CBSE NCERT Solutions for Class 12 Maths Chapter 08

Back of Chapter Questions

Exercise 8.1

1. Find the area of the region bounded by the curve $y^2 = x$ and the lines $x = 1, x = 4$ and the x-axis in the first quadrant.

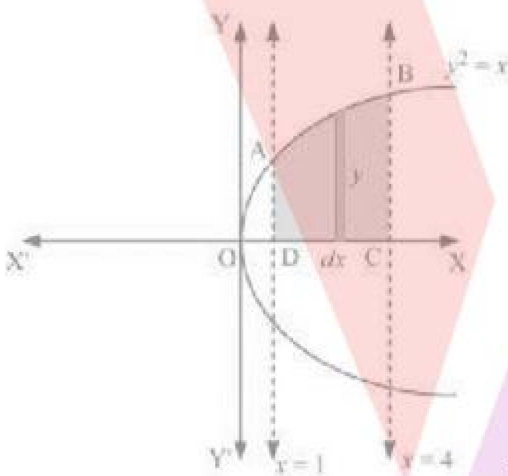
[2 Marks]

Solution:

Step 1:

Given: Equation of the curve $y^2 = x$ and the lines $x = 1, x = 4$ and the x-axis.

The given equations can be represented as follows:



[$\frac{1}{2}$Mark]

Step 2:

The area of the region bounded by the curve, $y^2 = x$, the lines, $x = 1$ and $x = 4$, and the x-axis is the area $ABCD$.

$$\text{Area of } ABCD = \int_1^4 y dx$$

$= \int_1^4 \sqrt{x} dx$ **[1 Mark]**

Step 3:

$$= \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^4$$

$$= \frac{2}{3}\left[(4)^{\frac{3}{2}} - (1)^{\frac{3}{2}}\right]$$

$= \frac{2}{3}[8-1]$

$= \frac{14}{3}$ square units [$\frac{1}{2}$ Mark]

Hence, the required area is $\frac{14}{3}$ square units.

2. Find the area of the region bounded by $y^2 = 9x, x = 2, x = 4$ and the x-axis in the first quadrant.

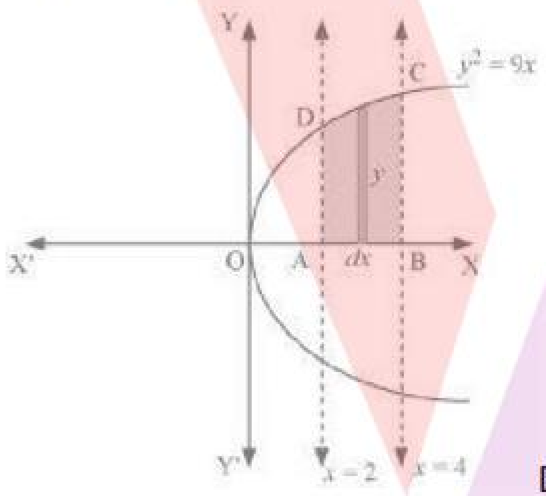
[2 Marks]

Solution:

Step 1:

Given: The equations of the curve $y^2 = 9x, x = 2, x = 4$ and the x-axis in the first quadrant.

The given equations can be represented as follows:



[$\frac{1}{2}$ Mark]

The area of the region bounded by the curve, $y^2 = 9x, x = 2$, and $x = 4$, and the x-axis is the area $ABCD$.

Step 2:

$$\text{Area of } ABCD = \int_2^4 y\,dx$$

$= \int_2^4 3\sqrt{x}dx$ **[1 Mark]**

$$= 3\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_2^4$$

$$= 2\left[x^{\frac{3}{2}}\right]_2^4$$

$= 2\left[(4)^{\frac{3}{2}} - (2)^{\frac{3}{2}}\right]$

$= 2[8 - 2\sqrt{2}]$

$= (16 - 4\sqrt{2})$ square units [$\frac{1}{2}$ Mark]

Hence, the required area is $(16 - 4\sqrt{2})$ square units.

3. Find the area of the region bounded by $x^2 = 4y, y = 2, y = 4$ and the y-axis in the first quadrant.

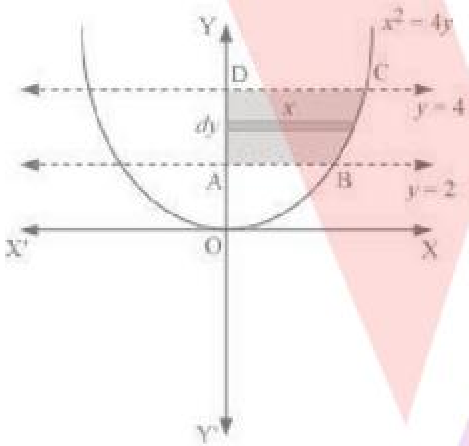
[2 Marks]

Solution:

Step 1:

Given: The equations of the curve $x^2 = 4y, y = 2, y = 4$ and the y-axis in the first quadrant.

The given equations can be represented as follows:



[$\frac{1}{2}$ Mark]

Step 2:

The area of the region bounded by the curve, $x^2 = 4y, y = 2$, and $y = 4$, and the y-axis is the area $ABCD$.

$$\text{Area of } ABCD = \int_2^4 x dy$$

$= \int_2^4 2\sqrt{y} dy$ **[1 Mark]**

Step 3:

$$= 2\int_2^4 \sqrt{y} dy$$

$= 2\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_2^4$

$= \frac{4}{3}\left[(4)^{\frac{3}{2}} - (2)^{\frac{3}{2}}\right]$

$= \frac{4}{3}\left[8 - 2\sqrt{2}\right]$

$= \left(\frac{32-8\sqrt{2}}{3}\right)$ square units **[$\frac{1}{2}$ Mark]**

Hence, the required area is $\left(\frac{32-8\sqrt{2}}{3}\right)$ square units.

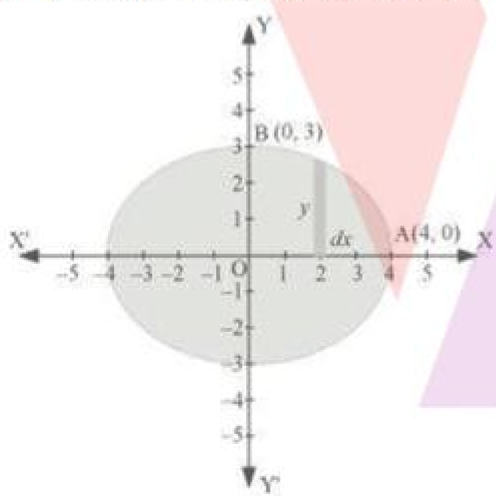
4. Find the area of the region bounded by the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$. **[2 Marks]**

Solution:

Step 1:

Given: The equation of the ellipse: $\frac{x^2}{16} + \frac{y^2}{9} = 1$

The given equation of the ellipse can be represented as follows:



[$\frac{1}{2}$ Mark]

Step 2:

It is observed that the ellipse is symmetrical about x-axis and y-axis. ∴ Area bounded by ellipse $= 4 \times$ Area of OAB

Area of $OAB = \int_0^4 y dx$

$= \int_0^4 3\sqrt{1 - \frac{x^3}{16}} dx$ **[1 Mark]**

Step 3:

$$= \frac{3}{4}\int_{0}^{4}\sqrt{16-x^2}\,dx$$

$$= \frac{3}{4}\left[\frac{x}{2}\sqrt{16-x^2} + \frac{16}{2}\sin^{-1}\frac{x}{4}\right]_0^4$$

$$= \frac{3}{4}\left[2\sqrt{16-16} + 18\sin^{-1}(1) - 0 - 8\sin^{-1}(0)\right]$$

$$= \frac{3}{4}\left[\frac{8\pi}{2}\right]$$

$$= \frac{3}{4}[4\pi]$$

$$= 3\pi$$

Hence, area bounded by the ellipse $= 4 \times 3\pi = 12\pi$ square units. **[$\frac{1}{2}$ Mark]**

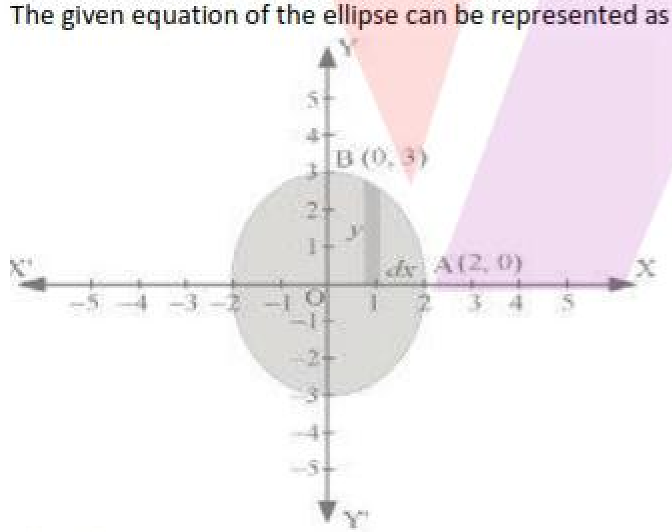
5. Find the area of the region bounded by the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$. **[2 Marks]**

Solution:

Step 1:

Given: The equation of the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$

The given equation of the ellipse can be represented as



$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

$\Rightarrow y = 3\sqrt{1 - \frac{x^2}{4}}$ ...(i) **[$\frac{1}{2}$ Mark]**

Step 2:

It is observed that the ellipse is symmetrical about x-axis and y-axis.

$\therefore$ Area bounded by ellipse $= 4 \times \text{Area } OAB$

$$\therefore \text{Area of } OAB = \int_0^2 y\,dx$$

$= \int_0^2 3\sqrt{1 - \frac{x^2}{4}}\,dx$ [Using (i)] **[1 Mark]**

Step 3:

$$= \frac{3}{2}\int_0^2 \sqrt{4 - x^2}\,dx$$

$$= \frac{3}{2}\left[\frac{x}{2}\sqrt{4 - x^2} + \frac{4}{2}\sin\frac{-x}{2}\right]_0^2$$

$$= \frac{3}{2}\left[\frac{2\pi}{2}\right]$$

$$= \frac{3\pi}{2}$$

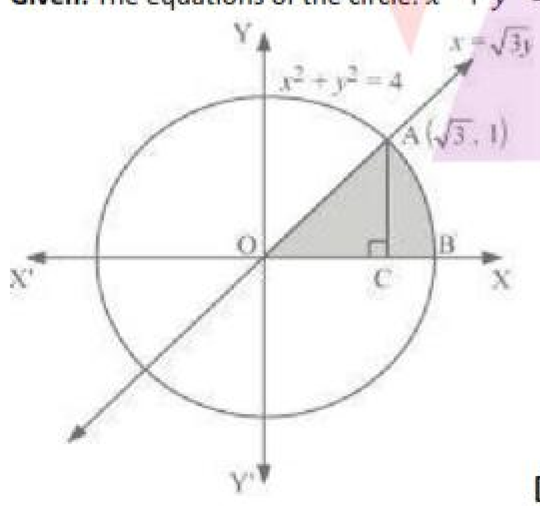
Hence, area bounded by the ellipse $= 4 \times \frac{3\pi}{2} = 6\pi$ square units. **[$\frac{1}{2}$ Mark]**

6. Find the area of the region in the first quadrant enclosed by x-axis, line $x = \sqrt{3}y$ and the circle $x^2 + y^2 = 4$. **[4 marks]**

Solution:

Step 1:

Given: The equations of the circle: $x^2 + y^2 = 4$ and x-axis, line $x = \sqrt{3}y$



[$\frac{1}{2}$ Mark]

Step 2:

The area of the region bounded by the circle, $x^2 + y^2 = 4, x = \sqrt{3}y$ and the x-axis is the area OAB.

The point of intersection of the line and the circle in the first quadrant is $(\sqrt{3}, 1)$

$\text{Area } OAB = \text{Area } \Delta OCA + \text{Area } ACB$ **[$\frac{1}{2}$ Mark]**

Step 3:

Area of $OAC = \frac{1}{2} \times OC \times AC = \frac{1}{2} \times \sqrt{3} \times 1 = \frac{\sqrt{3}}{2}$ ...(i) **[$\frac{1}{2}$ Mark]**

Step 4:

Area of $ABC = \int_{\sqrt{3}}^{2} y dx$ **[$\frac{1}{2}$ Mark]**

Step 5:

$$= \int_{\sqrt{3}}^{2} \sqrt{4 - x^2}\, dx$$

$$= \left[\frac{x}{2}\sqrt{4 - x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_{\sqrt{3}}^{2}$$ **[$\frac{1}{2}$ Mark]**

Step 5:

$$= \left[2 \times \frac{\pi}{2} - \frac{\sqrt{3}}{2}\sqrt{4 - 3} - 2\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]$$

$$= \left[\pi - \frac{\sqrt{3}}{2} - 2\left(\frac{\pi}{3}\right)\right]$$

$$= \left[\pi - \frac{\sqrt{3}}{2} - \frac{2\pi}{3}\right]$$

$= \left[\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right]$ ...(ii) **[$\frac{1}{2}$ Mark]**

Step 6:

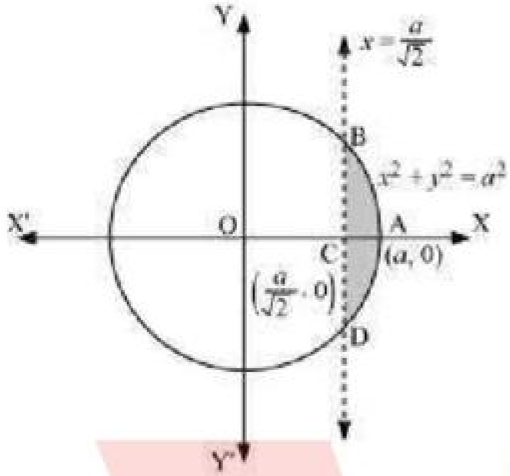
Hence, area enclosed by x-axis, the line $x = \sqrt{3}y$ and the circle $x^2 + y^2 = 4$ in the first quadrant $= \frac{\sqrt{3}\pi}{2} + \frac{\pi}{3} - \frac{3\sqrt{\pi}}{2} = \frac{\pi}{3}$ sq. units. **[1 Mark]**

7. Find the area of the smaller part of the circle $x^2 + y^2 = a^2$ cut off by the line $x = \frac{a}{\sqrt{2}}$. **[4 Marks]**

Solution:

Step 1:

Given: The equation of the circle: $x^2 + y^2 = a^2$ which is cut off by a line $x = \frac{a}{\sqrt{2}}$



[$\frac{1}{2}$ **Mark]**

Step 2:

The area of the smaller part of the circle, $x^2 + y^2 = a^2$, cut off by the line, $x = \frac{a}{\sqrt{2}}$, is the area $ABCDA$.

It is observed that the area $ABCD$ is symmetrical about x-axis.

$\therefore$ Area $ABCD = 2 \times$ Area ABC [$\frac{1}{2}$ **Mark]**

Step 3:

$$\text{Area of } ABC = \int_{\frac{a}{\sqrt{2}}}^{a} y dx$$

$= \int_{\frac{a}{\sqrt{2}}}^{a} \sqrt{a^2 - x^2}\, dx$ [$\frac{1}{2}$ **Mark]**

Step 3:

$= \left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_{\frac{a}{\sqrt{2}}}^{a}$ [$\frac{1}{2}$ **Mark]**

Step 4:

$= \left[\frac{a^2}{2}\left(\frac{\pi}{2}\right) - \frac{a}{2\sqrt{2}}\sqrt{a^2 - \frac{a^2}{2}} - \frac{a^2}{2}\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)\right]$

$= \frac{a^2\pi}{4} - \frac{a}{2\sqrt{2}}.\frac{a}{\sqrt{2}} - \frac{a^2}{2}\left(\frac{\pi}{4}\right)$

$= \frac{a^2\pi}{4} - \frac{a^2}{4} - \frac{a^2\pi}{8}$

$= \frac{a^2}{4}\left[\pi - 1 - \frac{\pi}{2}\right]$

$= \frac{a^2}{4}\left[\frac{\pi}{2} - 1\right]$ **[1 Mark]**

Step 5:

$\Rightarrow$ Area $ABCD = 2\left[\frac{a^2}{4}\left(\frac{\pi}{2} - 1\right)\right] = \frac{a^2}{2}\left(\frac{\pi}{2} - 1\right)$

Hence, the area of smaller part of the circle, $x^2 + y^2 = a^2$, cut off by the line, $x = \frac{a}{\sqrt{2}}$ is $\frac{a^2}{2}\left(\frac{\pi}{2} - 1\right)$ sq. units. **[1 Mark]**

8. The area between $x = y^2$ and $x = 4$ is divided into two equal parts by the line $x = a$, find the value of a. **[4 marks]**

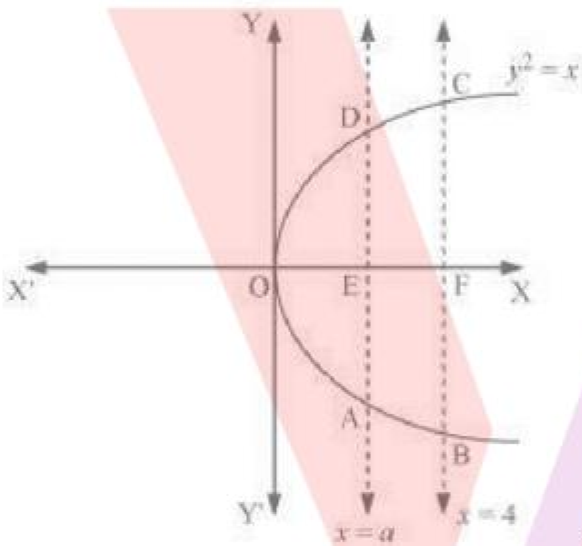
Solution:

Step 1:

Given: The area between $x = y^2$ and $x = 4$ is divided into two equal parts by the line $x = a$.

The line, $x = a$, divides the area bounded by the parabola and $x = 4$ into two equal parts.

$\therefore$ Area OAD = Area $ABCD$



[$\frac{1}{2}$ Mark]

Step 2:

It can be observed that the given area is symmetrical about x-axis.

$\Rightarrow$ Area OED = Area $EFCD$

Area $OED = \int_0^a y\,dx$ **[$\frac{1}{2}$ Mark]**

Step 3:

$= \int_0^a \sqrt{x}\,dx$

$= \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^a$

$= \frac{2}{3}(a)^{\frac{3}{2}}$ ...(i) **[$\frac{1}{2}$ Mark]**

Step 4:

Area of $EFCD = \int_a^4 \sqrt{x}\,dx$ **[$\frac{1}{2}$ Mark]**

Step 5:

$= \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_a^4$

$= \frac{2}{3}\left[8 - a^{\frac{3}{2}}\right]$ ...(ii) **[$\frac{1}{2}$ Mark]**

Step 6:

From (i) and (ii), we get

$\frac{2}{3}(a)^{\frac{3}{2}} = \frac{2}{3}\left[8 - (a)^{\frac{3}{2}}\right]$

$\Rightarrow 2.(a)^{\frac{3}{2}} = 8$

$\Rightarrow (a)^{\frac{3}{2}} = 4$

$\Rightarrow a = (4)^{\frac{2}{3}}$ **[1$\frac{1}{2}$ Mark]**

Hence, the value of a is $(4)^{\frac{2}{3}}$.

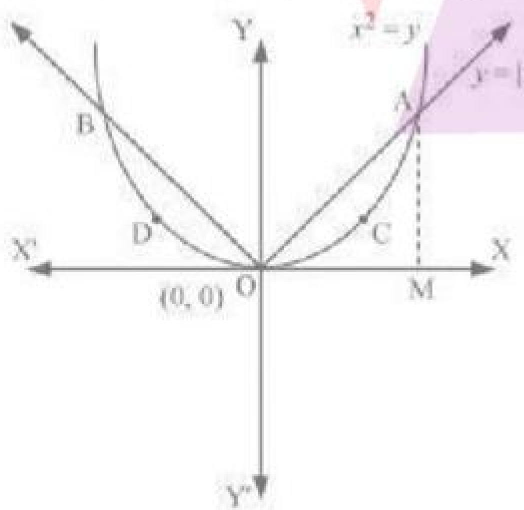
9. Find the area of the region bounded by the parabola $y = x^2$ and $y = |x|$. **[2 Marks]**

Solution:

Step 1:

Given: Equation of the parabola $y = x^2$ and $y = |x|$

The area bounded by the parabola, $x^2 = y$ and the line, $y = |x|$, can be represented as



[$\frac{1}{2}$ Mark]

Step 2:

The given area is symmetrical about y-axis.

$\therefore$ Area $OACO$ = Area $ODBO$

The point of intersection of parabola, $x^2 = y$ and line, $y = x$ is $A(1, 1)$.

Area of $OACO$ = Area ΔOAB − Area $OBACO$ **[$\frac{1}{2}$ Mark]**

Step 3:

$\therefore$ Area of $\Delta OAB = \frac{1}{2} \times OB \times AB = \frac{1}{2} \times 1 \times 1 = \frac{1}{2}$

Area of $OBACO = \int_0^1 y\,dx = \int_0^4 x^2\,dx = \left[\frac{x^3}{3}\right]_0^1 = \frac{1}{3}$ **[$\frac{1}{2}$ Mark]**

Step 4:

$\Rightarrow$ Area of $OACO$ = Area of ΔOAB − Area of $OBACO$

$= \frac{1}{2} - \frac{1}{3}$

$= \frac{1}{6}$

Hence, required area $= 2\left[\frac{1}{6}\right] = \frac{1}{3}$ sq. units **[$\frac{1}{2}$ Mark]**

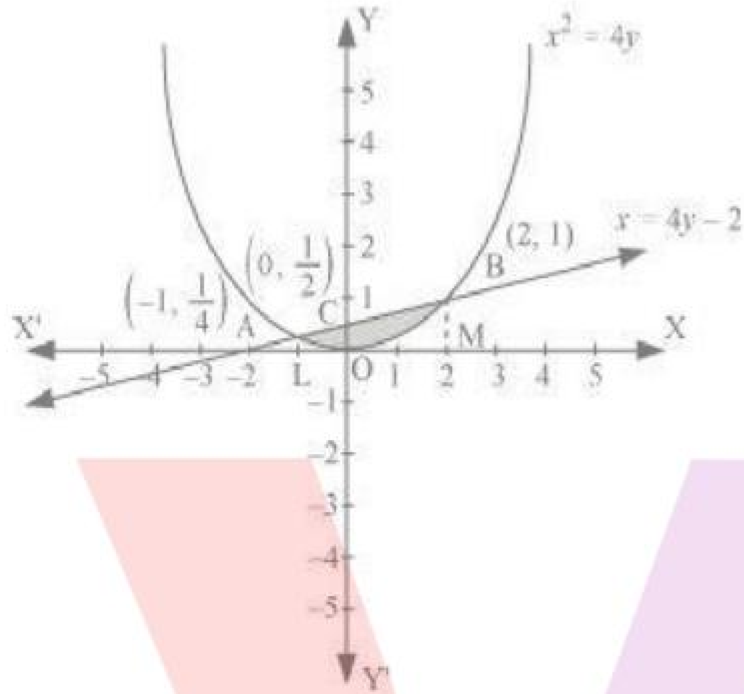
10. Find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$. **[4 Marks]**

Solution:

Step 1:

Given: Equation of the curve $x^2 = 4y$ and the line $x = 4y - 2$

The area bounded by the curve, $x^2 = 4y$ and line, $x = 4y - 2$, is represented by the shaded area $OBAO$.



[$\frac{1}{2}$ **Mark]**

Step 2:

Let A and B be the points of intersection of the line and parabola.

Coordinates of point A are $\left(-1, \frac{1}{4}\right)$.

Coordinates of point B are $(2, 1)$.

We draw AL and BM perpendicular to x-axis. [$\frac{1}{2}$ **Mark]**

Step 3:

It is observed that,

Area $OBAO$ = Area $OBCO$ + Area $OACO$ ...(i)

Then, Area $OBCO$ = Area $OMBC$ − Area $OMBO$

$= \int_0^2 \frac{x+2}{4}\, dx - \int_0^2 \frac{x^2}{4} dx$ [$\frac{1}{2}$ **Mark]**

Step 4:

$= \frac{1}{4}\left[\frac{x^2}{2} + 2x\right]_0^2 - \frac{1}{4}\left[\frac{x^3}{3}\right]_0^2$

$= \frac{1}{4}[2 + 4] - \frac{1}{4}\left[\frac{8}{3}\right]$

$= \frac{3}{2} - \frac{2}{3}$

$= \frac{5}{6}$ [$\frac{1}{2}$ **Mark]**

Step 5:

Similarly, Area $OACO$ = Area $OLAC$ − Area $OLAO$

$= \int_{-1}^0 \frac{x+2}{4} dx - \int_{-1}^0 \frac{x^2}{4} dx$ [$\frac{1}{2}$ **Mark]**

Step 6:

$= \frac{1}{4}\left[\frac{x^2}{2} + 2x\right]_{-1}^{0} - \frac{1}{4}\left[\frac{x^3}{3}\right]_{-1}^{0}$

$= -\frac{1}{4}\left[\frac{(-1)^2}{2} + 2(-1)\right] - \left[-\frac{1}{4}\left(\frac{(-1)^3}{3}\right)\right]$

$= -\frac{1}{4}\left[\frac{1}{2} - 2\right] - \frac{1}{12}$

$= \frac{1}{2} - \frac{1}{8} - \frac{1}{12}$

$= \frac{7}{24}$ **[$\frac{1}{2}$ Mark]**

Step 7:

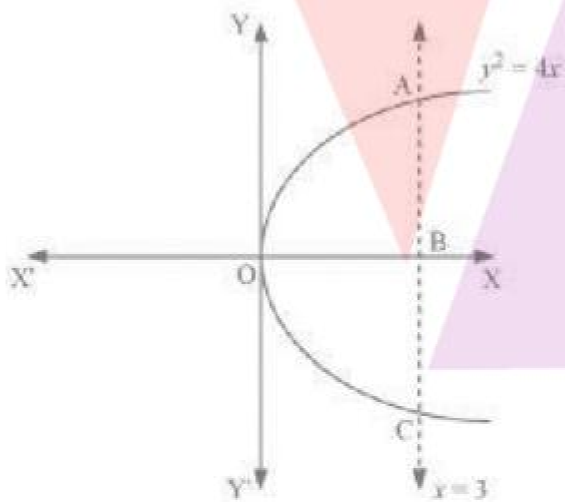
Hence, required area $= \left(\frac{5}{6} + \frac{7}{24}\right) = \frac{9}{8}$ sq. units **[1 Mark]**

11. Find the area of the region bounded by the curve $y^2 = 4x$ and the line $x = 3$. **[2 marks]**

Solution:

Step 1:

Given: Equation of the curve $y^2 = 4x$ and the line $x = 3$



The region bounded by the parabola, $y^2 = 4x$ and the line, $x = 3$ is the area $OACO$. **[$\frac{1}{2}$ Mark]**

Step 2:

The area $OACO$ is symmetrical about x-axis

$\therefore$ Area of $OACO = 2$ (Area of OAB)

Area $OACO = 2\left[\int_0^3 y dx\right]$ **[$\frac{1}{2}$ Mark]**

Step 3:

$$= 2\int_0^3 2\sqrt{x}dx$$

$$= 4\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^3$$ **[$\frac{1}{2}$ Mark]**

Step 4:

$$= \frac{8}{3}\left[(3)^{\frac{3}{2}}\right]$$

$$= 8\sqrt{3}$$

Hence, the required area is $8\sqrt{3}$ Square units. **[$\frac{1}{2}$ Mark]**

12. Area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$ and the lines $x = 0$ and $x = 2$ is **[2 Marks]**

(A) π

(B) $\frac{\pi}{2}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{4}$

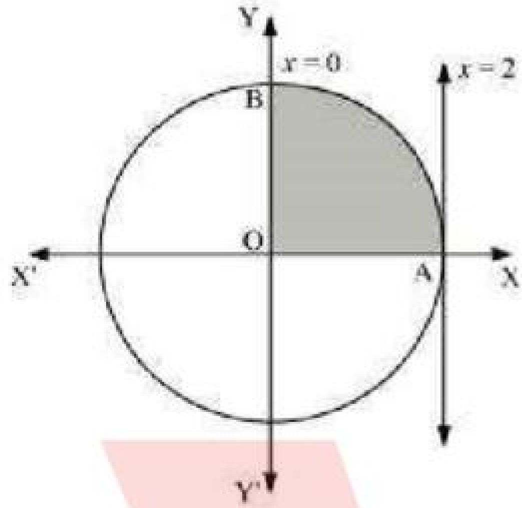
Solution:

(A)

Step 1:

Given: The equation of the circle $x^2 + y^2 = 4$ and the lines $x = 0$ and $x = 2$ is

The area bounded by the given circle and the lines, $x = 0$ and $x = 2$ in the first quadrant is represented as



$[\frac{1}{2}$ **Mark]**

Step 2:

$\therefore$ Area $OAB = \int_0^2 y dx$ $[\frac{1}{2}$ **Mark]**

Step 3:

$= \int_0^2 \sqrt{4 - x^2} dx$ $[\frac{1}{2}$ **Mark]**

Step 4:

$= \left[\frac{x}{2}\sqrt{4 - x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2$

$= 2\left(\frac{\pi}{2}\right)$

$= \pi$ units

Hence, (A) is the correct answer. $[\frac{1}{2}$ **Mark]**

13. Area of the region bounded by the curve $y^2 = 4x$, y-axis and the line $y = 3$ is **[2 Marks]**

(A) 2

(B) $\frac{9}{4}$

(C) $\frac{9}{3}$

(D) $\frac{9}{2}$

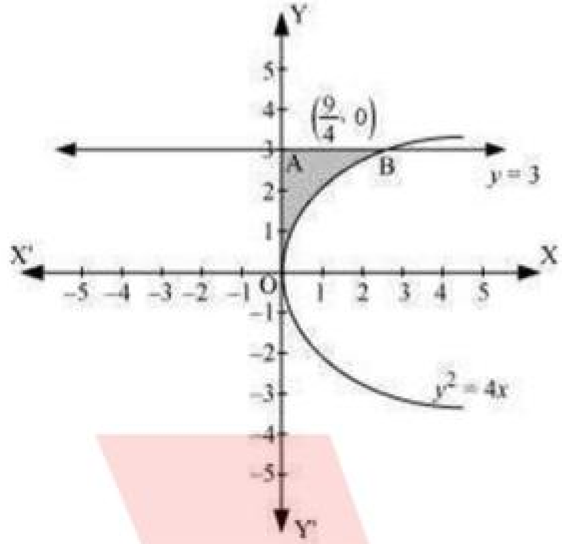
Solution:

(B)

Step 1:

Given: The equation of the curve $y^2 = 4x$, y-axis and the line $y = 3$

The area bounded by the curve, $y^2 = 4x$, y-axis and $y = 3$ is represented as



$\left[\frac{1}{2}\text{ Mark}\right]$

Step 2:

$\therefore$ Area $OAB = \int_0^3 x dy$ $\left[\frac{1}{2}\text{ Mark}\right]$

Step 3:

$= \int_0^3 \frac{y^3}{4} dy$ $\left[\frac{1}{2}\text{ Mark}\right]$

Step 4:

$= \frac{1}{4}\left[\frac{y^3}{3}\right]_0^3$

$= \frac{1}{12}(27)$

$= \frac{9}{4}$ units

Hence, (B) is the correct answer. $\left[\frac{1}{2}\text{ Mark}\right]$

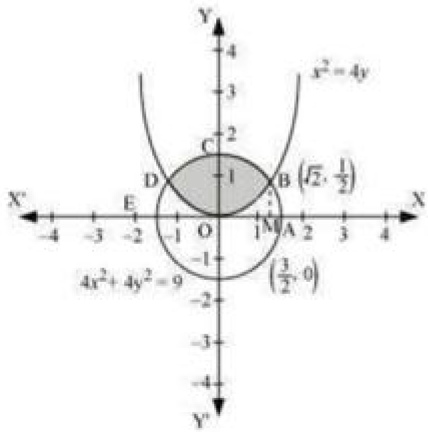
Exercise 8.2

1. Find the area of the circle $4x^2 + 4y^2 = 9$ which is interior to the parabola $x^2 = 4y$.

Solution:

Step 1:

Given: Equation of the circle $4x^2 + 4y^2 = 9$ and the equation of the parabola $x^2 = 4y$.



The required area is represented by the shaded area $OBCDO$. **[$\frac{1}{2}$ Mark]**

Step 2:

Solving the given equation of circle, $4x^2 + 4y^2 = 9$, and parabola, $x^2 = 4y$, we obtain the point of intersection as $B\left(\sqrt{2}, \frac{1}{2}\right)$ and $D\left(-\sqrt{2}, \frac{1}{2}\right)$

It is observed that the required area is symmetrical about y-axis.

$\therefore$ Area $OBCDO = 2 \times$ Area $OBCO$ **[$\frac{1}{2}$ Mark]**

Step 3:

We draw BM perpendicular to OA.

Hence, the coordinates of M are $(\sqrt{2}, 0)$.

Hence, Area $OBCO =$ Area $OMBCO -$ Area $OMBO$

$$= \int_0^{\sqrt{2}} \sqrt{\frac{(9 - 4x^2)}{4}}\, dx - \int_0^{\sqrt{2}} \sqrt{\frac{x^2}{4}}\, dx$$

$$= \frac{1}{2}\int_0^{\sqrt{2}} \sqrt{9 - 4x^2}\, dx - \frac{1}{4}\int_0^{\sqrt{2}} x^2 dx$$

$$= \frac{1}{4}\left[x\sqrt{9 - 4x^2} + \frac{9}{2}\sin^{-1}\frac{2x}{3}\right]_0^{\sqrt{2}} - \frac{1}{4}\left[\frac{x^3}{3}\right]_0^{\sqrt{2}}$$

$$= \frac{1}{4}\left[\sqrt{2}\sqrt{9 - 8} + \frac{9}{2}\sin^{-1}\frac{2\sqrt{2}}{3}\right] - \frac{1}{12}\left(\sqrt{2}\right)^3$$

$$= \frac{\sqrt{2}}{4} + \frac{9}{8}\sin^{-1}\frac{2\sqrt{2}}{3} - \frac{\sqrt{2}}{6}$$

$$= \frac{\sqrt{2}}{12} + \frac{9}{8}\sin^{-1}\frac{2\sqrt{2}}{3}$$

$$= \frac{1}{2}\left(\frac{\sqrt{2}}{6} + \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}\right)$$ **[2 Marks]**

Step 4:

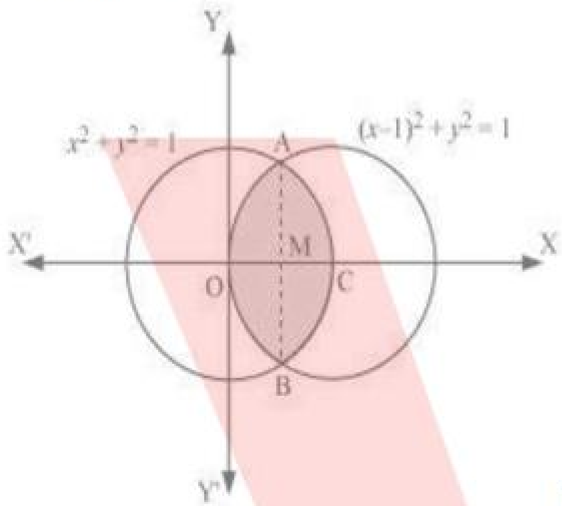
Hence, the required area $OBCDO$ is

$\left(2 \times \frac{1}{2}\left[\frac{\sqrt{2}}{6} + \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}\right]\right) = \left[\frac{\sqrt{2}}{6} + \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}\right]$ square units **[1 Mark]**

2. Find the area bounded by curves $(x-1)^2+y^2=1$ and $x^2+y^2=1$. **[4 Marks]**

Solution:

Step 1:
Given: The equations of the curves $(x-1)^2+y^2=1$ and $x^2+y^2=1$.
The area bounded by the curves, $(x-1)^2+y^2=1$ and $x^2+y^2=1$ is represented by the shaded area as



[$\frac{1}{2}$ Mark]

Step 2:
After solving the equations $(x-1)^2+y^2=1$ and $x^2+y^2=1$, we get the point of intersection as $A\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$ and $B\left(\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$.
It is observed that the required area is symmetrical about x-axis.

$\therefore$ Area $OBCAO = 2\times$ Area $OCAO$ **[$\frac{1}{2}$ Mark]**

Step 3:
We join AB, which intersects OC at M, such that AM is perpendicular to OC.
The coordinates of M are $\left(\frac{1}{2},0\right)$.

$\Rightarrow$ Area $OCAO =$ Area $OMAO +$ Area $MCAM$ **[$\frac{1}{2}$ Mark]**

Step 4:

$$=\left[\int_0^{\frac{1}{2}}\sqrt{1-(x-1)^2}\,dx+\int_{\frac{1}{2}}^{1}\sqrt{1-x^2}\,dx\right]$$

$$=\left[\frac{x-1}{2}\sqrt{1-(x-1)^2}+\frac{1}{2}\sin^{-1}(x-1)\right]_0^{\frac{1}{2}}+\left[\frac{x}{2}\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}x\right]_{\frac{1}{2}}^{1}$$

$$= \left[-\frac{1}{4}\sqrt{1-\left(-\frac{1}{2}\right)^2} + \frac{1}{2}\sin^{-1}\left(\frac{1}{2}-1\right) - \frac{1}{2}\sin^{-1}(-1)\right] + \left[\frac{1}{2}\sin^{-1}(1) - \frac{1}{4}\sqrt{1-\left(\frac{1}{2}\right)^2} - \frac{1}{2}\sin^{-1}\left(\frac{1}{2}\right)\right]$$

$$= \left[-\frac{\sqrt{3}}{8} + \frac{1}{2}\left(-\frac{\pi}{6}\right) - \frac{1}{2}\left(-\frac{\pi}{2}\right)\right] + \left[\frac{1}{2}\left(\frac{\pi}{2}\right) - \frac{\sqrt{3}}{8} - \frac{1}{2}\left(\frac{\pi}{6}\right)\right]$$

$$= \left[-\frac{\sqrt{3}}{4} - \frac{\pi}{12} + \frac{\pi}{4} + \frac{\pi}{4} - \frac{\pi}{12}\right]$$

$$= \left[-\frac{\sqrt{3}}{4} - \frac{\pi}{6} + \frac{\pi}{2}\right]$$

$= \left[\frac{2\pi}{6} - \frac{\sqrt{3}}{4}\right]$ **[1 $\frac{1}{2}$ Mark]**

Step 5:

Hence, required area $OBCAO = 2 \times \left(\frac{2\pi}{6} - \frac{\sqrt{3}}{4}\right) = \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$ sq. units **[1 Mark]**

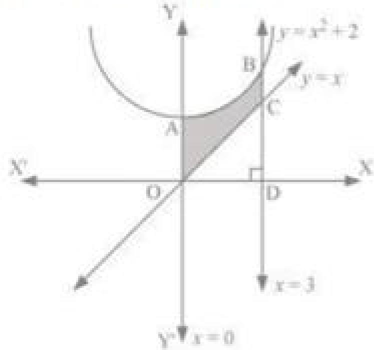
3. Find the area of the region bounded by the curves $y = x^2 + 2, y = x, x = 0$ and $x = 3$ **[2 Marks]**

Solution:

Step 1:

Given: The equations $y = x^2 + 2, y = x, x = 0$ and $x = 3$

The area bounded by the curves $y = x^2 + 2, y = x, x = 0$ and $x = 3$ is represented by the shaded area $OCBAO$ as



[$\frac{1}{2}$ Mark]

Step 2:

Then, Area $OCBAO$ = Area $ODBAO$ − Area $ODCO$

$= \int_0^3 (x^2 + 2)dx - \int_0^3 x dx$ **[1 Mark]**

Step 4:

$= \left[\frac{x^3}{3} + 2x\right]_0^3 - \left[\frac{x^2}{2}\right]_0^3$

$= [9 + 6] - \left[\frac{9}{2}\right]$

$= 15 - \frac{9}{2}$

$= \frac{21}{2}$ square units **[$\frac{1}{2}$ Mark]**

Hence, the required area is $\frac{21}{2}$ square units.

4. Using integration find the area of the region bounded by the triangle whose vertices are $(-1, 0), (1, 3)$ and $(3, 2)$. **[6 Marks]**

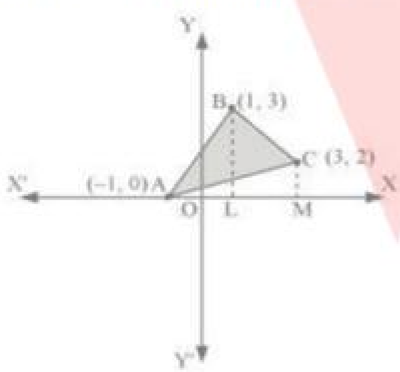
Solution:

Step 1:

Given: The vertices of a triangle $(-1, 0), (1, 3)$ and $(3, 2)$.

Step 2:

BL and CM are drawn perpendicular to x-axis.



It is observed in the figure that

$\text{Area}(\Delta ACB) = \text{Area}(ALBA) + \text{Area}(BLMCB) - \text{Area}(AMCA)$ ...(i) **[1 Mark]**

Step 3:

Equation of line segment AB is

$y - 0 = \frac{3 - 0}{1 + 1}(x + 1)$

$y = \frac{3}{2}(x + 1)$ **[$\frac{1}{2}$ Mark]**

Step 4:

$\therefore \text{Area(ALBA)} = \int_{-1}^{1} \frac{3}{2}(x + 1)dx = \frac{3}{2}\left[\frac{x^2}{2} + x\right]_{-1}^{1} = \frac{3}{2}\left[\frac{1}{2} + 1 - \frac{1}{2} + 1\right] = 3$ sq. units. **[1 Mark]**

Step 5:

Equation of line segment BC is

$$y-3=\frac{2-3}{3-1}(x-1)$$

$y=\frac{1}{2}(-x+7)$ **[$\frac{1}{2}$ Mark]**

Step 6:

$\therefore \text{Area}(BLMCB)=\int_1^3 \frac{1}{2}(-x+7)dx=\frac{1}{2}\left[-\frac{x^2}{2}+7x\right]_1^3=\frac{1}{2}\left[-\frac{9}{2}+21+\frac{1}{2}-7\right]=5$ square units.

[1 Mark]

Step 7:

Equation of line segment AC is

$$y-0=\frac{2-0}{3+1}(x+1)$$

$y=\frac{1}{2}(x+1)$ **[$\frac{1}{2}$ Mark]**

$\therefore \text{Area}(AMCA)=\frac{1}{2}\int_{-1}^3(x+1)dx=\frac{1}{2}\left[\frac{x^2}{2}+x\right]_{-1}^3=\frac{1}{2}\left[\frac{9}{2}+3-\frac{1}{2}+1\right]=4$ square units. **[1 Mark]**

Step 8:

Hence, from equation (i), we obtain $\text{Area}(\Delta ABC)=(3+5-4)=4$ square units. **[$\frac{1}{2}$ Mark]**

5. Using integration find the area of the triangular region whose sides have the equations $y=2x+1, y=3x+1$ and $x=4$. **[4 Marks]**

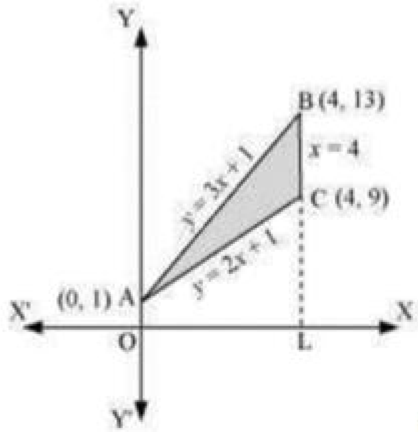
Solution:

Step 1:

Given: The equations of the sides of a triangle: $y=2x+1, y=3x+1$ and $x=4$.

Step 2:

On solving these equations, we obtain the vertices of triangle as $A(0,1), B(4,13)$ and $C(4,9)$.



$[1\frac{1}{2}$ **mark]**

Step 3:

It is observed that,

$\text{Area}(\Delta ACB) = \text{Area}(OLBAO) - \text{Area}(OLCAO)$ **[$\frac{1}{2}$ Mark]**

Step 4:

$= \int_0^4 (3x+1)dx - \int_0^4 (2x+1)dx$ **[1 Mark]**

Step 5:

$= \left[\frac{3x^2}{2} + x\right]_0^4 - \left[\frac{2x^2}{2} + x\right]_0^4$

$= (24+4) - (16+4)$

$= 28 - 20$

$= 8$ square units **[1 Mark]**

Hence, the area of the triangular region is 8 square units.

6. Smaller area enclosed by the circle $x^2 + y^2 = 4$ and the line $x + y = 2$ is **[2 Marks]**

(A) $2(\pi - 2)$

(B) $\pi - 2$

(C) $2\pi - 1$

(D) $2(\pi + 2)$

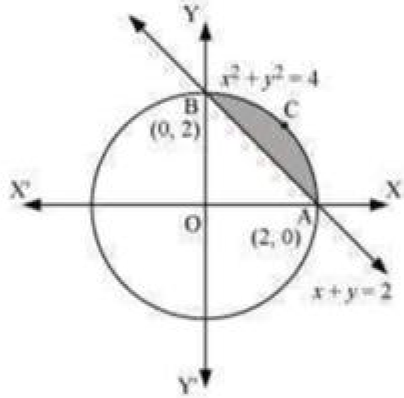
Solution:

(B)

Step 1:

Given: The equation of the circle $x^2 + y^2 = 4$ and the line $x + y = 2$.

Step 2:



The smaller area enclosed by the circle, $x^2 + y^2 = 4$ and the line $x + y = 2$ is represented by the shaded area $ACBA$. **[$\frac{1}{2}$ Mark]**

Step 3:

It is observed that,

Area $ACBA$ = Area $OACBO$ − Area (ΔOAB) **[$\frac{1}{2}$ Mark]**

Step 4:

$= \int_0^2 \sqrt{4-x^2}dx - \int_0^2 (2-x)dx$

$= \left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2 - \left[2x - \frac{x^2}{2}\right]_0^2$ **[$\frac{1}{2}$ Mark]**

Step 5:

$= \left[2.\frac{\pi}{2}\right] - [4-2]$

$= (\pi - 2)$ square units

Hence, (B) is the correct answer. **[$\frac{1}{2}$ Mark]**

7. Area lying between the curve $y^2 = 4x$ and $y = 2x$ is **[2 Marks]**

(A) $\frac{2}{3}$

(B) $\frac{1}{3}$

(C) $\frac{1}{4}$

(D) $\frac{3}{4}$

Solution:

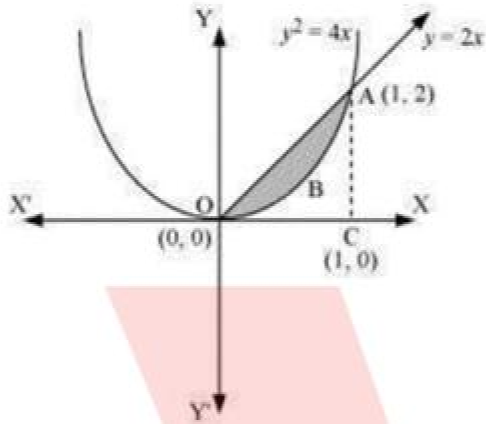
(B)

Step 1:

Given: The equations of the curve $y^2 = 4x$ and $y = 2x$

Step 2:

The area lying between the curve $y^2 = 4x$ and $y = 2x$ is represented by the shaded area $OBAO$ as



The points of intersection of these curves are $O(0,0)$ and $A(1,2)$. **[$\frac{1}{2}$ Mark]**

Step 3:

We draw AC perpendicular to x-axis such that the coordinates of C are $(1,0)$.

$\therefore$ Area $OBAO = \text{Area } (\Delta OCA) - \text{Area } (OCABO)$ **[$\frac{1}{2}$ Mark]**

Step 4:

$= \int_0^1 2x dx - \int_0^1 2\sqrt{x}\ dx$ **[$\frac{1}{2}$ Mark]**

Step 5:

$$= 2\left[\frac{x^2}{2}\right]_0^1 - 2\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1$$

$$= \left|1 - \frac{4}{3}\right|$$

$$= \left|-\frac{1}{3}\right|$$

$= \frac{1}{3}$ square units

hence, (B) is the correct answer. **[$\frac{1}{2}$ Mark]**

Miscellaneous

1. Find the area under the given curves and given lines:

(i) $y = x^2, x = 1, x = 2$ and x-axis **[2 Marks]**

(ii) $y = x^4, x = 1, x = 5$ and x-axis **[2 Marks]**

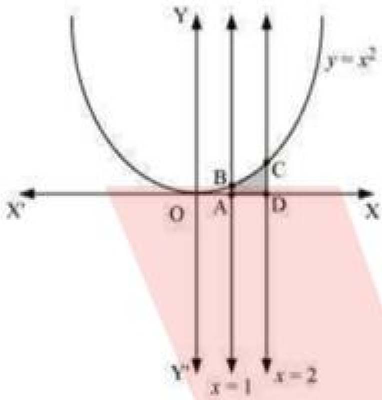
Solution:

(i) **Step 1:**

Given: The equations of the curves: $y = x^2, x = 1, x = 2$ and x-axis

Step 2:

The required area is represented by the shaded area $ADCBA$ as



[$\frac{1}{2}$ **Mark]**

Step 3:

Area $ADCBA = \int_1^2 y dx$ [$\frac{1}{2}$ **Mark]**

Step 4:

$= \int_1^2 x^2 dx$

$= \left[\frac{x^3}{3}\right]_1^2$

$= \frac{8}{3} - \frac{1}{3}$

$= \frac{7}{3}$ square units **[1 Mark]**

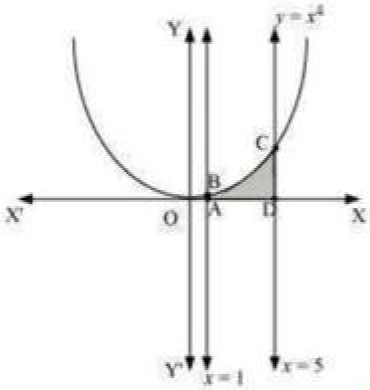
Hence, the required area is $\frac{7}{3}$ square units.

(ii)**Step 1:**

Given: The equations of the curves are: $y = x^4, x = 1, x = 5$ and x-axis.

Step 2:

The required area is represented by the shaded area $ADCBA$



[$\frac{1}{2}$ **Mark]**

Step 3:

Area $ADCBA = \int_1^5 x^4 dx$ [$\frac{1}{2}$ **Mark]**

Step 4:

$= \left[\frac{x^5}{5}\right]_1^5$

$= \frac{(5)^5}{5} - \frac{1}{5}$

$= (5)^4 - \frac{1}{5}$

$= 625 - \frac{1}{5}$

$= 624.8$ square units **[1 Mark]**

Hence, the required area is 624.8 square units

2. Find the area between the curves $y = x$ and $y = x^2$. **[4 Marks]**

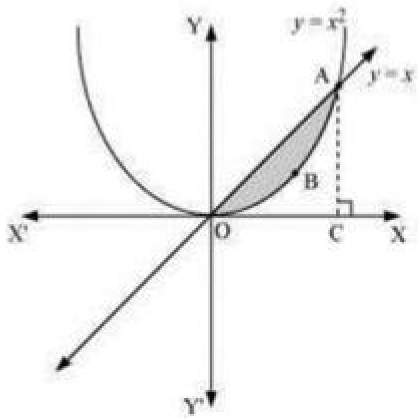
Solution:

Step 1:

Given: The equation of the curve are $y = x$ and $y = x^2$

Step 2:

The required area is represented by the shaded area $OBAO$ as



[$\frac{1}{2}$ Mark]

Step 3:

The points of intersection of the curves, $y = x$ and $y = x^2$ is $A(1, 1)$ and $O(0, 0)$.

We draw AC perpendicular to x-axis. [$\frac{1}{2}$ **Mark]**

Step 4:

$\therefore$ Area $(OBAO)$ = Area (ΔOCA) − Area $(OCABO)$ ...(i) **[1 Mark]**

Step 5:

$= \int_0^1 xdx - \int_0^1 x^2 dx$ **[1 Mark]**

Step 6:

$$= \left[\frac{x^2}{2}\right]_0^1 - \left[\frac{x^3}{3}\right]_0^1$$

$$= \frac{1}{2} - \frac{1}{3}$$

$= \frac{1}{6}$ square units. **[1 Mark]**

3. Find the area of the region lying in the first quadrant and bounded by $y = 4x^2, x = 0, y = 1$ and $y = 4$. **[2 Marks]**

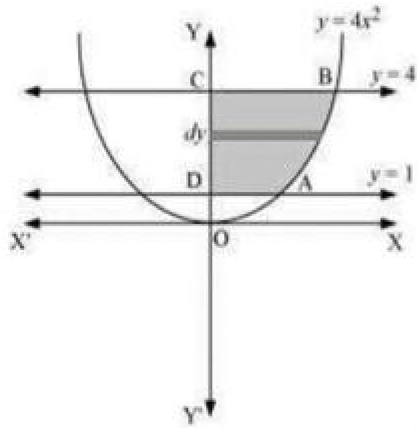
Solution:

Step 1:

Given: The equations of the curves are: $y = 4x^2, x = 0, y = 1$ and $y = 4$.

Step 2:

The area in the first quadrant bounded by $y = 4x^2, x = 0, y = 1$ and $y = 4$ is represented by the shaded area $ABCDA$ as



[$\frac{1}{2}$ **Mark]**

Step 3:

$\therefore$ Area $ABCD = \int_1^4 x dy$ [$\frac{1}{2}$ **Mark]**

Step 4:

$= \int_1^4 \frac{\sqrt{y}}{2} dy$ [$\frac{1}{2}$ **Mark]**

Step 5:

$= \frac{1}{2}\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^4$

$= \frac{1}{3}\left[(4)^{\frac{3}{2}} - 1\right]$

$= \frac{1}{3}[8 - 1]$

$= \frac{7}{3}$ square units [$\frac{1}{2}$ **Mark]**

Hence, the required area is $\frac{7}{3}$ square units.

4. Sketch the graph of $y = |x + 3|$ and evaluate $\int_{-6}^{0} |x + 3| dx$. **[4 Marks]**

Solution:

Step 1:

Given: The equation is $y = |x + 3|$

Step 2:

Some corresponding values of x and y are given in the table.

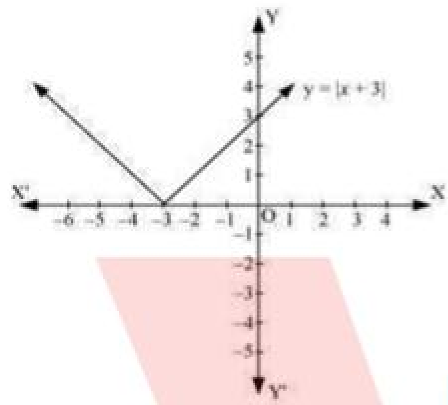
x	-6	-5	-4	-3	-2	-1	0

y	3	2	1	0	1	2	3

[$\frac{1}{2}$ Mark]

Step 3:

After plotting these points, we obtain the graph of $y = |x + 3|$ as follows.



[$1\frac{1}{2}$ Marks]

Step 4:

As we know that $(x + 3) \le 0$ for $-6 \le x \le -3$ and $(x + 3) \ge 0$ for $-3 \le x \le 0$

$\therefore \int_{-6}^{0} |(x + 3)| dx = -\int_{-6}^{-3} (x + 3) dx + \int_{-3}^{0} (x + 3) dx$ **[1 mark]**

Step 5:

$$= -\left[\frac{x^2}{2} + 3x\right]_{-6}^{-3} + \left[\frac{x^2}{2} + 3x\right]_{-3}^{0}$$

$$= -\left[\left(\frac{(-3)^2}{2} + 3(-3)\right) - \left(\frac{(-6)^2}{2} + 3(-6)\right)\right] + \left[0 - \left(\frac{(-3)^2}{2} + 3(-3)\right)\right]$$

$$= -\left[-\frac{9}{2}\right] - \left[-\frac{9}{2}\right]$$

$= 9$ **[1 Mark]**

Hence, the required area is 9 square units.

5. Find the area bounded by the curve $y = \sin x$ between $x = 0$ and $x = 2\pi$. **[2 Marks]**

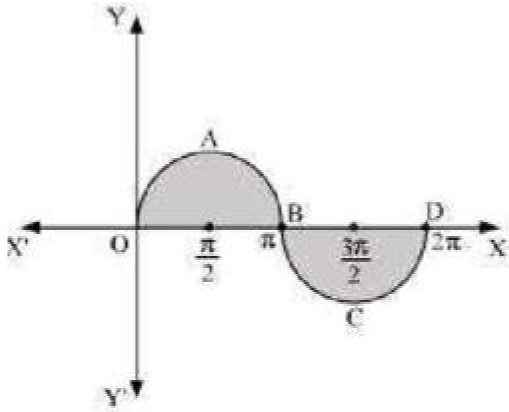
Solution:

Step 1:

Given: The equation of the curve: $y = \sin x$ bounded between $x = 0$ and $x = 2\pi$

Step 2:

The graph of $y = \sin x$ is drawn as



$\therefore$ Required area = Area $OABO$ + Area $BCDB$ **[$\frac{1}{2}$ Mark]**

Step 3:

$= \int_0^{\pi} \sin x\, dx + \left|\int_{\pi}^{2\pi} \sin x\, dx\right|$ **[$\frac{1}{2}$ Mark]**

Step 4:

$= [-\cos x]_0^{\pi} + |[-\cos x]_x^{2\pi}|$

$= [-\cos \pi + \cos 0] + |-\cos 2\pi + \cos \pi|$ **[$\frac{1}{2}$ Mark]**

Step 5:

$= 1 + 1 + |(-1 - 1)|$

$= 2 + |-2|$

$= 2 + 2 = 4$ square units **[$\frac{1}{2}$ Mark]**

Hence, the required area is 4 square units.

6. Find the area enclosed between the parabola $y^2 = 4ax$ and the line $y = mx$. **[6 marks]**

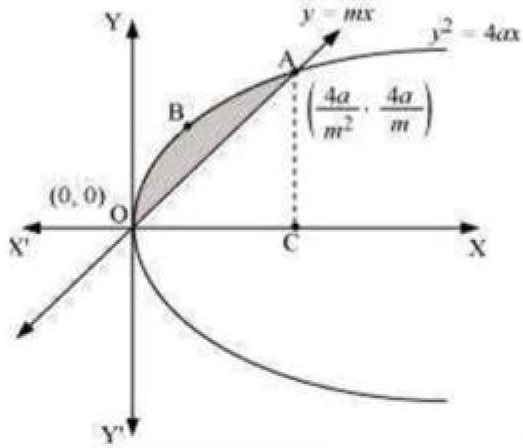
Solution:

Step 1:

Given: The equation of parabola: $y^2 = 4ax$ and the line $y = mx$

Step 2:

The area enclosed between the parabola, $y^2 = 4ax$ and the line $y = mx$ is represented by the shaded area $OABO$ as



[$\frac{1}{2}$ Mark]

Step 3:

The points of intersection of both the curves are $(0, 0)$ and $\left(\frac{4a}{m^2}, \frac{4a}{m}\right)$. **[$\frac{1}{2}$ Mark]**

Step 4:

We draw AC perpendicular to x-axis.

$\therefore$ Area $OABO =$ Area $OCABO -$ Area (ΔOCA) **[1 Mark]**

Step 5:

$= \int_0^{\frac{4a}{m^2}} 2\sqrt{ax}\ dx - \int_0^{\frac{4a}{m^2}} mx\,dx$ **[1 Mark]**

Step 6:

$$= 2\sqrt{a}\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^{\frac{4a}{m^2}} - m\left[\frac{x^2}{2}\right]_0^{\frac{4a}{m^2}}$$

$$= \frac{4}{3}\sqrt{a}\left(\frac{4a}{m^2}\right)^{\frac{3}{2}} - \frac{m}{2}\left[\left(\frac{4a}{m^2}\right)^2\right]$$

$$= \frac{32a^2}{3m^3} - \frac{m}{2}\left(\frac{16a^2}{m^4}\right)$$

$= \frac{32a^2}{3m^3} - \frac{8a^2}{m^3}$ **[2 Marks]**

Step 7:

$= \frac{8a^2}{3m^3}$ square units **[1 Mark]**

Hence, the required area is $\frac{8a^2}{3m^3}$ square units

7. Find the area enclosed by the parabola $4y = 3x^2$ and the line $2y = 3x + 12$. **[4 Marks]**

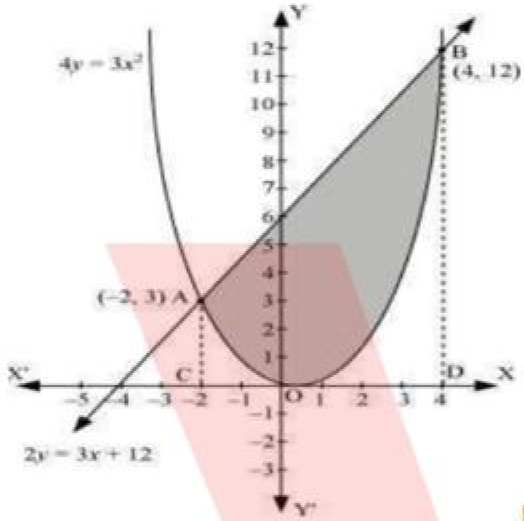
Solution:

Step 1:

Given: Equation of the parabola: $4y = 3x^2$. Equation of the line: $2y = 3x + 12$

Step 2:

The area enclosed between the parabola, $4y = 3x^2$ and the line $2y = 3x + 12$ is represented by the shaded area $OBAO$ as



[1 Mark]

Step 3:

The points of intersection of the given curves are $A(-2, 3)$ and $B(4, 12)$. **[$\frac{1}{2}$ Mark]**

Step 4:

We draw AC and BD perpendicular to x-axis.

$\therefore$ Area $OBAO =$ Area $CDBA -$ (Area $ODBO +$ Area $OACO$) **[$\frac{1}{2}$ Mark]**

Step 5:

$$= \int_{-2}^{1} \frac{1}{2}(3x + 12)dx - \int_{-2}^{1} \frac{3x^2}{4} dx$$

$$= \frac{1}{2}\left[\frac{3x^2}{2} + 12x\right]_{-2}^{4} - \frac{3}{4}\left[\frac{x^3}{3}\right]_{-2}^{4}$$

$$= \frac{1}{2}[24 + 48 - 6 + 24] - \frac{1}{4}[64 + 8]$$

$$= \frac{1}{2}[90] - \frac{1}{4}[72]$$

$= 45 - 18$

$= 27$ square units **[2 Marks]**

Hence, the required area is 27 square units.

8. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line $\frac{x}{3} + \frac{y}{2} = 1$.

[6 Marks]

Solution:

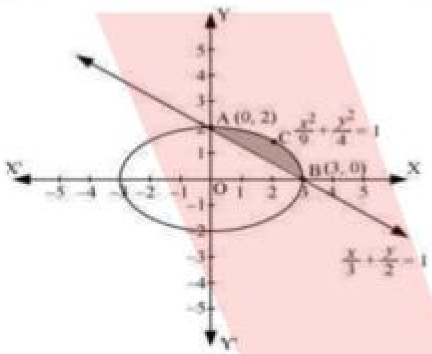
Step 1:

Given: Equation of the ellipse: $\frac{x^2}{9}+\frac{y^2}{4}=1.$

Equation of the line: $\frac{x}{3}+\frac{y}{2}=1$

Step 2:

The area of the smaller region bounded by the ellipse, $\frac{x^2}{9}+\frac{y^2}{4}=1$ and the line, $\frac{x}{3}+\frac{y}{2}=1$ is represented by the shaded region $BCAB$



[1 Mark]

Step 3:

The points of intersection obtained after solving the two equations are $A(0,2)$ and $B(3,0)$. **[$\frac{1}{2}$ Mark]**

Step 4:

$\therefore$ Area $BCAB=$ Area $(OBCAO)-$ Area $(OBAO)$ **[$\frac{1}{2}$ Mark]**

Step 5:

$\frac{x^2}{9}+\frac{y^2}{4}=1$

Hence, $y=2\sqrt{1-\frac{x^2}{9}}=y_1$

$\frac{x}{3}+\frac{y}{2}=1$

Hence, $y=2\left(1-\frac{x}{3}\right)=y_2$ **[1 Mark]**

Step 6:

$=\int_0^3 y_1 dx-\int_0^3 y_2 dx$ **[1 Mark]**

Step 7:

$=\int_0^3 2\sqrt{1-\frac{x^2}{9}}dx-\int_0^3 2\left(1-\frac{x}{3}\right)dx$

$$=\frac{2}{3}\left[\int_0^3 \sqrt{9-x^2}\,dx\right]-\frac{2}{3}\int_0^3 (3-x)dx$$

$= \frac{2}{3}\left[\frac{x}{2}\sqrt{9-x^2}+\frac{9}{2}\sin^{-1}\frac{x}{3}\right]_0^3 - \frac{2}{3}\left[3x-\frac{x^2}{2}\right]_0^3$

$= \frac{2}{3}\left[\frac{9}{2}\left(\frac{\pi}{2}\right)\right] - \frac{2}{3}\left[9-\frac{9}{2}\right]$

$= \frac{2}{3}\left[\frac{9\pi}{4}-\frac{9}{2}\right]$

$= \frac{2}{3}\times\frac{9}{4}(\pi-2)$ **[1$\frac{1}{2}$ Marks]**

Step 8:

$= \frac{3}{2}(\pi-2)$ square units. **[$\frac{1}{2}$ Mark]**

Hence, the required area is $\frac{3}{2}(\pi-2)$ square units.

9. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the line $\frac{x}{a}+\frac{y}{b}=1$.
[6 Marks]

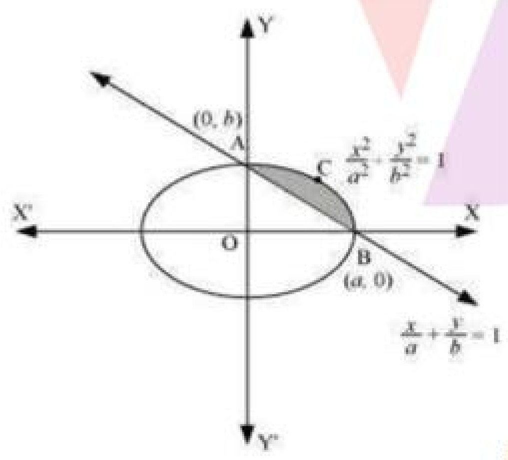
Solution:

Step 1:

Given: The equation of the ellipse: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the line $\frac{x}{a}+\frac{y}{b}=1$

Step 2:

The area of the smaller region bounded by the ellipse, $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the line, $\frac{x}{a}+\frac{y}{b}=1$ is represented by the shaded region $BCAB$ as



[1 Mark]

Step 3:

The points of intersection after solving the two given equations: $A(0,b)$ and $B(a,0)$ **[$\frac{1}{2}$ Mark]**

Step 4:

$\therefore$ Area $BCAB =$ Area $(OBCAO) -$ Area $(OBAO)$ **[$\frac{1}{2}$ Mark]**

Step 5:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

Hence, $y = b\sqrt{1-\frac{x^2}{a^2}} = y_1$

$$\frac{x}{a}+\frac{y}{b}=1$$

Hence, $y = b\left(1-\frac{x}{a}\right) = y_2$ **[1 Mark]**

Step 6:

$= \int_0^a y_1 \, dx - \int_0^a y_2 dx$ **[1 Mark]**

Step 7:

$$= \int_0^a b\sqrt{1-\frac{x^2}{a^2}}\,dx - \int_0^a b\left(1-\frac{x}{a}\right)dx$$

$$= \frac{b}{a}\int_0^a \sqrt{a^2-x^2}dx - \frac{b}{a}\int_0^a (a-x)dx$$

$$= \frac{b}{a}\left[\left\{\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right\}_0^a - \left\{ax-\frac{x^2}{2}\right\}_0^a\right]$$

$$= \frac{b}{a}\left[\left\{\frac{a^2}{2}\left(\frac{\pi}{2}\right)\right\} - \left\{a^2-\frac{a^2}{2}\right\}\right]$$

$$= \frac{b}{a}\left[\frac{a^2\pi}{4}-\frac{a^2}{2}\right]$$

$$= \frac{ba^2}{2a}\left[\frac{\pi}{2}-1\right]$$

$= \frac{ab}{2}\left[\frac{\pi}{2}-1\right]$ **[$1\frac{1}{2}$ Mark]**

Step 8:

$= \frac{ab}{4}(\pi-2)$ square units **[$\frac{1}{2}$ Mark]**

Hence, the required area is $\frac{ab}{4}(\pi-2)$ square units.

10. Find the area of the region enclosed by the parabola $x^2 = y$ the line $y = x + 2$ and the x-axis.
[6 Marks]

Solution:

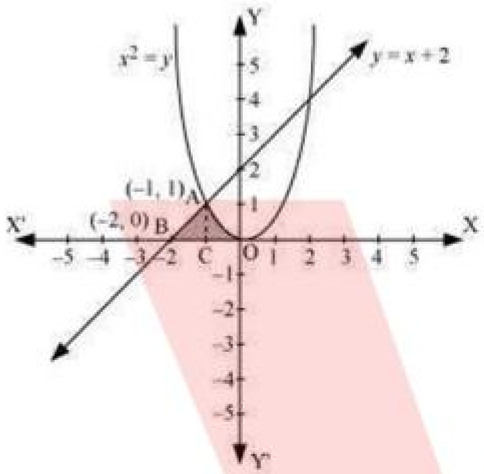
Step 1:

Given: Equation of the parabola $x^2 = y$

Equation of the line: $y = x + 2$ and x-axis.

Step 2:

The area of the region enclosed by the parabola $x^2 = y$ the line, $y = x + 2$ and x-axis is represented by the shaded region $OABCO$ as



[1 Mark]

Step 3:

The point of intersection of the parabola $x^2 = y$ and the line $y = x + 2$ is $A(-1, 1)$. **[$\frac{1}{2}$ Mark]**

Step 4:

$\therefore$ Area $OABCO =$ Area $(BCA) +$ Area $COAC$ **[$\frac{1}{2}$ Mark]**

Step 5:

Let $y_1 = x^2$ and $y_2 = x + 2$

Required area = $= \int_{-2}^{-1} y_2 dx + \int_{-1}^{0} y_1 dx$ **[1 Mark]**

Step 6:

$$= \int_{-2}^{-1} (x + 2)dx + \int_{-1}^{0} x^2 dx$$

$= \left[\frac{x^2}{2} + 2x\right]_{-2}^{-1} + \left[\frac{x^3}{3}\right]_{-1}^{0}$ **[1 Mark]**

Step 7:

$$= \left[\frac{(-1)^2}{2} + 2(-1) - \frac{(-2)^2}{2} - 2(-2)\right] + \left[-\frac{(-1)^3}{3}\right]$$

$= \left[\frac{1}{2} - 2 - 2 + 4 + \frac{1}{3}\right]$ **[1 Mark]**

Step 8:

$= \frac{5}{6}$ square units **[1 Mark]**

Hence, the required area is $\frac{5}{6}$ square units.

11. Using the method of integration find the area bounded by the curve $|x| + |y| = 1$. [Hint: The required region is bounded by lines $x + y = 1, x - y = 1, -x + y = 1$ and $-x - y = 1$]. **[4 Marks]**

Solution:

Step 1:

Given: Equation of the curve: $|x| + |y| = 1$

Step 2:

The given curve can be visualized as four different lines.

$x + y = 1$ for $x > 0, y > 0$

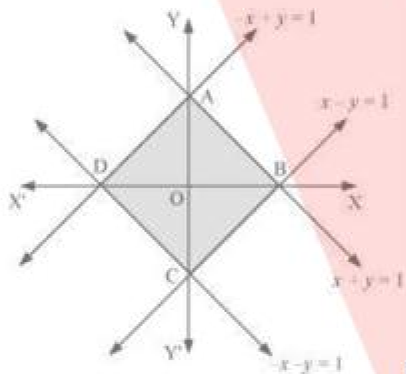
$-x + y = 1$ for $x < 0, y > 0$

$x - y = 1$ for $x > 0, y < 0$

$-x - y = 1$ for $x < 0, y < 0$ **[1 Mark]**

Step 3:

The area bounded by the curve, $|x| + |y| = 1$ is represented by the shaded region $ADCB$ as



[1 Mark]

Step 4:

The curve intersects the axes at points $A(0,1), B(1,0), C(0,-1)$ and $D(-1,0)$. It can be observed that the given curve is symmetrical about x-axis and y-axis.

$\therefore$ Area $ADCB = 4 \times$ Area $OBAO$ **[1 Mark]**

Step 5:

$$= 4\int_0^1 (1 - x)dx$$

$$= 4\left(x - \frac{x^2}{2}\right)_0^1$$

$$= 4\left[1 - \frac{1}{2}\right]$$

$$= 4\left(\frac{1}{2}\right)$$

$= 2$ square units **[1 Mark]**

Hence, the required area is 2 square units.

12. Find the area bounded by curves $\{(x, y): y \geq x^2 \text{ and } y = |x|\}$. **[6 Marks]**

Solution:

Step 1:

Given: The equations of the curves: $\{(x, y): y \geq x^2 \text{ and } y = |x|\}$

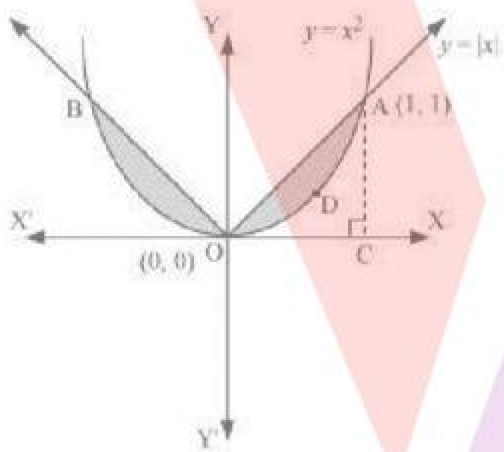
Step 2:

$y = |x|$

$= x, x \geq 0$

$\quad -x, x \leq 0$

Hence, the area bounded by the curves, $\{(x,\ y): y \geq x^2$ and $y = |x|\}$ is represented by the shaded region:



[$1\frac{1}{2}$ Mark]

Step 3: The points of intersection obtained by solving the given equations are: $A(1,1)$ and $B(-1,1)$.

It is observed that the required area is symmetrical about y-axis.

Required area $= 2[\text{Area } (OCAO) - \text{Area}(OCADO)]$ **[1 Mark]**

Step 4:

$y_1 = y = x$

$y_2 = y = x^2$

Step 5:

$= 2\left[\int_0^1 y_1 dx - \int_0^1 y_2 dx\right]$ **[1 mark]**

Step 6:

$$= 2\left[\int_0^1 x dx - \int_0^1 x^2 dx\right]$$

$= 2\left[\left[\frac{x^2}{2}\right]_0^1 - \left[\frac{x^3}{3}\right]_0^1\right]$

$= 2\left[\frac{1}{2} - \frac{1}{3}\right]$

$= 2\left[\frac{1}{6}\right] = \frac{1}{3}$ square units **[$2\frac{1}{2}$ Marks]**

Hence, the required area is $\frac{1}{3}$ square units

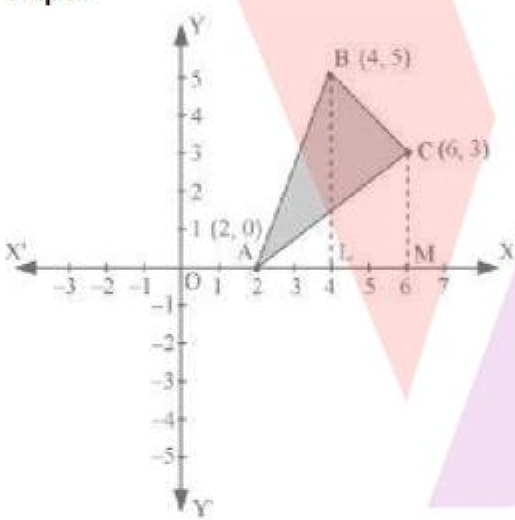
13. Using the method of integration find the area of the triangle ABC, coordinates of whose vertices are $A(2,0), B(4,5)$ and $C(6,3)$. **[6 Marks]**

Solution:

Step 1:

Given: The vertices of ΔABC are $A(2,0), B(4,5)$ and $C(6,3)$.

Step 2:



[1 Mark]

Step 2:

Equation of line segment AB is

$y - 0 = \frac{5-0}{4-2}(x-2)$

$2y = 5x - 10$

$y = \frac{5}{2}(x-2)$ ...(i) **[$\frac{1}{2}$ Mark]**

Step 3:

Equation of line segment BC is

$y - 5 = \frac{3-5}{6-4}(x-4)$

$2y - 10 = -2x + 8$
$2y = -2x + 18$
$y = -x + 9$ ...(ii) **[$\frac{1}{2}$ Mark]**

Step 4:

Equation of line segment CA is

$y - 3 = \frac{0-3}{2-6}(x - 6)$
$-4y + 12 = -3x + 18$
$4y = 3x - 6$
$y = \frac{3}{4}(x - 2)$ ...(iii) **[$\frac{1}{2}$ Mark]**

Step 5:

Area $(\Delta ABC) = \text{Area }(ABLA) + \text{Area }(BLMCB) - \text{Area }(ACMA)$ **[1 Mark]**

Step 6:

$= \int_2^4 \frac{5}{2}(x-2)dx + \int_4^6(-x+9)\,dx - \int_2^6 \frac{3}{4}(x-2)dx$ **[1 Mark]**

Step 7:

$= \frac{5}{2}\left[\frac{x^2}{2} - 2x\right]_2^4 + \left[\frac{-x^2}{2} + 9x\right]_4^6 - \frac{3}{4}\left[\frac{x^2}{2} - 2x\right]_2^6$ **[1 Mark]**

$= \frac{5}{2}[8 - 8 - 2 + 4] + [-18 + 54 + 8 - 36] - \frac{3}{4}[18 - 12 - 2 + 4]$

$= 5 + 8 - \frac{3}{4}(8)$

$= 13 - 6$

Step 8:

$= 7$ square units **[$\frac{1}{2}$ Mark]**

Hence, the required area is 7 square units.

14. Using the method of integration find the area of the region bounded by lines: $2x + y = 4, 3x - 2y = 6$ and $x - 3y + 5 = 0$ **[6 Marks]**

Solution:

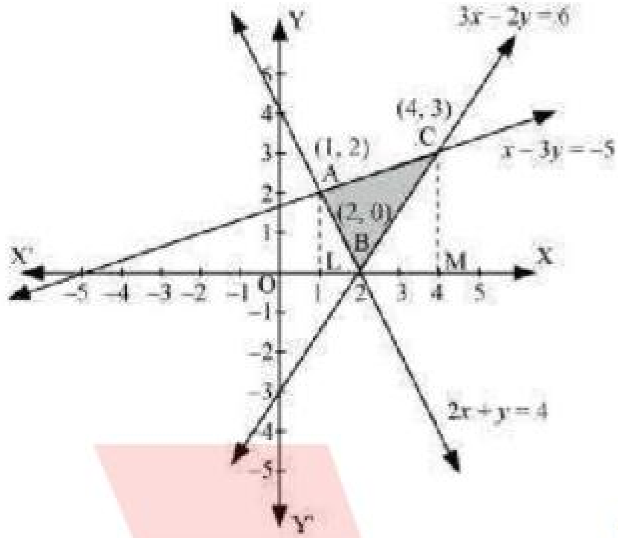
Step 1:

Given:

The given equations of lines are

$2x + y = 4$ ...(i)
$3x - 2y = 6$ ...(ii)
And $x - 3y + 5 = 0$ ...(iii)

Step 2:



[1 Mark]

Step 3: The points of intersection of the three lines obtained by solving them simultaneously are: $A(1,2)$, $B(2,0)$ and $C(4,3)$

The area of the region bounded by the lines is the area of ΔABC.

AL and CM are the perpendiculars on x-axis. **[1 Mark]**

Step 4:

$\text{Area}(\Delta ABC) = \text{Area}(ALMCA) - \text{Area}(ALB) - \text{Area}(CMB)$ **[1 Mark]**

Step 5:

$= \int_2^4 \left(\frac{x+5}{3}\right) dx - \int_1^2 (4-2x)\, dx - \int_2^4 \left(\frac{3x-6}{2}\right) dx$ **[1 Mark]**

Step 6:

$= \frac{1}{3}\left[\frac{x^2}{2} + 5x\right]_1^4 - [4x - x^2]_1^2 - \frac{1}{2}\left[\frac{3x^2}{2} - 6x\right]_2^4$ **[1 Mark]**

$= \frac{1}{3}\left[8 + 20 - \frac{1}{2} - 5\right] - [8 - 4 - 4 + 1] - \frac{1}{2}[24 - 24 - 6 + 12]$

Step 7:

$= \left(\frac{1}{3} \times \frac{45}{2}\right) - (1) - \frac{1}{2}(6)$

$= \frac{15}{2} - 1 - 3$

$= \frac{15}{2} - 4 = \frac{15-8}{2} = \frac{7}{2}$ square units **[1 Mark]**

hence, the required area is $\frac{7}{2}$ square units

15. Find the area of the region $\{(x, y): y^2 \leq 4x, 4x^2 + 4y^2 \leq 9\}$ **[6 Marks]**

Solution:

Step 1:

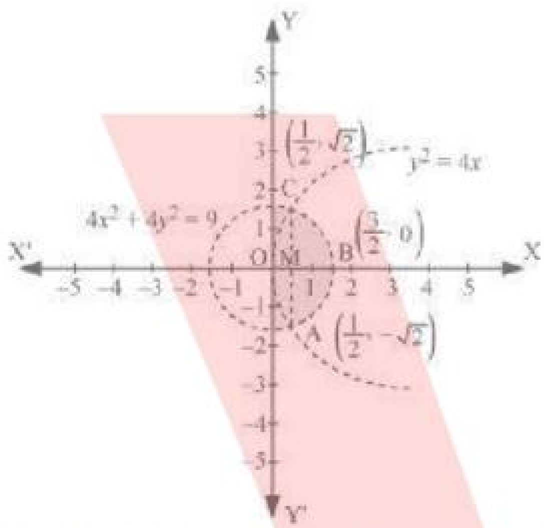
Given: Equation of the curve: $\{(x, y): y^2 \leq 4x, 4x^2 + 4y^2 \leq 9\}$

Step 2:

The equation $y^2 \leq 4x$ represents the region interior to a parabola, symmetric about x-axis.

The equation $4x^2 + 4y^2 \leq 9$ represents the region interior to a circle with center at the origin and radius $\frac{3}{2}$.

The area bounded by the curves $\{(x, y): y^2 \leq 4x, 4x^2 + 4y^2 \leq 9\}$ is represented as



The points of intersection of both the curves obtained by solving the given equations simultaneously are $\left(\frac{1}{2}, \sqrt{2}\right)$ and $\left(\frac{1}{2}, -\sqrt{2}\right)$. **[1 Mark]**

Step 3:

The required area is given by $OABCO$.

It is observed that area $OABCO$ is symmetrical about x-axis.

$\therefore$ Area $OABCO = 2 \times$ Area OBC **[1 Mark]**

Step 4:

Area $OBCO =$ Area $OMC +$ Area MBC **[1 mark]**

Step 5:

$$= \int_0^{\frac{1}{2}} 2\sqrt{x}\, dx + \int_{\frac{1}{2}}^{\frac{3}{2}} \sqrt{\frac{9}{4} - x^2}\, dx$$

$$= 2\int_0^{\frac{1}{2}} \sqrt{x}\, dx + \int_{\frac{1}{2}}^{\frac{3}{2}} \sqrt{(\frac{3}{2})^2 - (x)^2}\, dx$$

$= 2\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^{\frac{1}{2}} + [\frac{x}{2}\sqrt{\left(\frac{3}{2}\right)^2 - x^2} + \frac{\left(\frac{3}{2}\right)^2}{2}\sin^{-1}\frac{x}{\frac{3}{2}}]_{\frac{1}{2}}^{\frac{3}{2}}$

$= \frac{4}{3}\left[x^{\frac{3}{2}}\right]_0^{\frac{1}{2}} + [\frac{\frac{3}{2}}{2}\sqrt{\left(\frac{3}{2}\right)^2 - \frac{3}{2}^2} + \frac{\left(\frac{3}{2}\right)^2}{2}\sin^{-1}\frac{\frac{3}{2}}{\frac{3}{2}}] - [\frac{\frac{1}{2}}{2}\sqrt{\left(\frac{3}{2}\right)^2 - \frac{1}{2}^2} + \frac{\left(\frac{3}{2}\right)^2}{2}\sin^{-1}\frac{\frac{1}{2}}{\frac{3}{2}}]$

$= \frac{4}{3}\left[\left(\frac{1}{2}\right)^{\frac{3}{2}} - 0\right] + [\frac{3}{4}\sqrt{0} + \frac{9}{8}\sin^{-1}1] - [\frac{1}{4}\sqrt{2} + \frac{9}{8}\sin^{-1}\frac{1}{3}]$

$= \frac{\sqrt{2}}{3} + \frac{9\pi}{16} - \frac{\sqrt{2}}{4} - \frac{9}{8}\sin^{-1}\frac{1}{3}$ **[2 Marks]**

Step 6:

Hence, the required area = $2 \times [\frac{\sqrt{2}}{3} + \frac{9\pi}{16} - \frac{\sqrt{2}}{4} - \frac{9}{8}\sin^{-1}\frac{1}{3}]$

$= \frac{1}{3\sqrt{2}} + \frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\frac{1}{3}$ **[1 Mark]**

16. Area bounded by the curve $y = x^3$, the x-axis and the ordinates $x = -2$ and $x = 1$ is **[4 Marks]**

(A) -9

(B) $-\frac{15}{4}$

(C) $\frac{15}{4}$

(D) $\frac{17}{4}$

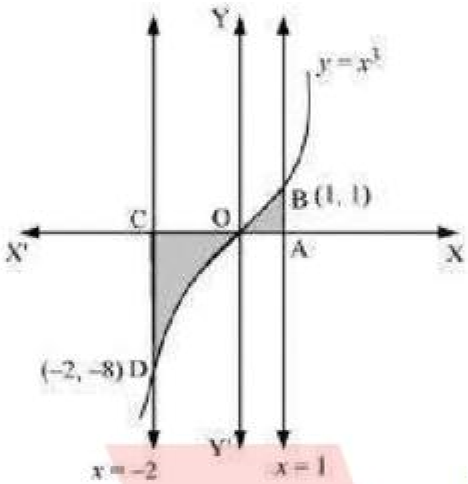
Solution:

(D)

Step 1:

Given: The equations of the curve is $y = x^3$ and the ordinates are $x = -2$ and $x = 1$.

Step 2: The given equations can be represented as follows:



[1 Mark]

Step 3:

Required area $= \int_{-2}^{1} |y| dx$

$= \int_{-2}^{1} |x^3| dx$ **[1 Mark]**

Step 4:

$= \int_{-2}^{0} |x^3| dx + \int_{0}^{1} |x^3| dx$

$= -\int_{-2}^{0} x^3 dx + \int_{0}^{1} x^3 dx$

$= -\left[\frac{x^4}{4}\right]_{-2}^{0} + \left[\frac{x^4}{4}\right]_{0}^{1}$

$= -\left[0 - \frac{(-2)^4}{4}\right] + \frac{1}{4} - 0$

$= 4 + \frac{1}{4} = \frac{17}{4}$ square units

Hence, (D) is the correct answer. **[2 Marks]**

17. The area bounded by the curve $y = x|x|$, x-axis and the ordinates $x = -1$ and $x = 1$ is given by [Hint: $y = x^2$ if $x > 0$ and $y = -x^2$ if $x < 0$]. **[4 Marks]**

(A) 0

(B) $\frac{1}{3}$

(C) $\frac{2}{3}$

(D) $\frac{4}{3}$

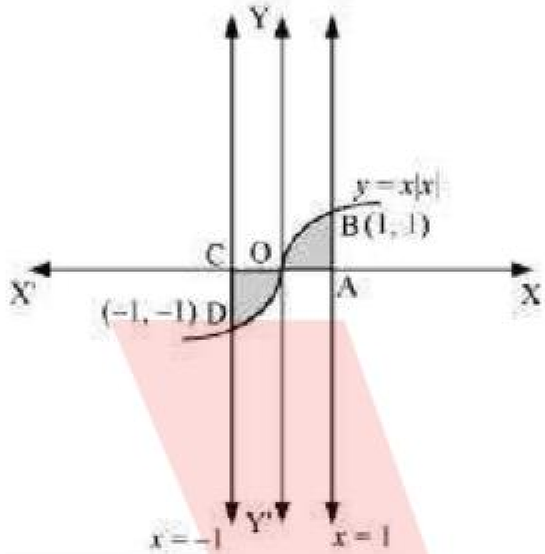
Solution:

(C)

Step 1:

Given: The equation of the curve: $y = x|x|$, x-axis and the ordinates $x = -1$ and $x = 1$

Step 2: The given equation can be represented as follows:



[1 Mark]

Step 3:

Required area $= \int_{-1}^{1} |y| dx$ **[1 Mark]**

Step 4:

$$= \int_{-1}^{1} \left| x|x| \right| dx$$

$= \int_{-1}^{0} x^2 dx + \int_{0}^{1} x^2 dx$ **[1 Mark]**

Step 5:

$$= \left[\frac{x^3}{3}\right]_{-1}^{0} + \left[\frac{x^3}{3}\right]_{0}^{1}$$

$$= -\left(-\frac{1}{3}\right) + \frac{1}{3}$$

$= \frac{2}{3}$ square units

Hence, (C) is the correct answer. **[1 Mark]**

18. The area of the circle $x^2 + y^2 = 16$ exterior to the parabola $y^2 = 6x$ is **[6 Marks]**

(A) $\frac{4}{3}\left(4\pi - \sqrt{3}\right)$

(B) $\frac{4}{3}\left(4\pi + \sqrt{3}\right)$

(C) $\frac{4}{3}\left(8\pi - \sqrt{3}\right)$

(D) $\frac{4}{3}\left(8\pi+\sqrt{3}\right)$

Solution:

(C)

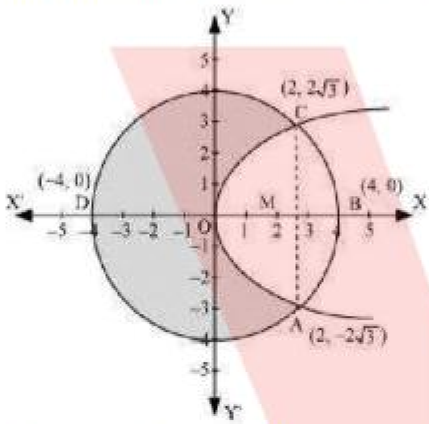
Step 1:

Given: Equation of the circle: $x^2+y^2=16$

Equation of the parabola: $y^2=6x$

Step 2:

The given equations is represented as follows:



The points of intersection are marked as shown.

Area bounded exterior to the parabola and the circle is as shaded above. **[1 Mark]**

Step 3: The area

$=2[\text{Area}(OMCO)+\text{Area}(CMBC)]$

$=2\left[\int_0^2\sqrt{6x}\,dx+\int_2^4\sqrt{16-x^2}dx\right]$ **[1 Mark]**

Step 4:

$=2\left[\sqrt{6}\left\{\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right\}_0^2\right]+2\left[\frac{x}{2}\sqrt{16-x^2}+\frac{16}{2}\sin^{-1}\frac{x}{4}\right]_2^4$

$=2\sqrt{6}\times\frac{2}{3}\left[x^{\frac{3}{2}}\right]_0^2+2\left[8.\frac{\pi}{2}-\sqrt{16-4}-8\sin^{-1}\left(\frac{1}{2}\right)\right]$

$=\frac{4\sqrt{6}}{3}(2\sqrt{2})+2\left[4\pi-\sqrt{12}-8\frac{\pi}{6}\right]$

$=\frac{16\sqrt{3}}{3}+8\pi-4\sqrt{3}-\frac{8}{3}\pi$

$=\frac{4}{3}[4\sqrt{3}+6\pi-3\sqrt{3}-2\pi]$

$=\frac{4}{3}[\sqrt{3}+4\pi]$

$=\frac{4}{3}[4\pi+\sqrt{3}]$ square units **[2 marks]**

Step 5:

Area of circle $= \pi(r)^2$
$= \pi(4)^2$
$= 16\pi$ square units **[$\frac{1}{2}$ Mark]**
$\therefore$ Required area $= 16\pi - \frac{4}{3}\left[4\pi + \sqrt{3}\right]$
$= \frac{4}{3}\left[4 \times 3\pi - 4\pi - \sqrt{3}\right]$
$= \frac{4}{3}\left(8\pi - \sqrt{3}\right)$ square units
Hence, (C) is the correct answer. **[1$\frac{1}{2}$ Mark]**

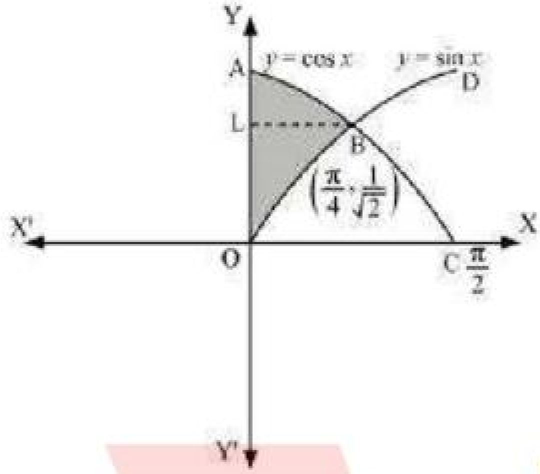
19. The area bounded by the y-axis, $y = \cos x$ and $y = \sin x$ when $0 \leq x \leq \frac{\pi}{2}$ **[4 Marks]**

(A) $2\left(\sqrt{2} - 1\right)$
(B) $\sqrt{2} - 1$
(C) $\sqrt{2} + 1$
(D) $\sqrt{2}$

Solution:

(B)

Step 1:
The given equations are $y = \cos x$ ...(i)
And $y = \sin x$ ....(ii)
Step 2: The given equations is represented as follows. The point of intersection for given curve is as:
$\sin x = \cos x$
$\Rightarrow \tan x = 1$
$\Rightarrow x = \frac{\pi}{4}$
$\Rightarrow y = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$



[1 Mark]

Step 3:

Required area = Area $(ABLA)$ + area $(OBLO)$

$= \int_{\frac{1}{\sqrt{2}}}^{1} xdy + \int_{0}^{\frac{1}{\sqrt{2}}} xdy$ **[1 mark]**

Step 4:

$$= \int_{\frac{1}{\sqrt{2}}}^{1} \cos^{-1} y dy + \int_{0}^{\frac{1}{\sqrt{2}}} \sin^{-1} y dy$$

Integrating by parts, we obtain

$$= \left[y \cos^{-1} y - \sqrt{1-y^2}\right]_{\frac{1}{\sqrt{2}}}^{1} + \left[y \sin^{-1} y + \sqrt{1-y^2}\right]_{0}^{\frac{1}{\sqrt{2}}}$$

$$= \left[\cos^{-1}(1) - \frac{1}{\sqrt{2}} \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) + \sqrt{1-\frac{1}{2}}\right] + \left[\frac{1}{\sqrt{2}} \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) + \sqrt{1-\frac{1}{2}} - 1\right]$$

$$= \frac{-\pi}{4\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{\pi}{4\sqrt{2}} + \frac{1}{\sqrt{2}} - 1$$

$$= \frac{2}{\sqrt{2}} - 1$$

$= \sqrt{2} - 1$ square units

Hence, (B) is the correct answer. **[2 marks]**