $\{x: x \text{ is an even natural number}\} \dots \{x: x \text{ is an integer}\}$

Solution:

(i) Step 1:

$$\{2,3, 4\} \subset \{1,2, 3,4,5\}$$

Hint: If every element of y is also the element of x ; We say that y is a subset of x . It is expressed as $y \subset x$.

(ii) Step 1:

$$\{a, b, c\} \not\subset \{b, c, d\}$$

Hint: If every element of y is also the element of x ; We say that y is a subset of x . It is expressed as $y \subset x$.

(iii) Step 1:

$$\{x: x \text{ is a student of class XI of your school}\} \subset \{x: x \text{ is a student of your school}\}$$

Hint: If every element of y is also the element of x ; We say that y is a subset of x . It is expressed as $y \subset x$.

(iv) Step 1:

$$\{x: x \text{ is a circle in the plane}\} \not\subset \{x: x \text{ is a circle in the same plane with radius 1 unit}\}$$

Hint: If every element of y is also the element of x ; We say that y is a subset of x . It is expressed as $y \subset x$.

(v) Step 1:

$$\{x: x \text{ is a triangle in a plane}\} \not\subset \{x: x \text{ is a rectangle in the plane}\}$$
 Hint:

(vi) Step 1:

$$\{x: x \text{ is an equilateral triangle in a plane}\} \subset \{x: x \text{ is a triangle in the same plane}\}$$

Hint: If every element of y is also the element of x ; We say that y is a subset of x . It is expressed as $y \subset x$.

(vii) Step 1:

$\{x: x \text{ is an even natural number}\} \subset \{x: x \text{ is an integer}\}$

Hint: If every element of y is also the element of x ; We say that y is a subset of x . It is expressed as $y \subset x$.

2. Examine whether the following statements are true or false:

(i) $\{a, b\} \not\subset \{b, c, a\}$

(ii) $\{a, e\} \subset \{x: x \text{ is a vowel in the English alphabet}\}$

(iii) $\{1, 2, 3\} \subset \{1, 3, 5\}$

(iv) $\{a\} \subset \{a, b, c\}$

(v) $\{a\} \in (a, b, c)$

(vi) $\{x: x \text{ is an even natural number less than } 6\} \subset \{x: x \text{ is a natural number which divides } 36\}$

Solution:

(i) Step 1:

False, as every element of $\{a, b\}$ is also an element of $\{b, c, a\}$.

Hint: If every element of y is also the element of x ; We say that y is a subset of x . It is expressed as $y \subset x$.

(ii) Step 1:

True, as a and e are two vowels of the English alphabet.

Hint: If every element of y is also the element of x ; We say that y is a subset of x . It is expressed as $y \subset x$.

(iii) Step 1:

False, as $2 \in \{1, 2, 3\}$ but $2 \notin \{1, 3, 5\}$

Hint: If every element of y is also the element of x ; We say that y is a subset of x . It is expressed as $y \subset x$.

(iv) Step 1:

True. Each element of $\{a\}$ is also an element of $\{a, b, c\}$

Hint: If every element of y is also the element of x ; We say that y is a subset of x . It is expressed as $y \subset x$.

(v) Step 1:

False. As the element of $\{a, b, c\}$ are a, b, c . Thus, $\{a\} \subset \{a, b, c\}$ Hint:

(vi) Step 1:

True. $\{x: x \text{ is an even natural number less than } 6\} = \{2, 4\}$

$\{x: x \text{ is a natural number which divides } 36\} = \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$

Therefore, $\{2, 4\} \subset \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$

Hint: If every element of y is also the element of x ; We say that y is a subset of x . It is expressed as $y \subset x$.

3. Let $A = \{1, 2, \{3, 4\}, 5\}$. Which of the following statements are incorrect and why?

(i) $\{3, 4\} \subset A$

(ii) $\{3, 4\} \in A$

(iii) $\{\{3, 4\}\} \subset A$

(iv) $1 \in A$

(v) $1 \subset A$

(vi) $\{1, 2, 5\} \subset A$

(vii) $\{1, 2, 5\} \in A$

(viii) $\{1, 2, 3\} \subset A$

(ix) $\phi \in A$

(x) $\phi \subset A$

(xi) $\{\phi\} \subset A$

Solution:

(i) Step 1:

The statement $\{3,4\} \subset A$ is wrong as $3 \in \{3,4\}$; but, $3 \notin A$.

Hint: If every element of y is also the element of x ; We say that y is a subset of x . It is expressed as $y \subset x$.

(ii) Step 1:

The statement $\{3,4\} \in A$ is correct as $\{3,4\}$ is an element of A .

:

(iii) Step 1:

The statement $\{\{3,4\}\} \subset A$ is correct as $\{3,4\} \in \{\{3,4\}\}$ and $\{3,4\} \in A$.

Hint: If every element of y is also the element of x ; We say that y is a subset of x . It is expressed as $y \subset x$.

(iv) Step 1:

The statement $1 \in A$ is correct as 1 is an element of A .

Hint: If every element of y is also the element of x ; We say that y is a subset of x . It is expressed as $y \subset x$.

(v) Step 1:

The statement $1 \subset A$ is incorrect as an element of a set can never be a subset of itself.

Hint: If every element of y is also the element of x ; We say that y is a subset of x . It is expressed as $y \subset x$.

(vi) Step 1:

The statement $\{1,2,5\} \subset A$ is correct because each element of set $\{1,2,5\}$ is an element of A . Hint:

If every element of y is also the element of x ; We say that y is a subset of x as $y \subset x$. It is expressed

(vii) Step 1:

The statement $\{1,2,5\} \in A$ is incorrect because $\{1,2,5\}$ is not an element of A .

Hint: If every element of y is also the element of x ; We say that y is a subset of x . It is expressed

as $y \subset x$.

(viii) Step 1:

The statement $\{1, 2, 3\} \subset A$ is incorrect as 3 is not an element of A .

Hint: If every element of y is also the element of x ; We say that y is a subset of x . It is expressed as $y \subset x$.

(ix) Step 1:

The statement $\phi \in A$ is incorrect because ϕ is not an element of A .

Hint: If every element of y is also the element of x ; We say that y is a subset of x . It is expressed as $y \subset x$.

(x) Step 1:

The statement $\phi \subset A$ is correct because ϕ is a subset of every set.

Hint: If every element of y is also the element of x ; We say that y is a subset of x . It is expressed as $y \subset x$.

(xi) Step 1:

The statement $\{\phi\} \subset A$ is incorrect because ϕ is a subset of A , not an element of A .

Hint: If every element of y is also the element of x ; We say that y is a subset of x . It is expressed as $y \subset x$.

4. Write down all the subsets of the following sets:

(i) $\{a\}$

(ii) $\{a, b\}$

(iii) $\{1, 2, 3\}$

(iii) ϕ

Solution:

(i) Step 1:

The subsets of $\{a\}$ are ϕ and $\{a\}$.

Hint: If every element of y is also the element of x ; We say that y is a subset of x . It is expressed as $y \subset x$.

(ii) Step 1:

The subsets of $\{a, b\}$ are ϕ , $\{a\}$, $\{b\}$ and $\{a, b\}$.

Hint: If every element of y is also the element of x ; We say that y is a subset of x . It is expressed as $y \subset x$.

(iii) Step 1:

The subsets of $\{1, 2, 3\}$ are ϕ , $\{1\}$, $\{2\}$, $\{3\}$, $\{1, 2\}$, $\{2, 3\}$, $\{3, 1\}$ and $\{1, 2, 3\}$.

Hint: If every element of y is also the element of x ; We say that y is a subset of x . It is expressed as $y \subset x$.

(iv) Step 1:

The subsets of ϕ is ϕ alone.

Hint: If every element of y is also the element of x ; We say that y is a subset of x . It is expressed as $y \subset x$.

5. How many elements has $P(A)$, if $A = \phi$?

Solution:

Step 1:

As we know, if A is a set with m elements then number of elements in $P(A)$ is 2^m .

Given, $A = \phi$

Number of elements in $A = 0$

So, number of elements in $P(A) = 2^0 = 1$ Hence,
 $P(A)$ has one element.

The collection of all subsets of a set A is called the power set of A . It is denoted by $P(A)$. In $P(A)$, every element is a set. $n[P(A)] = 2^m$ where, $n(A) = m$

6. Write the following as intervals:

- (i) $\{x: x \in R, -4 < x \leq 6\}$
- (ii) $\{x: x \in R, -12 < x < -10\}$
- (iii) $\{x: x \in R, 0 \leq x < 7\}$
- (v) $\{x: x \in R, 3 \leq x \leq 4\}$

Solution:

(i) Step 1:

$$\{x: x \in R, -4 < x \leq 6\} = (-4, 6]$$

Hint: The interval which contains the end points also is called closed interval and is denoted by $[a, b]$. We also have intervals closed at one end and open at the other, i.e., $[a, b) = \{x: a \leq x < b\}$ is an open interval from a to b , including a but excluding b .

(ii) Step 1:

$$\{x: x \in R, -12 < x < -10\} = (-12, -10)$$

Hint: The interval which contains the end points also is called closed interval and is denoted by $[a, b]$. We also have intervals closed at one end and open at the other, i.e., $[a, b) = \{x: a \leq x < b\}$ is an open interval from a to b , including a but excluding b .

(iii) Step 1:

$$\{x: x \in R, 0 \leq x < 7\} = [0, 7)$$

Hint: The interval which contains the end points also is called closed interval and is denoted by $[a, b]$. We also have intervals closed at one end and open at the other, i.e., $[a, b) = \{x: a \leq x < b\}$ is an open interval from a to b , including a but excluding b .

(iv) Step 1:

$$\{x: x \in R, 3 \leq x \leq 4\} = [3, 4]$$

The interval which contains the end points also is called closed interval and is denoted by $[a, b]$. We also have intervals closed at one end and open at the other, i.e., $[a, b) = \{x: a \leq x < b\}$ is an open interval from a to b , including a but excluding b .

7. Write the following intervals in set-builder form:

(i) $(-3, 0)$

(ii) $[6, 12]$

(iii) $(6, 12]$

(v) $[-23, 5)$

Solution:

(i) Step 1:

$$(-3, 0) = \{x: x \in R, -3 < x < 0\}$$

Hint: In set builder form, all the elements of a set possess a single common property which is not possessed by any element outside the set.

(ii) Step 1:

$$[6, 12] = \{x: x \in R, 6 \leq x \leq 12\}$$

Hint: In set builder form, all the elements of a set possess a single common property which is not possessed by any element outside the set.

(iii) Step 1:

$$(6, 12] = \{x: x \in R, 6 < x \leq 12\}$$

Hint: In set builder form, all the elements of a set possess a single common property which is not possessed by any element outside the set.

(iv) Step 1:

$$[-23,5) = \{x: x \in R, -23 \leq x < 5\}$$

In set builder form, all the elements of a set possess a single common property which is not possessed by any element outside the set.

8. What universal set(s) would you propose for each of the following?

- (i) The set of right triangles
- (ii) The set of isosceles triangles

Solution:

1. Step 1:

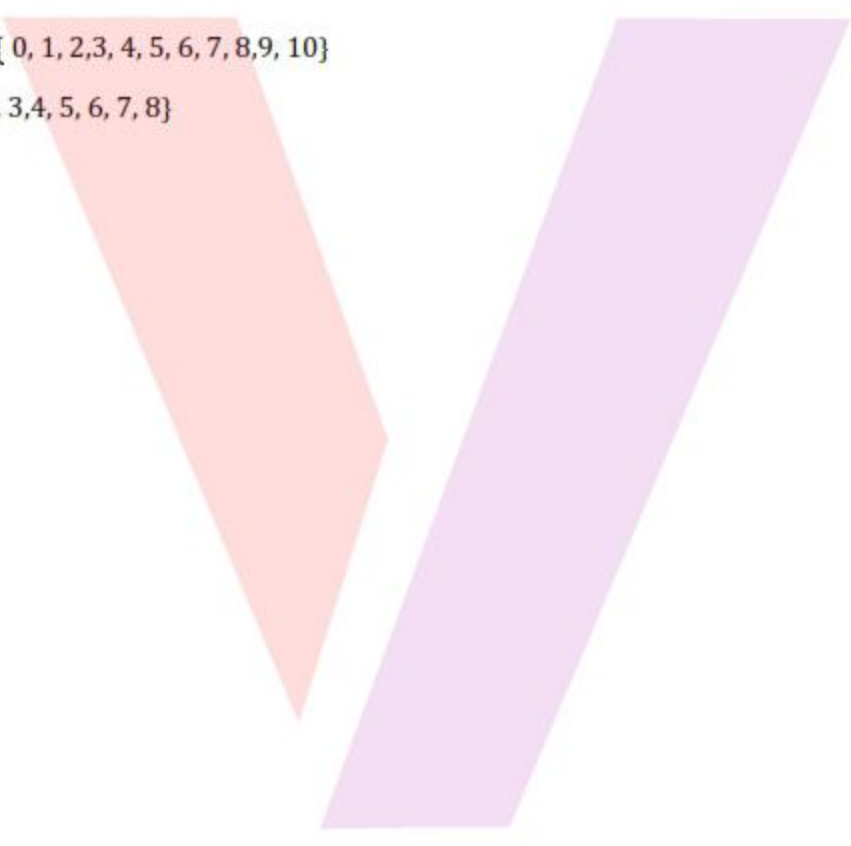
The universal set for the set of right triangles can be the set of triangles or the set of polygons.

Hint: Universal set is a set containing all objects or elements and of which all other sets are subsets. It is denoted by U .

2. Step 1:

The universal set for the set of isosceles triangles can be the set of triangles or the set of polygons or the set of two dimensional figures.

Hint: Universal set is a set containing all objects or elements and of which all other sets are subsets. It is denoted by U .

9. Given the sets $A = \{1,3, 5\}$, $B = \{2,4, 6\}$ and $C = \{0,2, 4, 6, 8\}$, which of the following may be considered as universal set (s) for all the three sets A, B and C
- (i) $\{0, 1, 2,3, 4, 5, 6\}$
 - (ii) ϕ
 - (iii) $\{0, 1, 2,3, 4, 5, 6, 7, 8,9, 10\}$
 - (iv) $\{1, 2, 3,4, 5, 6, 7, 8\}$
- 

Solution:

(i) Step 1:

We can see that $A \subset \{0,1, 2, 3, 4, 5,6\}$

$B \subset \{0,1, 2, 3, 4, 5, 6\}$

But, $C \not\subset \{0, 1, 2,3, 4, 5, 6\}$ as 8 does not exist in $\{0, 1, 2,3, 4, 5, 6\}$

Thus, the set $\{0,1, 2, 3, 4,5, 6\}$ cannot be the universal set for all the three sets A, B and C .

Hint: Universal set is a set containing all objects or elements and of which all other sets are subsets. It is denoted by U .

(ii) Step 1:

$A \not\subset \phi, B \not\subset \phi, C \not\subset \phi$

Thus, ϕ cannot be the universal set for all the three sets A, B and C .

Hint: Universal set is a set containing all objects or elements and of which all other sets are subsets. It is denoted by U .

(iii) Step 1:

We can see that $A \subset \{0,1, 2, 3, 4, 5,6, 7, 8, 9, 10\}$

$B \subset \{0,1, 2, 3, 4, 5, 6,7, 8, 9, 10\}$

$C \subset \{0,1, 2, 3, 4, 5, 6,7, 8, 9, 10\}$

Thus, the set $\{0,1, 2, 3, 4,5, 6, 7, 8, 9, 10\}$ is the universal set for all the three sets A, B and C .

Hint: Universal set is a set containing all objects or elements and of which all other sets are subsets. It is denoted by U .

(iv) Step 1:

We can see that $A \subset \{1,2, 3, 4, 5, 6,7, 8\}$

$B \subset \{1,2, 3, 4, 5, 6, 7,8\}$

But, $C \not\subset \{1, 2, 3,4, 5, 6, 7, 8\}$ as 0 does not exist in $\{1, 2,3, 4, 5, 6, 7, 8\}$

Therefore, the set $\{1, 2, 3, 4, 5, 6, 7, 8\}$ cannot be the universal set for all the three sets A , B and C .

Hint: Universal set is a set containing all objects or elements and of which all other sets are subsets. It is denoted by U .

Exercise 11.4

1. Find the union of each of the following pairs of sets:

(i). $X = \{1, 3, 5\}; Y = \{1, 2, 3\}$

(ii). $A = \{a, e, i, o, u\}; B = \{a, b, c\}$

(iii) $A = \{x: x \text{ is a natural number and multiple of } 3\}; B = \{x: x \text{ is a natural number less than } 6\}$

(v) $A = \{x: x \text{ is a natural number and } 1 < x \leq 6\}; B = \{x: x \text{ is a natural number and } 6 < x < 10\}$

(vi) $A = \{1, 2, 3\}; B = \phi$

Solution:

(i) Step 1:

$$X = \{1, 3, 5\}$$

$$Y = \{1, 2, 3\}$$

$$\text{Therefore, } X \cup Y = \{1, 2, 3, 5\}$$

Hint: If A and B are any two sets. The union of A and B is a set which consists of all the elements of A and all the elements of B . A union B is represented as $A \cup B$.

(ii) Step 1:

$$A = \{a, e, i, o, u\}$$

$$B = \{a, b, c\}$$

$$\text{Therefore, } A \cup B = \{a, b, c, e, i, o, u\}$$

Hint: If A and B are any two sets. The union of A and B is a set which consists of all the elements of A and all the elements of B . A union B is represented as $A \cup B$.

(iii) Step 1:

$$A = \{x: x \text{ is a natural number and multiple of } 3\}$$

$$B = \{x: x \text{ is a natural number less than } 6\}$$

$$A = \{3, 6, 9, 12, \dots\}$$

$$B = \{1, 2, 3, 4, 5\}$$

$$\text{Therefore, } A \cup B = \{1, 2, 3, 4, 5, 6, 9, 12, \dots\}$$

Hint: If A and B are any two sets. The union of A and B is a set which consists of all the elements of A and all the elements of B . A union B is represented as $A \cup B$.

(iv) Step 1:

$$A = \{x: x \text{ is a natural number and } 1 < x \leq 6\}$$

$$B = \{x: x \text{ is a natural number and } 6 < x < 10\}$$

$$A = \{2, 3, 4, 5, 6\}$$

$$B = \{7, 8, 9\}$$

$$A \cup B = \{2, 3, 4, 5, 6, 7, 8, 9\}$$

$$\text{Therefore, } A \cup B = \{x: x \text{ is a natural number and } 1 < x < 10\}$$

Hint: If A and B are any two sets. The union of A and B is a set which consists of all the elements of A and all the elements of B . A union B is represented as $A \cup B$.

(v) Step 1:

$$A = \{1, 2, 3\}$$

$$B = \phi$$

$$\text{Therefore, } A \cup B = \{1, 2, 3\}$$

Hint: If A and B are any two sets. The union of A and B is a set which consists of all the elements of A and all the elements of B . A union B is represented as $A \cup B$.

2. Let $A = \{a, b\}$, $B = \{a, b, c\}$. Is $A \subset B$? What is $A \cup B$?

Solution:

Here, $A = \{a, b\}$ and $B = \{a, b, c\}$ Step

1:

Every element in set A exist in set B

Therefore it is clearly seen that, $A \subset B$

$$\text{And, } A \cup B = \{a, b, c\} = B$$

Hint: If A and B are any two sets. The union of A and B is a set which consists of all the elements of A and all the elements of B . A union B is represented as $A \cup B$.

3. If A and B are two sets such that $A \subset B$, then what is $A \cup B$?

Solution:

Step 1:

If A and B are two sets such that $A \subset B$, then $A \cup B = B$.

Hint: If A and B are any two sets. The union of A and B is a set which consists of all the elements of A and all the elements of B . A union B is represented as $A \cup B$.

4. If $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, $C = \{5, 6, 7, 8\}$ and $D = \{7, 8, 9, 10\}$; find

- (i) $A \cup B$
- (ii) $A \cup C$
- (iii) $B \cup C$
- (iv) $B \cup D$
- (v) $A \cup B \cup C$
- (vi) $A \cup B \cup D$
- (vi) $B \cup C \cup D$

Solution:

$A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, $C = \{5, 6, 7, 8\}$ and $D = \{7, 8, 9, 10\}$

(i) Step 1:

$$A \cup B = \{1, 2, 3, 4, 5, 6\}$$

Hint: If A and B are any two sets. The union of A and B is a set which consists of all the elements of A and all the elements of B . A union B is represented as $A \cup B$.

(ii) Step 1:

$$A \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

Hint: If A and B are any two sets. The union of A and B is a set which consists of all the elements of A and all the elements of B . A union B is represented as $A \cup B$.

(iii) Step 1:

$$B \cup C = \{3, 4, 5, 6, 7, 8\}$$

Hint: If A and B are any two sets. The union of A and B is a set which consists of all the elements of A and all the elements of B . A union B is represented as $A \cup B$.

(iv) Step 1:

$$B \cup D = \{3, 4, 5, 6, 7, 8, 9, 10\}$$

Hint: If A and B are any two sets. The union of A and B is a set which consists of all the elements of A and all the elements of B . A union B is represented as $A \cup B$.

(v) Step 1:

$$A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

Hint: If A and B are any two sets. The union of A and B is a set which consists of all the elements of A and all the elements of B . A union B is represented as $A \cup B$.

(vi) Step 1:

$$A \cup B \cup D = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

Hint: If A and B are any two sets. The union of A and B is a set which consists of all the elements of A and all the elements of B . A union B is represented as $A \cup B$.

(vii) Step 1:

$$B \cup C \cup D = \{3, 4, 5, 6, 7, 8, 9, 10\}$$

Hint: If A and B are any two sets. The union of A and B is a set which consists of all the elements of A and all the elements of B . A union B is represented as $A \cup B$.

5. Find the intersection of each pair of sets:

(i) $X = \{1, 3, 5\}$ $Y = \{1, 2, 3\}$

(ii) $A = \{a, e, i, o, u\}$ $B = \{a, b, c\}$

(iii) $A = \{x: x \text{ is a natural number and multiple of } 3\}$

$$B = \{x: x \text{ is a natural number less than } 6\}$$

(iv) $A = \{x: x \text{ is a natural number and } 1 < x \leq 6\}$

$$B = \{x: x \text{ is a natural number and } 6 < x < 10\}$$

(vii) $A = \{1, 2, 3\}$, $B = \phi$

Solution:

(i) Step 1:

$$X = \{1, 3, 5\}$$

$$Y = \{1, 2, 3\}$$

$$\text{Therefore, } X \cap Y = \{1, 3\}$$

Hint: If A and B are any two sets. The intersection of A and B is a set all the elements which are common in both A and B . A Intersection B is represented as $A \cap B$.

(ii) Step 1:

$$A = \{a, e, i, o, u\}$$

$$B = \{a, b, c\}$$

$$\text{Therefore, } A \cap B = \{a\}$$

Hint: If A and B are any two sets. The intersection of A and B is a set all the elements which are common in both A and B . A Intersection B is represented as $A \cap B$.

(iii) Step 1:

$$A = \{x: x \text{ is a natural number and multiple of } 3\}$$

$$B = \{x: x \text{ is a natural number less than } 6\}$$

$$A = \{3, 6, 9, 12 \dots\}$$

$$B = \{1, 2, 3, 4, 5\}$$

$$\text{Therefore, } A \cap B = \{3\}$$

Hint: If A and B are any two sets. The intersection of A and B is a set all the elements which are common in both A and B . A Intersection B is represented as $A \cap B$.

(iv) Step 1:

$$A = \{x: x \text{ is a natural number and } 1 < x \leq 6\}$$

$$B = \{x: x \text{ is a natural number and } 6 < x < 10\}$$

$$A = \{2, 3, 4, 5, 6\}$$

$$B = \{7, 8, 9\}$$

$$\text{Therefore, } A \cap B = \phi$$

Hint: If A and B are any two sets. The intersection of A and B is a set all the elements which are common in both A and B . A Intersection B is represented as $A \cap B$.

(v) Step 1:

$$A = \{1, 2, 3\}$$

$$B = \phi$$

$$\text{Therefore, } A \cap B = \phi$$

Hint: If A and B are any two sets. The intersection of A and B is a set all the elements which are common in both A and B . A Intersection B is represented as $A \cap B$.

6. If $A = \{3, 5, 7, 9, 11\}$, $B = \{7, 9, 11, 13\}$, $C = \{11, 13, 15\}$ and $D = \{15, 17\}$; find

(i) $A \cap B$

(ii) $B \cap C$

(iii) $A \cap C \cap D$

(iv) $A \cap C$

(v) $B \cap D$

(vi) $A \cap (B \cup C)$ (vii) $A \cap D$

(viii) $A \cap (B \cup D)$

(ix) $(A \cap B) \cap (B \cup C)$

(x) $(A \cup D) \cap (B \cup C)$

Solution:

$$A = \{3, 5, 7, 9, 11\}, B = \{7, 9, 11, 13\}, C = \{11, 13, 15\} \text{ and } D = \{15, 17\}$$

(i) Step 1:

$$A = \{3, 5, 7, 9, 11\}, B = \{7, 9, 11, 13\},$$

$$A \cap B = \{7, 9, 11\}$$

Hint: If A and B are any two sets. The intersection of A and B is a set all the elements which are common in both A and B . A Intersection B is represented as $A \cap B$.

(ii) Step 1:

$$B = \{7, 9, 11, 13\}, C = \{11, 13, 15\}$$

$$B \cap C = \{11, 13\}$$

Hint: If A and B are any two sets. The intersection of A and B is a set all the elements which are common in both A and B . A Intersection B is represented as $A \cap B$.

(iii) Step 1:

$$A = \{3, 5, 7, 9, 11\}, C = \{11, 13, 15\} \text{ And } D = \{15, 17\}$$

$$A \cap C \cap D = \phi$$

Hint: If A and B are any two sets. The intersection of A and B is a set all the elements which are common in both A and B . A Intersection B is represented as $A \cap B$.

(iv) Step 1:

$$A = \{3, 5, 7, 9, 11\}, C = \{11, 13, 15\}$$

$$A \cap C = \{11\}$$

Hint: If A and B are any two sets. The intersection of A and B is a set all the elements which are common in both A and B . A Intersection B is represented as $A \cap B$.

(v) Step 1:

$$B = \{7, 9, 11, 13\}, D = \{15, 17\}$$

$$B \cap D = \phi$$

Hint: If A and B are any two sets. The intersection of A and B is a set all the elements which are common in both A and B . A Intersection B is represented as $A \cap B$.

(vi) Step 1:

$$B = \{7, 9, 11, 13\}, C = \{11, 13, 15\}$$

$$B \cup C = \{7, 9, 11, 13, 15\}$$

$$A = \{3, 5, 7, 9, 11\}$$

$$A \cap (B \cup C) = \{7, 9, 11\}$$

Hint: : If A and B are any two sets. The intersection of A and B is a set all the elements which are common in both A and B . A Intersection B is represented as $A \cap B$.

If A and B are any two sets. The union of A and B is a set which consists of all the elements of A and all the elements of B . A union B is represented as $A \cup B$.

(vii) Step 1:

$$A = \{3, 5, 7, 9, 11\}, D = \{15, 17\}$$

$$A \cap D = \phi$$

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Hint: If A and B are any two sets. The intersection of A and B is a set all the elements which are common in both A and B . A Intersection B is represented as $A \cap B$.

(viii) Step 1:

$$B = \{7, 9, 11, 13\}, D = \{15, 17\}$$

$$B \cup D = \{7, 9, 11, 13, 15, 17\}$$

$$A = \{3, 5, 7, 9, 11\}$$

$$A \cap (B \cup D) = \{7, 9, 11\}$$

Hint: If A and B are any two sets. The intersection of A and B is a set all the elements which are common in both A and B . A Intersection B is represented as $A \cap B$.

If A and B are any two sets. The union of A and B is a set which consists of all the elements of A and all the elements of B . A union B is represented as $A \cup B$.

(ix) Step 1:

$$A = \{3, 5, 7, 9, 11\}, B = \{7, 9, 11, 13\}, C = \{11, 13, 15\}$$

$$A \cap B = \{7, 9, 11\}$$

$$B \cup C = \{7, 9, 11, 13, 15\}$$

$$(A \cap B) \cap (B \cup C) = \{7, 9, 11\}$$

Hint: If A and B are any two sets. The intersection of A and B is a set all the elements which are common in both A and B . A Intersection B is represented as $A \cap B$.

If A and B are any two sets. The union of A and B is a set which consists of all the elements of A and all the elements of B . A union B is represented as $A \cup B$.

(x) Step 1:

$$A = \{3, 5, 7, 9, 11\}, D = \{15, 17\}, C = \{11, 13, 15\}, B = \{7, 9, 11, 13\}$$

$$A \cup D = \{3, 5, 7, 9, 11, 15, 17\}$$

$$B \cup C = \{7, 9, 11, 13, 15\}$$

$$(A \cup D) \cap (B \cup C) = \{7, 9, 11, 15\}$$

Hint: If A and B are any two sets. The intersection of A and B is a set all the elements which are common in both A and B . A Intersection B is represented as $A \cap B$.

If A and B are any two sets. The union of A and B is a set which consists of all the elements of A and all the elements of B . A union B is represented as $A \cup B$.

7. If $A = \{x: x \text{ is a natural number}\}$, $B = \{x: x \text{ is an even natural number}\}$
 $C = \{x: x \text{ is an odd natural number}\}$ and $D = \{x: x \text{ is a prime number}\}$, find

(i) $A \cap B$

(ii) $A \cap C$

(iii) $A \cap D$

(iv) $B \cap C$

(v) $B \cap D$

(vi) $C \cap D$

Solution:

$$A = \{x: x \text{ is a natural number}\} = \{1, 2, 3, 4 \dots\}$$

$$B = \{x: x \text{ is an even natural number}\} = \{2, 4, 6, 8 \dots\}$$

$$C = \{x: x \text{ is an odd natural number}\} = \{1, 3, 5, 7 \dots\}$$

$$D = \{x: x \text{ is a prime number}\} = \{2, 3, 5, 7 \dots\}$$

(i) Step 1:

$$A = \{1, 2, 3, 4 \dots\}, B = \{2, 4, 6, 8 \dots\}$$

$$A \cap B = \{2, 4, 6, 8 \dots\} = \{x: x \text{ is an even natural number}\} = B$$

Hint: If A and B are any two sets. The intersection of A and B is a set all the elements which are common in both A and B . A Intersection B is represented as $A \cap B$.

(ii) Step 1:

$$A = \{1, 2, 3, 4 \dots\}, C = \{1, 3, 5, 7 \dots\}$$

$$A \cap C = \{1, 3, 5, 7 \dots\} = \{x: x \text{ is an odd natural number}\} = C$$

Hint: If A and B are any two sets. The intersection of A and B is a set all the elements which are common in both A and B . A Intersection B is represented as $A \cap B$.

(i) Step 1:

$$A = \{1, 2, 3, 4, \dots\}, D = \{2, 3, 5, 7, \dots\}$$

$$A \cap D = \{2, 3, 5, 7, \dots\} = \{x: x \text{ is a prime number}\} = D$$

Hint: If A and B are any two sets. The intersection of A and B is a set all the elements which are common in both A and B . A Intersection B is represented as $A \cap B$.

(ii) Step 1:

$$B = \{2, 4, 6, 8, \dots\}, C = \{1, 3, 5, 7, \dots\}$$

$$B \cap C = \phi$$

Hint: If A and B are any two sets. The intersection of A and B is a set all the elements which are common in both A and B . A Intersection B is represented as $A \cap B$.

(iii) Step 1:

$$B = \{2, 4, 6, 8, \dots\}, D = \{2, 3, 5, 7, \dots\}$$

$$B \cap D = \{2\}$$

Hint: If A and B are any two sets. The intersection of A and B is a set all the elements which are common in both A and B . A Intersection B is represented as $A \cap B$.

(iv) Step 1:

$$C = \{1, 3, 5, 7, \dots\}, D = \{2, 3, 5, 7, \dots\}$$

$$C \cap D = \{3, 5, 7, 11, 13, 17, \dots\} = \{x: x \text{ is an odd prime number}\} = D$$

Hint: If A and B are any two sets. The intersection of A and B is a set all the elements which are common in both A and B . A Intersection B is represented as $A \cap B$.

8. Which of the following pairs of sets are disjoint

- (i) $\{1, 2, 3, 4\}$ and $\{x: x \text{ is a natural number and } 4 \leq x \leq 6\}$
- (ii) $\{a, e, i, o, u\}$ and $\{c, d, e, f\}$
- (iii) $\{x: x \text{ is an even integer}\}$ and $\{x: x \text{ is an odd integer}\}$

Solution:

- (i) Step 1:

Consider, $\{1, 2, 3, 4\}$

$\{x: x \text{ is a natural number and } 4 \leq x \leq 6\} = \{4, 5, 6\}$

Now, $\{1, 2, 3, 4\} \cap \{4, 5, 6\} = \{4\}$

Thus, this pair of sets is not disjoint.

Hint: If A and B are two sets such that $A \cap B = \phi$, then A and B are called disjoint sets.

- (ii) Step 1:

Given, $\{a, e, i, o, u\} \cap \{c, d, e, f\} = \{e\}$

Thus, this pair of sets is not disjoint.

Hint: If A and B are two sets such that $A \cap B = \phi$, then A and B are called disjoint sets.

- (iii) Step 1:

$\{x: x \text{ is an even integer}\} \cap \{x: x \text{ is an odd integer}\} = \phi$

Thus, this pair of sets is disjoint.

Hint: If A and B are two sets such that $A \cap B = \phi$, then A and B are called disjoint sets.

9. If $A = \{3, 6, 9, 12, 15, 18, 21\}$, $B = \{4, 8, 12, 16, 20\}$, $C = \{2, 4, 6, 8, 10, 12, 14, 16\}$,

$D = \{5, 10, 15, 20\}$; Find

(i) $A - B$

(ii) $A - C$

(iii) $A - D$

(iv) $B - A$

(v) $C - A$

(vi) $D - A$

(vii) $B - C$

(viii) $B - D$

(ix) $C - B$

(x) $D - B$

(xi) $C - D$

(xii) $D - C$

Solution:

(i) Step 1:

$$A = \{3, 6, 9, 12, 15, 18, 21\}, B = \{4, 8, 12, 16, 20\},$$

$$A - B = \{3, 6, 9, 15, 18, 21\}$$

Hint: Difference of the sets A and B in this order is the set of elements which belong to A but not to B . Symbolically, we write $A - B$ and read as A minus B .

(ii) Step 1:

$$A = \{3, 6, 9, 12, 15, 18, 21\}, C = \{2, 4, 6, 8, 10, 12, 14, 16\}$$

$$A - C = \{3, 9, 15, 18, 21\}$$

Hint: Difference of the sets A and B in this order is the set of elements which belong to A but not to B . Symbolically, we write $A - B$ and read as A minus B .

(iii) Step 3:

$$A = \{3, 6, 9, 12, 15, 18, 21\}, D = \{5, 10, 15, 20\}$$

$$A - D = \{3, 6, 9, 12, 18, 21\}$$

Hint: Difference of the sets A and B in this order is the set of elements which belong to A but not to B . Symbolically, we write $A - B$ and read as A minus B .

(iv) Step 1:

$$A = \{3, 6, 9, 12, 15, 18, 21\}, B = \{4, 8, 12, 16, 20\}$$

$$B - A = \{4, 8, 16, 20\}$$

Hint: Difference of the sets A and B in this order is the set of elements which belong to A but not to B . Symbolically, we write $A - B$ and read as A minus B .

(v) Step 1:

$$A = \{3, 6, 9, 12, 15, 18, 21\}, C = \{2, 4, 6, 8, 10, 12, 14, 16\}$$

$$C - A = \{2, 4, 8, 10, 14, 16\}$$

Hint: Difference of the sets A and B in this order is the set of elements which belong to A but not to B . Symbolically, we write $A - B$ and read as A minus B .

(vi) Step 1:

$$A = \{3, 6, 9, 12, 15, 18, 21\}, D = \{5, 10, 15, 20\}$$

$$D - A = \{5, 10, 20\}$$

Hint: Difference of the sets A and B in this order is the set of elements which belong to A but not to B . Symbolically, we write $A - B$ and read as A minus B .

(vii) Step 1:

$$B = \{4, 8, 12, 16, 20\}, C = \{2, 4, 6, 8, 10, 12, 14, 16\}$$

$$B - C = \{20\}$$

Hint: Difference of the sets A and B in this order is the set of elements which belong to A but not to B . Symbolically, we write $A - B$ and read as A minus B .

(viii) Step 1:

$$B = \{4, 8, 12, 16, 20\}, D = \{5, 10, 15, 20\}$$

$$B - D = \{4, 8, 12, 16\}$$

Hint: Difference of the sets A and B in this order is the set of elements which belong to A but not to B . Symbolically, we write $A - B$ and read as A minus B .

(ix) Step 1:

$$C = \{2, 4, 6, 8, 10, 12, 14, 16\}, B = \{4, 8, 12, 16, 20\}$$

$$C - B = \{2, 6, 10, 14\}$$

Hint: Difference of the sets A and B in this order is the set of elements which belong to A but not to B . Symbolically, we write $A - B$ and read as A minus B .

(x) Step 1:

$$B = \{4, 8, 12, 16, 20\}, D = \{5, 10, 15, 20\}$$

$$D - B = \{5, 10, 15\}$$

Hint: Difference of the sets A and B in this order is the set of elements which belong to A but not to B . Symbolically, we write $A - B$ and read as A minus B .

(xi) Step 1:

$$C = \{2, 4, 6, 8, 10, 12, 14, 16\}, D = \{5, 10, 15, 20\}$$

$$C - D = \{2, 4, 6, 8, 12, 14, 16\}$$

Hint: Difference of the sets A and B in this order is the set of elements which belong to A but not to B . Symbolically, we write $A - B$ and read as A minus B .

(xii) Step 1:

$$D = \{5, 10, 15, 20\}, C = \{2, 4, 6, 8, 10, 12, 14, 16\}$$

$$D - C = \{5, 15, 20\}$$

Hint: Difference of the sets A and B in this order is the set of elements which belong to A but not to B . Symbolically, we write $A - B$ and read as A minus B .

10. If $X = \{a, b, c, d\}$ and $Y = \{f, b, d, g\}$, find

(i) $X - Y$

(ii) $Y - X$

(iii) $X \cap Y$

Solution:

(i) Step 1:

$$X = \{a, b, c, d\} \text{ and } Y = \{f, b, d, g\}$$

$$X - Y = \{a, c\}$$

Hint: Difference of the sets A and B in this order is the set of elements which belong to A but not to B . Symbolically, we write $A - B$ and read as A minus B .

(ii) Step 2:

$$X = \{a, b, c, d\} \text{ and } Y = \{f, b, d, g\}$$

$$Y - X = \{f, g\}$$

Mark 1: 1

Hint: Difference of the sets A and B in this order is the set of elements which belong to A but not to B . Symbolically, we write $A - B$ and read as A minus B .

(iii) Step 1:

$$X = \{a, b, c, d\} \text{ and } Y = \{f, b, d, g\}$$

$$X \cap Y = \{b, d\}$$

Hint: Difference of the sets A and B in this order is the set of elements which belong to A but not to B . Symbolically, we write $A - B$ and read as A minus B .

11. If R is the set of real numbers and Q is the set of rational numbers, then what is $R - Q$?

Solution:

Step 1:

R : set of real numbers

Q : set of rational numbers

$R - Q$: set of irrational numbers

Hint: Difference of the sets A and B in this order is the set of elements which belong to A but not to B . Symbolically, we write $A - B$ and read as A minus B .

12. State whether each of the following statement is true or false. Justify your answer.

- (i) $\{2, 3, 4, 5\}$ and $\{3, 6\}$ are disjoint sets
- (ii) $\{a, e, i, o, u\}$ and $\{a, b, c, d\}$ are disjoint sets
- (iii) $\{2, 6, 10, 14\}$ and $\{3, 7, 11, 15\}$ are disjoint sets
- (iv) $\{2, 6, 10\}$ and $\{3, 7, 11\}$ are disjoint sets

Solution:

(i) Step 1:

False.

As $3 \in \{2, 3, 4, 5\}$ and $3 \in \{3, 6\}$

$$\Rightarrow \{2, 3, 4, 5\} \cap \{3, 6\} = \{3\}$$

Therefore, they are not disjoint sets.

Hint: If A and B are two sets such that $A \cap B = \phi$, then A and B are called disjoint sets.

(ii) Step 1:

False.

As $a \in \{a, e, i, o, u\}$ and $a \in \{a, b, c, d\}$

$$\Rightarrow \{a, e, i, o, u\} \cap \{a, b, c, d\} = \{a\}$$

Therefore, they are not disjoint sets.

Hint: If A and B are two sets such that $A \cap B = \phi$, then A and B are called disjoint sets.

(iii) Step 1:

True.

As $\{2, 6, 10, 14\} \cap \{3, 7, 11, 15\} = \phi$

Therefore, they are disjoint sets

Hint: If A and B are two sets such that $A \cap B = \phi$, then A and B are called disjoint sets.

(iv) Step 1: True.

As $\{2, 6, 10\} \cap \{3, 7, 11\} = \phi$

Therefore, they are disjoint sets

: If A and B are two sets such that $A \cap B = \phi$, then A and B are called disjoint sets.**Exercise: 11.5**1. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6, 8\}$ and $C = \{3, 4, 5, 6\}$. Find(i) A' (ii) B'

(iii) $(A \cup C)'$ (iv) $(A \cup B)'$

(v) $(A)'$

(vi) $(B - C)'$

Solution:

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

$$A = \{1, 2, 3, 4\}$$

$$B = \{2, 4, 6, 8\}$$

$$C = \{3, 4, 5, 6\}$$

(i) Step 1:

$$A' = U - A$$

$$\Rightarrow U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - A = \{1, 2, 3, 4\}$$

$$A' = \{5, 6, 7, 8, 9\}$$

Hint: If A is a subset of the universal set U then its complement A' is also a subset of U .

(ii) Step 1:

$$B' = U - B$$

$$\Rightarrow U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - B = \{2, 4, 6, 8\}$$

$$B' = \{1, 3, 5, 7, 9\}$$

Hint:

(iii) Step 1: $A = \{1,$

$$2, 3, 4\}, C = \{3, 4, 5, 6\}$$

$$A \cup C = \{1, 2, 3, 4, 5, 6\} (A \cup C)' =$$

$$U - (A \cup C) = \{7, 8, 9\}$$

Mark 1: 1

Hint: If A is a subset of the universal set U then its complement A' is also a subset of U .

(iv) Step 1:

$$A = \{1, 2, 3, 4\}, B = \{2, 4, 6, 8\}$$

$$A \cup B = \{1, 2, 3, 4, 6, 8\}$$

$$(A \cup B)' = U - (A \cup B) = \{5, 7, 9\}$$

Hint: If A is a subset of the universal set U then its complement A' is also a subset of U .

(v) Step 1:

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

$$A = \{1, 2, 3, 4\}$$

$$(A)' = U - A$$

$$A' = \{5, 6, 7, 8, 9\}$$

$$= \{1, 2, 3, 4\}$$

Hint: If A is a subset of the universal set U then its complement A' is also a subset of U .

(vi) Step 1:

$$B = \{2, 4, 6, 8\}$$

$$C = \{3, 4, 5, 6\}$$

$$B - C = \{2, 8\}$$

$$(B - C)' = \{1, 3, 4, 5, 6, 7, 9\}$$

Hint: If A is a subset of the universal set U then its complement A' is also a subset of U .

2. If $U = \{a, b, c, d, e, f, g, h\}$, find the complements of the following sets:

(i) $A = \{a, b, c\}$

(ii) $B = \{d, e, f, g\}$

(iii) $C = \{a, c, e, g\}$

(v) $D = \{f, g, h, a\}$

Solution:

$U = \{a, b, c, d, e, f, g, h\}$ (i)

Step 1:

$$A = \{a, b, c\}$$

$$A' = U - A = \{d, e, f, g, h\}$$

Hint: If A is a subset of the universal set U then its complement A' is also a subset of U .

(ii) Step 1:

$$B = \{d, e, f, g\}$$

$$B' = U - B = \{a, b, c, h\}$$

Hint: If A is a subset of the universal set U then its complement A' is also a subset of U .

(iii) Step 1:

$$C = \{a, c, e, g\}$$

$$C' = U - C = \{b, d, f, h\}$$

Hint: If A is a subset of the universal set U then its complement A' is also a subset of U .

(iv) Step 1:
 $D = \{f, g, h, a\}$
 $D' = U - D = \{b, c, d, e\}$

Hint: If A is a subset of the universal set U then its complement A' is also a subset of U .

3. Taking the set of natural numbers as the universal set, write down the complements of the following sets:
- (i) $\{x: x \text{ is an even natural number}\}$
 - (ii) $\{x: x \text{ is an odd natural number}\}$
 - (iii) $\{x: x \text{ is a positive multiple of 3}\}$
 - (iv) $\{x: x \text{ is a prime number}\}$
 - (v) $\{x: x \text{ is a natural number divisible by 3 and 5}\}$
 - (vi) $\{x: x \text{ is a perfect square}\}$
 - (vii) $\{x: x \text{ is perfect cube}\}$
 - (viii) $\{x: x + 5 = 8\}$
 - (ix) $\{x: 2x + 5 = 9\}$
 - (x) $\{x: x \geq 7\}$
 - (xi) $\{x: x \in N \text{ and } 2x + 1 > 10\}$

Solution:

- i. Step 1:
 $\{x: x \text{ is an even natural number}\}' = \{x: x \text{ is an odd natural number}\}$
Hint: If A is a subset of the universal set U then its complement A' is also a subset of U .
- ii. Step 1:
 $\{x: x \text{ is an odd natural number}\}' = \{x: x \text{ is an even natural number}\}$
Hint: If A is a subset of the universal set U then its complement A' is also a subset of U .
- iii. Step 1:
 $\{x: x \text{ is a positive multiple of 3}\}' = \{x: x \in N \text{ and } x \text{ is not a multiple of 3}\}$

Hint: If A is a subset of the universal set U then its complement A' is also a subset of U .

iv. Step 1:

$$\{x: x \text{ is a prime number}\}' = \{x: x \text{ is a positive composite number and } x \neq 1\}$$

Hint: If A is a subset of the universal set U then its complement A' is also a subset of U . v.

Step 1:

$$\{x: x \text{ is a natural number divisible by 3 and 5}\}' = \{x: x \text{ is a natural number that is not divisible by 3 and 5}\}$$

Hint: If A is a subset of the universal set U then its complement A' is also a subset of U .

vi. Step 1:

$$\{x: x \text{ is a perfect square}\}' = \{x: x \in N \text{ and } x \text{ is not a perfect square}\}$$

Hint: If A is a subset of the universal set U then its complement A' is also a subset of U .

vii. Step 1:

$$\{x: x \text{ is a perfect cube}\}' = \{x: x \in N \text{ and } x \text{ is not a perfect cube}\}$$

Hint: If A is a subset of the universal set U then its complement A' is also a subset of U .

viii. Step 1:

$$\{x: x + 5 = 8\}' = \{x: x \in N \text{ and } x \text{ is a natural number and } x \neq 3\}$$

Hint: If A is a subset of the universal set U then its complement A' is also a subset of U .

ix. p 1:

$$\{x: 2x + 5 = 9\}' = \{x: x \in N \text{ and } x \text{ is a natural number and } x \neq 2\}$$

Mark 1: 1

Hint: If A is a subset of the universal set U then its complement A' is also a subset of U . x.

Step 1:

$$\{x: x \geq 7\}' = \{x: x \in N \text{ and } x < 7\}$$

Mark 1: 1

Hint: If A is a subset of the universal set U then its complement A' is also a subset of U .

xi. Step 1:

$$\{x: x \in N \text{ and } 2x + 1 > 10\}' = \left\{x: x \in N \text{ and } x \leq \frac{9}{2}\right\}$$

Hint: If A is a subset of the universal set U then its complement A' is also a subset of U .

4. If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$,

$$A = \{2, 4, 6, 8\} \text{ and } B = \{2, 3, 5, 7\}.$$

Verify that

(i) $(A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

Solution:

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}, A = \{2, 4, 6, 8\} \text{ and } B = \{2, 3, 5, 7\}.$$

(i) Step 1:

$$A \cup B = \{2, 3, 4, 5, 6, 7, 8\}$$

$$(A \cup B)' = U - A \cup B = \{1, 9\}$$

$$A' = U - A = \{1, 3, 5, 7, 9\}$$

$$B' = U - B = \{1, 4, 6, 8, 9\}$$

$$A' \cap B' = \{1, 9\} \therefore (A \cup B)' = A' \cap B'$$

Hint: If A is a subset of the universal set U then its complement A' is also a subset of U .

(ii) Step 2:

$$A \cap B = \{2\}$$

$$(A \cap B)' = \{1, 3, 4, 5, 6, 7, 8, 9\}$$

$$A' = \{1, 3, 5, 7, 9\}$$

$$B' = \{1, 4, 6, 8, 9\}$$

$$A' \cup B' = \{1, 3, 4, 5, 6, 7, 8, 9\} \therefore$$

$$(A \cap B)' = A' \cup B'$$

Hint: If A is a subset of the universal set U then its complement A' is also a subset of U .

5. Draw appropriate Venn diagram for each of the following:

(i) $(A \cup B)'$

(ii) $A' \cap B'$

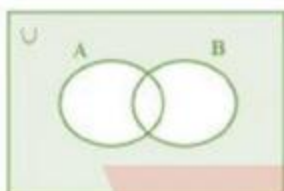
(iii) $(A \cap B)'$

(iv) $A' \cup B'$

Solution:

(i) Step 1:

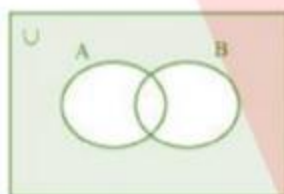
$(A \cup B)'$



Hint: If A is a subset of the universal set U then its complement A' is also a subset of U .

(ii) Step 1:

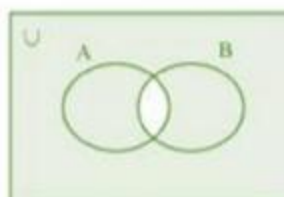
$A' \cap B'$



Hint: If A is a subset of the universal set U then its complement A' is also a subset of U .

(iii) Step 1:

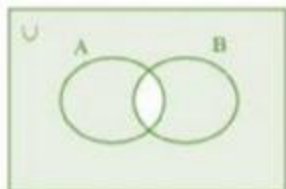
$(A \cap B)'$



Hint: If A is a subset of the universal set U then its complement A' is also a subset of U .

(iv) Step 1:

$$A' \cup B'$$



Hint: If A is a subset of the universal set U then its complement A' is also a subset of U .

6. Let U be the set of all triangles in a plane. If A is the set of all triangles with at least one angle different from 60° , what is A' ?

Solution:

Step 1:

U is the set of all triangles which is a universal set

$A' = U - A$, is the set of all equilateral triangles.

Hint: If A is a subset of the universal set U then its complement A' is also a subset of U .

7. Fill in the blanks to make each of the following a true statement:

(i) $A \cup A' = \dots$

(ii) $\phi' \cap A = \dots$

(iii) $A \cap A' = \dots$

(iv) $U' \cap A = \dots$

Solution:

i. Step 1:

$$A \cup A' = A \cup (U - A) = U$$

Hint: If A is a subset of the universal set U then its complement A' is also a subset of U .

ii. Step 2:

$$\phi' \cap A = U \cap A = A$$

Hint: If A is a subset of the universal set U then its complement A' is also a subset of U . iii.

Step 1:

$$A \cap A' = A \cap (U - A) = \phi$$

Hint: If A is a subset of the universal set U then its complement A' is also a subset of U . iv.

Step 1:

$$U' \cap A = \phi \cap A = \phi$$

Hint: If A is a subset of the universal set U then its complement A' is also a subset of U .

Exercise 11.6

1. If X and Y are two sets such that $n(X) = 17$, $n(Y) = 23$ and $n(X \cup Y) = 38$, find $n(X \cap Y)$.

Solution:

Given, Step

1:

$$n(X) = 17, n(Y) = 23, n(X \cup Y) = 38$$

We know that;

$$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$$

Step 2:

$$38 = 17 + 23 - n(X \cap Y)$$

$$38 = 40 - n(X \cap Y)$$

$$n(X \cap Y) = 40 - 38$$

$$n(X \cap Y) = 2$$

Hint: Use,

$$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$$

2. If X and Y are two sets such that $X \cup Y$ has 18 elements, X has 8 elements and Y has 15 elements; how many elements does $X \cap Y$ have?

Solution:

Given

Step 1:

$$n(X) = 8, n(Y) = 15, n(X \cup Y) = 18$$

It is known that,

$$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$$

Step 2:

$$18 = 8 + 15 - n(X \cap Y)$$

$$18 = 23 - n(X \cap Y)$$

$$n(X \cap Y) = 23 - 18$$

$$n(X \cap Y) = 5$$

Hint: Use,

$$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$$

3. In a group of 400 people, 250 can speak Hindi and 200 can speak English. How many people can speak both Hindi and English?

Solution:

Let E be the set of people who speak English and H be the set of people who speak Hindi.

Given, Step

1:

$$n(H) = 250, n(E) = 200, n(H \cup E) = 400$$

It is known that,

$$n(H \cup E) = n(H) + n(E) - n(H \cap E)$$

Step 2:

$$400 = 250 + 200 - n(H \cap E)$$

$$400 = 450 - n(H \cap E)$$

$$n(H \cap E) = 450 - 400$$

$$n(H \cap E) = 50$$

Hint: Use,

$$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$$

4. If S and T are two sets such that S has 21 elements, T has 32 elements and $S \cap T$ has 11 elements, how many elements does $S \cup T$ have?

Solution:

Given, Step

1:

$$n(S) = 21, n(T) = 32, n(S \cap T) = 11$$

It is known that,

$$\begin{aligned} n(S \cup T) &= n(S) + n(T) - n(S \cap T) \\ &= 21 + 32 - 11 \end{aligned}$$

$$= 42$$

Therefore, the set $S \cup T$ has 42 elements.

Hint: Use,

$$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$$

5. If X and Y are two sets such that X has 40 elements, $X \cup Y$ has 60 elements and $X \cap Y$ has 10 elements, how many elements does Y have?

Solution:

Given, Step

1:

$$n(X) = 40, n(X \cap Y) = 10, n(X \cup Y) = 60$$

It is known that,

$$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$$

$$60 = 40 + n(Y) - 10$$

$$60 = 30 + n(Y)$$

$$n(Y) = 30$$

Therefore, the set Y has 30 elements. Hint:

Use,

$$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$$

6. In a group of 70 people, 37 like coffee, 52 like tea and each person likes at least one of the two drinks. How many people like both coffee and tea?

Solution:

Considering C as the set of people who like coffee and T as the set of people who like tea.

Given, Step

1:

$$n(C) = 37, n(T) = 52, n(C \cup T) = 70$$

We know that;

$$n(C \cup T) = n(C) + n(T) - n(C \cap T)$$

Step 2:

$$70 = 37 + 52 - n(C \cap T)$$

$$70 = 89 - n(C \cap T)$$

$$n(C \cap T) = 89 - 70$$

$$n(C \cap T) = 19$$

Hint: Use,

$$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$$

7. In a group of 65 people, 40 like cricket, 10 like both cricket and tennis. How many like tennis only and not cricket? How many like tennis?

Solution:

Considering C as the set of people who like cricket and T as the set of people who like tea.

Given, Step

1:

$$n(C) = 40, n(C \cup T) = 65, n(C \cap T) = 10$$

We know that;

$$n(C \cup T) = n(C) + n(T) - n(C \cap T)$$

$$65 = 40 + n(T) - 10$$

$$65 = 30 + n(T)$$

$$n(T) = 35$$

Therefore, 35 people like tennis.

Step 2:

$$\text{Now, } (T - C) \cup (C \cap T) = T$$

$$\text{And also, } (T - C) \cap (C \cap T) = \phi$$

$$\text{Therefore, } n(T) = n(T - C) + n(T \cap C)$$

$$\Rightarrow 35 = n(T - C) + 10$$

$$\Rightarrow n(T - C) = 35 - 10 = 25$$

Therefore, 25 people like only tennis.

Hint: Use,

$$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$$

8. In a committee, 50 people speak French, 20 speak Spanish and 10 speak both Spanish and French. How many speak at least one of these two languages?

Solution:

Consider F as the set of people who speak French and S as the set of the people who speak Spanish

Given,

Step 1:

$$n(S) = 20, n(F) = 50, n(S \cap F) = 10$$

It is known that;

$$n(S \cup F) = n(S) + n(F) - n(S \cap F)$$

$$= 20 + 50 - 10$$

$$= 60$$

Therefore, 60 people in the committee speak at least one of the two languages.

Hint: Use,

$$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$$

Miscellaneous

1. Decide, among the following sets, which sets are subsets of one and another:

$$A = \{x: x \in R \text{ and } x \text{ satisfy } x^2 - 8x + 12 = 0\}, B = \{2, 4, 6\}, C = \{2, 4, 6, 8, \dots\}, D = \{6\}.$$

Solution:

Step 1:

$$A = \{x: x \in R \text{ and } x \text{ satisfy } x^2 - 8x + 12 = 0\}$$

2 and 6 are the only solution of $x^2 - 8x + 12 = 0$

$$\therefore A = \{2, 6\}$$

Step 2:

$$B = \{2, 4, 6\}, C = \{2, 4, 6, 8, \dots\}, D = \{6\}$$

Therefore, we can write it as $D \subset A \subset B \subset C$ Thus,

$$A \subset B, A \subset C, B \subset C, D \subset A, D \subset B, D \subset C$$

Hint: A set A is said to be a subset of a set B if every element of A is also an element of B .

2. In each of the following, determine whether the statement is true or false. If it is true, prove it. If it is false, give an example.
- If $x \in A$ and $A \in B$, then $x \in B$
 - If $A \subset B$ and $B \in C$, then $A \in C$
 - If $A \subset B$ and $B \subset C$, then $A \subset C$
 - If $A \not\subset B$ and $B \not\subset C$, then $A \not\subset C$
 - If $x \in A$ and $A \not\subset B$, then $x \in B$
 - If $A \subset B$ and $x \notin B$, then $x \notin A$

Solution:

- (i) Step 1:
False.

Let $A = \{1, 2\}$ and $B = \{1, \{1, 2\}, \{3\}\}$

Now, $2 \in \{1, 2\}$ and $\{1, 2\} \in \{1, \{1, 2\}, \{3\}\}$

$$\therefore A \in B$$

But, $2 \notin \{1, \{1, 2\}, \{3\}\}$

Hint: A set A is said to be a subset of a set B if every element of A is also an element of B .

Step 1:

False.

Let $A = \{2\}$, $B = \{0, 2\}$ and $C = \{1, \{0, 2\}, \{3\}\}$

Clearly, $A \subset B$ and $B \in C$, but $A \notin C$

Hint: A set A is said to be a subset of a set B if every element of A is also an element of B .

(ii) Step

True 1:

Let $A \subset B$ and $B \subset C$.

Let $x \in A$

$\Rightarrow x \in B$ [$\because A \subset B$]

$\Rightarrow x \in C$ [$\because B \subset C$]

Therefore, $A \subset C$

Hint: A set A is said to be a subset of a set B if every element of A is also an element of B .

Step 1:

False.

Let $A = \{2\}$, $B = \{4, 6\}$ and $C = \{2, 3, 5\}$ Here

$A \not\subset B$ and $B \not\subset C$.

But $A \subset C$

Hint: A set A is said to be a subset of a set B if every element of A is also an element of B .

Step 1:

False.

Let $A = \{1, 4, 7\}$ and $B = \{3, 4, 6\}$

Now, $1 \in A$ and $1 \notin B$

But, $1 \notin B$

Hint: A set A is said to be a subset of a set B if every element of A is also an element of B .

is also an element

(iii) Step 1:

True

Considering $A \subset B$ and $x \notin B$.

We need to show: $x \notin A$

Let us assume $x \in A$, then $x \in B$, which is a contradiction as $x \notin B$

$\therefore x \notin A$

Hint: A set A is said to be a subset of a set B if every element of A is also an element of B .

3. Let A , B and C be the sets such that $A \cup B = A \cup C$ and $A \cap B = A \cap C$. Show that $B = C$.

Solution:

Given

Step 1:

A , B , and C are the sets such that $A \cup B = A \cup C$ and $A \cap B = A \cap C$.

We need to show: $B = C$ Consider,

$x \in B$

$\Rightarrow x \in A \cup B$ [$B \subset A \cup B$]

$\Rightarrow x \in A \cup C$ [$A \cup B = A \cup C$]

$\Rightarrow x \in A$ or $x \in C$

Also, $x \in B$

$\therefore x \in A \cap B$

$\Rightarrow x \in A \cap C$

$\therefore x \in A$ and $x \in C$

$\therefore x \in C$ \therefore

$B \subset C$

Similarly, we can show that $C \subset B$.

Therefore, $B = C$

Hint: A set A is said to be a subset of a set B if every element of A is also an element of B .

4. Show that the following four conditions are equivalent:

(i) $A \subset B$

(ii) $A - B = \phi$

(iii) $A \cup B = B$

(iv) $A \cap B = A$ **Solution:**

Step 1:

Let us first show that (i) is equivalent to (ii)

Let $A \subset B$

To prove: $A - B = \phi$

Let us suppose $A - B \neq \phi$

This means that there exists a x such that $x \in A$ but $x \notin B$ which is not possible as A

\subset

B .

$$\therefore A - B = \phi \therefore A \subset B$$

$$\Rightarrow A - B = \phi$$

Let $A - B = \phi$ Step 2:

To show: $A \subset B$

Let $x \in A$

Clearly, $x \in B$ because if $x \notin B$, then $A - B \neq \phi$

$$\therefore A - B = \phi \Rightarrow A \subset B$$

$\therefore (i) \Leftrightarrow (ii)$ Step

3:

Let $A \subset B$

To show: $A \cup B = B$

Clearly, $B \subset A \cup B$

Let $x \in A \cup B$

$$\Rightarrow x \in A \text{ or } x \in B$$

Case I: $x \in A$

$\Rightarrow x \in B$ [$\because A \subset B$]

$\therefore A \cup B \subset B$

Case II: $x \in B$

Then, $A \cup B = B$

Conversely, let $A \cup B = B$

Let $x \in A$

$\Rightarrow x \in A \cup B$ [$\because A \subset A \cup B$]

$\Rightarrow x \in B$ [$\because A \cup B = B$]

$\therefore A \subset B$

Therefore, (i) \Leftrightarrow (iii) Step

4:

Now, we have to show that (i) \Leftrightarrow (iv)

Let $A \subset B$

Clearly $A \cap B \subset A$

Let $x \in A$

We have to show that $x \in A \cap B$

As $A \subset B$, $x \in B$

$\therefore x \in A \cap B$

$\therefore A \subset A \cap B$

Therefore, $A = A \cap B$

Step 5:

Conversely, suppose $A \cap B = A$

Let $x \in A$

$\Rightarrow x \in A \cap B$

$$\Rightarrow x \in A \text{ and } x \in B$$

$$\Rightarrow x \in B$$

$$\therefore A \subset B$$

Therefore, (i) \Leftrightarrow (iv).

Hint: A set A is said to be a subset of a set B if every element of A is also an element of B .

5. Show that if $A \subset B$, then $C - B \subset C - A$.

Solution:

Let $A \subset B$

We have to prove that $C - B \subset C - A$

Let $x \in C - B$

$$\Rightarrow x \in C \text{ and } x \notin B$$

$$\Rightarrow x \in C \text{ and } x \notin A \quad [\because A \subset B]$$

$$\Rightarrow x \in C - A \therefore C$$

$$- B \subset C - A$$

6. Assume that $P(A) = P(B)$. Show that $A = B$.

Solution:

Step 1:

Let $P(A) = P(B)$

We have to prove that $A = B$

Let $x \in A$

$A \in P(A) = P(B)$

$\therefore x \in C$, for some $C \in P(B)$

Step 2:

Now, $C \subset B$

$\therefore x \in B \therefore$

$A \subset B$

Similarly, $B \subset A$

$\therefore A = B$

Hint: A set A is said to be a subset of a set B if every element of A is also an element of B .

7. Is it true that for any set A and B , $P(A) \cup P(B) = P(A \cup B)$? Justify your answer.

Solution:

Step 1:

False.

Let $A = \{0, 1\}$ and $B = \{1, 2\}$

$\therefore A \cup B = \{0, 1, 2\}$

$P(A) = \{\phi, \{0\}, \{1\}, \{0, 1\}\}$

$P(B) = \{\phi, \{1\}, \{2\}, \{1, 2\}\}$

Step 2:

$P(A \cup B) = \{\phi, \{0\}, \{1\}, \{2\}, \{0, 1\}, \{1, 2\}, \{0, 2\}, \{0, 1, 2\}\}$

$P(A) \cup P(B) = \{\phi, \{0\}, \{1\}, \{2\}, \{0, 1\}, \{1, 2\}\}$

$\therefore P(A) \cup P(B) \neq P(A \cup B)$

Hint: If A and B are the two sets then A union B consists of the elements from both the sets.

8. Show that for any set A and B ,

$A = (A \cap B) \cup (A - B)$ and $A \cup (B - A) = (A \cup B)$ **Solution:**

Step 1:

Let $x \in A$

We have to prove that $x \in (A \cap B) \cup (A - B)$ Case

1

$$x \in A \cap B$$

$$\text{Then, } x \in (A \cap B) \subset (A \cup B) \cup (A - B)$$

Case 2

$$x \notin A \cap B$$

$$\Rightarrow x \notin A \text{ or } x \notin B$$

$$\therefore x \notin B [x \notin A]$$

$$\therefore x \in A - B \subset (A \cup B) \cup (A - B)$$

Step 2:

$$\therefore A \subset (A \cap B) \cup (A - B) \dots\dots\dots(1)$$

Clearly we get,

$$A \cap B \subset A \text{ and } (A - B) \subset A \therefore (A \cap B) \cup$$

$$(A - B) \subset A \dots\dots\dots(2)$$

From (1) and (2), we get,

$$A = (A \cap B) \cup (A - B)$$

Step 3:

To prove

$$A \cup (B - A) \subset (A \cup B)$$

$$\text{Let } x \in A \cup (B - A)$$

$$\Rightarrow x \in A \text{ or } (x \in B \text{ and } x \notin A)$$

$$\Rightarrow (x \in A \text{ or } x \in B) \text{ and } (x \in A \text{ or } x \notin A)$$

$$\Rightarrow x \in A \cup B$$

$$\therefore A \cup (B - A) \subset (A \cup B) \dots\dots\dots(3)$$

Step 4:

Now, we have to show that $(A \cup B) \subset A \cup (B - A)$

$$\text{Let } y \in A \cup B$$

$$\Rightarrow y \in A \text{ or } y \in B$$

$$\Rightarrow$$

$$\Rightarrow$$

$$\Rightarrow$$

$$(y \in A \text{ or } y \in B) \text{ and } (y \in A \text{ or } y \notin A)$$

$$y \in A \text{ or } (y \in B \text{ and } y \notin A)$$

$$y \in A \cup (B - A)$$

$$\therefore (A \cup B) \subset (A \cup (B - A)) \dots\dots\dots (4)$$

Hence, from (3) and (4), we obtain,

$$A \cup (B - A) = (A \cup B)$$

Hint: Use the concepts of union of the sets and subtracting of a set from another set.

9. Using properties of sets show that

$$(i) A \cup (A \cap B) = A \quad (ii)$$

$$A \cap (A \cup B) = A.$$

Solution:

$$(i) \quad \text{To show, } A \cup (A \cap B) = A$$

Step 1:

We know that, $A \subset A$

$$A \cap B \subset A$$

$$\therefore A \cup (A \cap B) \subset A \dots\dots\dots(1)$$

$$\text{Also, } A \subset A \cup (A \cap B) \dots\dots\dots(2)$$

$$\therefore \text{From (1) \& (2), } A \cup (A \cap B) = A$$

Hint: Union of two sets consists of all the elements from both the sets whereas intersection of any two sets consists of common elements from both the sets.

$$(ii) \quad \text{To show, } A \cap (A \cup B) = A$$

Step 1:

$$A \cap (A \cup B) = (A \cap A) \cup (A \cap B)$$

$$= A \cup (A \cap B)$$

$$= A \quad (\text{from (1)})$$

Hint: Union of two sets consists of all the elements from both the sets whereas intersection of any two sets consists of common elements from both the sets.

10. Show that $A \cap B = A \cap C$ need not imply $B = C$.

Solution:

Step 1:

Let $A = \{0, 1\}$, $B = \{0, 2, 3\}$, and $C = \{0, 4, 5\}$ We can see that $A \cap B = \{0\}$ and $A \cap C = \{0\}$ Therefore, $A \cap B = A \cap C$ But, $B \neq C$ as $2 \in B$ and $2 \notin C$

Hint: Intersection of any two sets consists of common elements from both the sets.

11. Let A and B be sets. If $A \cap X = B \cap X = \phi$ and $A \cup X = B \cup X$ for some set X , show that $A = B$.

(Hints $A = A \cap (A \cup X)$, $B = B \cap (B \cup X)$ and the use distributive law) **Solution:**

Step 1:

Let A and B be two sets such that $A \cap X = B \cap X = \phi$ and $A \cup X = B \cup X$ for some set X .We can see that $A = A \cap (A \cup X) = A \cap (B \cup X)$ $= (A \cap B) \cup (A \cap X)$ [Distributive Law] $= (A \cap B) \cup \phi$ [$A \cap X = \phi$] $= A \cap B \dots\dots\dots (1)$

Step 2:

Now, $B = B \cap (B \cup X)$ $= B \cap (A \cup X)$ [$A \cup X = B \cup X$] $= (B \cap A) \cup (B \cap X)$ [Distributive Law] $= (B \cap A) \cup \phi$ [$B \cap X = \phi$] $= A \cap B \dots\dots\dots (2)$ Therefore, from (1) and (2), we obtain $A = B$.

Hint: Intersection of any two sets consists of common elements from both the sets.

12. Find sets A , B and C such that $A \cap B$, $B \cap C$ and $A \cap C$ are non-empty sets and $A \cap B \cap C = \phi$.

Solution:

Step 1:

Let $A = \{0, 1\}$, $B = \{1, 2\}$, and $C = \{2, 0\}$ Also, $A \cap B = \{1\}$, $B \cap C = \{2\}$, $C \cap A = \{0\}$ $\therefore A \cap B, B \cap C, C \cap A$ are non-empty setsHowever, $A \cap B \cap C = \phi$

Hint: Intersection of any two sets consists of common elements from both the sets.

13. In a survey of 600 students in a school, 150 students were found to be taking tea and 225 taking coffee, 100 were taking both tea and coffee. Find how many students were taking neither tea nor coffee?

Solution:

Step 1:

Considering U as the set of students who took part in the survey.Let C be the set of students who take coffee and T be the set of students who take tea.Given, $n(U) = 600$, $n(C) = 225$, $n(T) = 150$, $n(C \cap T) = 100$

We know that;

$$\begin{aligned}n(C \cup T) &= n(C) + n(T) - n(C \cap T) \\ &= 225 + 150 - 100 \\ &= 275\end{aligned}$$

Step 2:

Now, Number of students taking neither tea nor coffee is $n(C' \cap T')$

$$\begin{aligned}n(C' \cap T') &= n(C \cup T)' \\ &= n(U) - n(C \cup T) \\ &= 600 - 275 \\ &= 325\end{aligned}$$

Therefore, 325 students were taking neither tea nor coffee.

Hint: Use $n(C \cup T) = n(C) + n(T) - n(C \cap T)$ and $n(C' \cap T') = n(C \cup T)'$

14. In a group of students 100 students know Hindi, 50 know English and 25 know both. Each of the students knows either Hindi or English. How many students are there in the group?

Solution:

Step 1:

Considering U as the set of all the students in the group.

Considering E as the set of students who know English and H as the set of students who know Hindi.

$$\therefore H \cup E = U$$

$$\text{Given, } n(H) = 100, n(E) = 50, n(H \cap E) = 25$$

We know that;

$$n(H \cup E) = n(H) + n(E) - n(H \cap E) \text{ Step}$$

2:

$$= 100 + 50 - 25$$

$$= 125$$

$$\text{So, } n(U) = 125$$

Hence, there are 125 students in the group.

Hint: Use $n(C \cup T) = n(C) + n(T) - n(C \cap T)$

15. In a survey of 60 people, it was found that 25 people read newspaper H , 26 read newspaper T , 26 read newspaper I , 9 read both H and I , 11 read both H and T , 8 read both T and I , 3 read all three newspapers. Find:

(i) the number of people who read at least one of the newspapers.

(ii) the number of people who read exactly one newspaper.

Solution:

Let U be the set of people who took part in survey.

Let A be the set of people who read newspaper H .

Let B be the set of people who read newspaper T . Let

C be the set of people who read newspaper I .

$$\text{Given, } n(A) = 25, n(B) = 26, n(C) = 26, n(A \cap B) = 11, n(B \cap C) = 8, n(C \cap A) = 9$$

$$n(A \cap B \cap C) = 3 \text{ (i)}$$

Step 1:

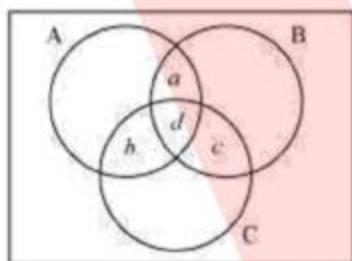
$$\begin{aligned} n(A \cup B \cup C) &= n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + \\ &n(A \cap B \cap C) \\ &= 25 + 26 + 26 - 11 - 8 - 9 + 3 \\ &= 52 \end{aligned}$$

Hence, 52 people read at least one of the newspapers.

Hint: Use, $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

Step 1:

Let a be the number of people who read newspaper H and T only.



Let b be the number of people who read newspaper H and I only. Let c be the number of people who read newspaper T and I only. Let d be the number of people who read all the newspapers.

Step 2:

$$\text{So, } d = n(A \cap B \cap C) = 3$$

$$\text{Now, } n(A \cap B) = a + d$$

$$n(B \cap C) = c + d$$

$$n(C \cap A) = b + d$$

$$\text{Now, } a + d + c + d + b + d = 11 + 8 + 9 = 28$$

$$\Rightarrow a + b + c + d = 28 - 2d = 28 - 6 = 22$$

Hence, $(52 - 22) = 30$ people read exactly one newspaper.

Hint: Use Venn diagram concept and intersection of the two sets.

16. In a survey it was found that 21 people liked product A , 26 liked product B and 29 liked product C . If 14 people liked products A and B , 12 people liked products C and A , 14 people liked products B and C and 8 liked all the three products. Find how many liked product C only.

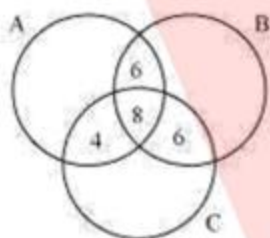
Solution:

Step 1:

Considering A , B and C as the set of people who like product A , product B and product C respectively.

Given, $n(A) = 21$, $n(B) = 26$, $n(C) = 29$, $n(A \cap B) = 14$, $n(B \cap C) = 14$, $n(C \cap A) = 12$
 $n(A \cap B \cap C) = 8$

The Venn diagram for the given problem can be drawn as



Step 2:

It is seen that number of people who like product C only is $\{29 - (4 + 8 + 6)\} = 11$ Hint: Use the Venn diagram and concept of complement of a set.