CBSE NCERT Solutions for Class 12 Maths Chapter 03

Back of Chapter Questions

EXERCISE 3.1

1. In the matrix $A = \begin{bmatrix} 2 & 5 & 19 & -7 \\ 35 & -2 & \frac{5}{2} & 12 \\ \sqrt{3} & 1 & -5 & 17 \end{bmatrix}$, write:

(i) The order of the matrix,

(ii) The number of elements,

(iii) Write the elements $a_{13}, a_{21}, a_{33}, a_{24}, a_{23}$.

Solution:

Given:

$$A = \begin{bmatrix} 2 & 5 & 19 & -7 \\ 35 & -2 & \frac{5}{2} & 12 \\ \sqrt{3} & 1 & -5 & 17 \end{bmatrix}$$

Number of rows $= 3$

Number of columns $= 4$

Step 1:

(i) The order of the matrix=number of rows × number of columns

Hence the order of the given matrix $= 3 \times 4$

Step 2:

(ii) Since, the order of matrix is 3×4

So, the number of elements $= 3 \times 4 = 12$

(iii) From the given matrix, we can observe the elements:

$a_{13} = 19$

$a_{21} = 35$

$a_{33} = -5$

$a_{24} = 12$

$a_{23} = \frac{5}{2}$

2. If a matrix has 24 elements, what are the possible orders it can have? What, if it has 13 elements?

Solution:

Given:

The number of elements $= 24$

Step 1:

Therefore, the possible orders are as follows:

$1 \times 24, 2 \times 12, 3 \times 8, 4 \times 6, 6 \times 4, 8 \times 3, 12 \times 2$ and 24×1

Step 2:

Now, if it has 13 elements,

then the possible orders are 13×1 and 1×13

3. If a matrix has 18 elements, what are the possible orders it can have? What, if it has 5 elements?

Solution:

Given:

Total number of elements in matrix $= 18$

Step 1:

Therefore, the possible orders are as follows:

$1 \times 18, 2 \times 9, 3 \times 6, 6 \times 3, 9 \times 2$ and 18×1

Step 2:

So, if it has 5 elements,

then the possible orders are 5×1 and 1×5

4. Construct a 2×2 matrix, $A = [a_{ij}]$, whose elements are given by:

(i) $a_{ij} = \frac{(i+j)^2}{2}$ **[2 marks]**

(ii) $a_{ij} = \frac{i}{j}$ **[2 marks]**

(iii) $a_{ij} = \frac{(i+2j)^2}{2}$ **[2 marks]**

Solution:

Given:

$A = [a_{ij}]$

Since, it is a 2×2 matrix

$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

Step 1:

(i) Here, $a_{ij} = \frac{(i+j)^2}{2}$,

So, the elements of matrix are:

$a_{11} = \frac{(1+1)^2}{2} = 2$

$a_{12} = \frac{(1+2)^2}{2} = \frac{9}{2}$

$a_{21} = \frac{(2+1)^2}{2} = \frac{9}{2}$

$a_{22} = \frac{(2+2)^2}{2} = 8$

Therefore, the required matrix $= \begin{bmatrix} 2 & \frac{9}{2} \\ \frac{9}{2} & 8 \end{bmatrix}$

Step 2:

(ii) Here , $a_{ij} = \frac{i}{j}$

So, the elements of matrix are:

$a_{11} = \frac{1}{1} = 1$

$a_{12} = \frac{1}{2}$

$a_{21} = \frac{2}{1} = 2$

$a_{22} = \frac{2}{2} = 1$

Therefore, the required matrix$=\begin{bmatrix}1 & \frac{1}{2}\\ 2 & 1\end{bmatrix}$ **[2 marks]**

Step 3:

(iii) Here, $a_{ij}=\frac{(i+2j)^2}{2}$,

The elements of matrix are:

$$a_{11}=\frac{(1+2(1))^2}{2}=\frac{(1+2)^2}{2}=\frac{3^2}{2}=\frac{9}{2}$$

$$a_{12}=\frac{(1+2(2))^2}{2}=\frac{(1+4)^2}{2}=\frac{5^2}{2}=\frac{25}{2}$$

$$a_{21}=\frac{(2+2(1))^2}{2}=\frac{(2+2)^2}{2}=\frac{(4)^2}{2}=\frac{16}{2}=8$$

$$a_{22}=\frac{(2+2(2))^2}{2}=\frac{(2+4)^2}{2}=\frac{(6)^2}{2}=\frac{36}{2}=18$$

Therefore, the required matrix $=\begin{bmatrix}\frac{9}{2} & \frac{25}{2}\\ 8 & 18\end{bmatrix}$ **[2 marks]**

5. Construct a 3×4 matrix, whose elements are given by:

(i) $a_{ij}=\frac{1}{2}|-3i+j|$ **[3 marks]**

(ii) $a_{ij}=2i-j$ **[3 marks]**

Solution:

Given:

Since, it is a 3×4 matrix

$$A=\begin{bmatrix}a_{11} & a_{12} & a_{13} & a_{14}\\ a_{21} & a_{22} & a_{23} & a_{24}\\ a_{31} & a_{32} & a_{33} & a_{34}\end{bmatrix}$$

Step 1:

(i) Here, $a_{ij}=\frac{1}{2}|-3i+j|$

So, the elements of matrix are:

$a_{11}=\frac{1}{2}|-3(1)+1|=\frac{1}{2}|-3+1|=\frac{1}{2}|-2|=\frac{1}{2}(2)=1$ **[$\frac{1}{4}$ Mark]**

$a_{12} = \frac{1}{2}|-3(1)+2| = \frac{1}{2}|-3+2| = \frac{1}{2}|-1| = \frac{1}{2}(1) = \frac{1}{2}$ **[$\frac{1}{4}$ Mark]**

$a_{13} = \frac{1}{2}|-3(1)+3| = \frac{1}{2}|0| = \frac{1}{2}(0) = 0$ **[$\frac{1}{4}$ Mark]**

$a_{14} = \frac{1}{2}|-3(1)+4| = \frac{1}{2}|-3+4| = \frac{1}{2}|1| = \frac{1}{2}(1) = \frac{1}{2}$ **[$\frac{1}{4}$ Mark]**

$a_{21} = \frac{1}{2}|-3(2)+1| = \frac{1}{2}|-6+1| = \frac{1}{2}|-5| = \frac{1}{2}(5) = \frac{5}{2}$ **[$\frac{1}{4}$ Mark]**

$a_{22} = \frac{1}{2}|-3\times 2+2| = \frac{1}{2}|-6+2| = \frac{1}{2}|-4| = \frac{1}{2}(4) = 2$ **[$\frac{1}{4}$ Mark]**

$a_{23} = \frac{1}{2}|-3(2)+1| = \frac{1}{2}|-6+3| = \frac{1}{2}|-3| = \frac{1}{2}(3) = \frac{3}{2}$ **[$\frac{1}{4}$ Mark]**

$a_{24} = \frac{1}{2}|-3\times 2+2| = \frac{1}{2}|-6+4| = \frac{1}{2}|-2| = \frac{1}{2}(2) = 1$ **[$\frac{1}{4}$ Mark]**

$a_{31} = \frac{1}{2}|-3(3)+1| = \frac{1}{2}|-9+1| = \frac{1}{2}|-8| = \frac{1}{2}(8) = 4$ **[$\frac{1}{4}$ Mark]**

$a_{32} = \frac{1}{2}|-3(3)+2| = \frac{1}{2}|-9+2| = \frac{1}{2}|-7| = \frac{1}{2}(7) = \frac{7}{2}$ **[$\frac{1}{4}$ Mark]**

$a_{33} = \frac{1}{2}|-3\times 3+3| = \frac{1}{2}|-9+3| = \frac{1}{2}|-6| = \frac{1}{2}(6) = 3$ **[$\frac{1}{4}$ Mark]**

$a_{34} == \frac{1}{2}|-3\times 3+4| = \frac{1}{2}|-9+4| = \frac{1}{2}|-5| = \frac{1}{2}(5) = \frac{5}{2}$ **[$\frac{1}{4}$ Mark]**

Therefore, the required matrix A = $\begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & 2 & \frac{3}{2} & 1 \\ 4 & \frac{7}{2} & 3 & \frac{5}{2} \end{bmatrix}$

Step 2:

(ii) Here, $a_{ij} = 2i - j,$

So, the elements of matrix are:

$a_{11} = 2(1) - 1 = 2 - 1 = 1$ **[$\frac{1}{4}$ Mark]**

$a_{12} = 2(1) - 2 = 2 - 2 = 0$ **[$\frac{1}{4}$ Mark]**

$a_{13} = 2(1) - 3 = 2 - 3 = -1$ **[$\frac{1}{4}$ Mark]**

$a_{14} = 2(1) - 4 = 2 - 4 = -2$ **[$\frac{1}{4}$ Mark]**

$a_{21} = 2(2) - 1 = 4 - 1 = 3$ **[$\frac{1}{4}$ Mark]**

$a_{22} = 2(2) - 2 = 4 - 2 = 2$ **[$\frac{1}{4}$ Mark]**

$a_{23} = 2(2) - 3 = 4 - 3 = 1$ **[$\frac{1}{4}$ Mark]**

$a_{24} = 2(2) - 4 = 4 - 4 = 0$ **[$\frac{1}{4}$ Mark]**

$a_{31} = 2(3) - 1 = 6 - 1 = 5$ **[$\frac{1}{4}$ Mark]**

$a_{32} = 2(3) - 2 = 6 - 2 = 4$ **[$\frac{1}{4}$ Mark]**

$a_{33} = 2(3) - 3 = 6 - 3 = 3$ **[$\frac{1}{4}$ Mark]**

$a_{34} = 2(3) - 4 = 6 - 4 = 2$ **[$\frac{1}{4}$ Mark]**

Therefore, the required matrix $\mathrm{A} = \begin{bmatrix} 1 & 0 & -1 & -2 \\ 3 & 2 & 1 & 0 \\ 5 & 4 & 3 & 2 \end{bmatrix}$

6. Find the values of x, y and z from the following equations:

(i) $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$ **[2 Marks]**

(ii) $\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$ **[2 Marks]**

(iii) $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$ **[2 Marks]**

Solution:

(i) **Step 1:**

Given: $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$.

If two matrices are equal, then their corresponding elements are also equal. **[$\frac{1}{2}$ Mark]**

Step 2:

Comparing the corresponding elements, we get

$4 = y$

$3 = z$

$x = 1$

Therefore, $4 = y, 3 = z$ and $x = 1$ **[$1\frac{1}{2}$ Mark]**

(ii)**Step 1:**

Given: $\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$

If two matrices are equal, then their corresponding elements are also equal. **[$\frac{1}{2}$ Mark]**

Step 2:

$x + y = 6$

$5 + z = 5$

$xy = 8$ **[$\frac{1}{2}$ Mark]**

Step 3:

By solving these above equations, we get

$x = 2, y = 4$ and $z = 0$ OR

$x = 4, y = 2$ and $z = 0$

Therefore, $x = 2, y = 4$ and $z = 0$ or $x = 4, y = 2$ and $z = 0$ **[1 Mark]**

(iii) **Step 1:**

Given: $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$

If two matrices are equal, then their corresponding elements are also equal. **[$\frac{1}{2}$ Mark]**

Step 2:

Comparing the corresponding elements, we get

$x + y + z = 9$

$x + z = 5$

$y + z = 7$ **[$\frac{1}{2}$ Mark]**

Step 3:

By solving these above equations, we get

$x = 2,$

$y = 4$

$z = 3$

Therefore, $x = 2, y = 4$ and $z = 3$ **[1 Mark]**

7. Find the value of a, b, c and d from the equation:

$\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$ **[3 marks]**

Solution:

Step 1:

If two matrices are equal, then their corresponding elements are also equal.

Therefore, on comparing the corresponding elements, we get

$a - b = -1$... (1)

$2a - b = 0$... (2)

$2a + c = 5$... (3)

$3c + d = 13$... (4) **[$1\frac{1}{2}$ Marks]**

Step 2:

Now,

By solving equation (1) and (2), we get

$a = 1, b = 2$

Putting the value of a in equation (3), we get

$c = 3$

Putting the value of c in the equation (4), we get

$d = 4$

Hence, the required values are $a = 1, b = 2, c = 3$ and $d = 4$ **[$1\frac{1}{2}$ Marks]**

8. $A = [a_{ij}]_{m \times n}$ is a square matrix, if

(A) $m < n$

(B) $m > n$

(C) $m = n$

(D) None of these

Solution:

(C)

Step 1:

$A = [a_{ij}]_{m \times n}$ means matrix A is of order $\mathrm{m} \times \mathrm{n}$

In a square matrix the number of column is same as the number of rows.

Number of rows = Number of columns

$m = n$

Hence, the option (C) is correct.

9. Which of the given values of x and y make the following pair of matrices equal

$\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix}, \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$ **[2 marks]**

(A) $x = \frac{-1}{3}, y = 7$

(B) Not possible to find

(C) $y = 7, x = \frac{-2}{3}$

(D) $x = \frac{-1}{3}, y = \frac{-2}{3}$

Solution:

(B)

Step 1:

Here $\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix} = \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$

If two matrices are equal, then their corresponding elements are also equal. **[$\frac{1}{2}$ Mark]**

Step 2:

Comparing the corresponding elements, we get

$3x + 7 = 0$

$5 = y - 2$

$y - 1 = 8$

$2 - 3x = 4$ **[$\frac{1}{2}$ Mark]**

Step 3:

By solving these above equations, we get

$y = 7, x = -\frac{7}{3}$ and $x = -\frac{2}{3}$,

Here, the value of x is not unique.

Therefore, the option (B) is correct.

10. The number of all possible matrices of order 3×3 with each entry 0 or 1 is:

(A) 27

(B) 18

(C) 81

(D) 512 **[2 Marks]**

Solution:

(D)

Given:

It is a 3×3 matrix.

Step 1:

$$\mathrm{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

The total number of elements in a matrix of order $3 \times 3 = 9$

Step 2:

If each entry is 0 or 1, then total number of permutations for each element $= 2$

Therefore, the total permutation for 9 elements $= 2^9 = 512$

Hence, the option (D) is correct.

EXERCISE 3.2

1. Let $A=\begin{bmatrix}2 & 4\\3 & 2\end{bmatrix}, B=\begin{bmatrix}1 & 3\\-2 & 5\end{bmatrix}, C=\begin{bmatrix}-2 & 5\\3 & 4\end{bmatrix}$

Find each of the following:

(i) $A+B$ **[2 marks]**

(ii) $A-B$ **[2 marks]**

(iii) $3A-C$ **[3 marks]**

(iv) AB **[3 marks]**

(v) BA **[3 marks]**

Solution:

Given:

$$A=\begin{bmatrix}2 & 4\\3 & 2\end{bmatrix}, B=\begin{bmatrix}1 & 3\\-2 & 5\end{bmatrix}, C=\begin{bmatrix}-2 & 5\\3 & 4\end{bmatrix}$$

(i) $A+B$

Step 1:

$$A+B=\begin{bmatrix}2 & 4\\3 & 2\end{bmatrix}+\begin{bmatrix}1 & 3\\-2 & 5\end{bmatrix}$$

Step 2:

$$=\begin{bmatrix}2+1 & 4+3\\3-2 & 2+5\end{bmatrix}=\begin{bmatrix}3 & 7\\1 & 7\end{bmatrix}$$

Hence, $A+B=\begin{bmatrix}3 & 7\\1 & 7\end{bmatrix}$

(ii) $\mathrm{A}-\mathrm{B}$

Step 1:

$$\mathrm{A}-\mathrm{B}=\begin{bmatrix}2 & 4\\3 & 2\end{bmatrix}-\begin{bmatrix}1 & 3\\-2 & 5\end{bmatrix}$$

Step 2:

$$=\begin{bmatrix}2-1 & 4-3\\3+2 & 2-5\end{bmatrix}=\begin{bmatrix}1 & 1\\5 & -3\end{bmatrix}$$

Hence, $\mathrm{A} - \mathrm{B} = \begin{bmatrix} 1 & 1 \\ 5 & -3 \end{bmatrix}$

(iii) $3A - C$

Step 1:

$3A - C = 3\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$

Step 2:

$= \begin{bmatrix} 6 & 12 \\ 9 & 6 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$

Step 3:

$= \begin{bmatrix} 6+2 & 12-5 \\ 9-3 & 6-4 \end{bmatrix} = \begin{bmatrix} 8 & 7 \\ 6 & 2 \end{bmatrix}$

Hence, $3A - C = \begin{bmatrix} 8 & 7 \\ 6 & 2 \end{bmatrix}$

(iv) AB

Step 1:

$AB = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$

$= \begin{bmatrix} 2 \times 1 + 4 \times -2 & 2 \times 3 + 4 \times 5 \\ 3 \times 1 + 2 \times -2 & 3 \times 3 + 2 \times 5 \end{bmatrix}$

Step 2:

$= \begin{bmatrix} 2-8 & 6+20 \\ 3-4 & 9+10 \end{bmatrix}$

Step 3:

$= \begin{bmatrix} -6 & 26 \\ -1 & 19 \end{bmatrix}$

Hence, $AB = \begin{bmatrix} -6 & 26 \\ -1 & 19 \end{bmatrix}$

(v) BA

Step 1:

$BA = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$

$= \begin{bmatrix} 1 \times 2 + 3 \times 3 & 1 \times 4 + 3 \times 2 \\ -2 \times 2 + 5 \times 3 & -2 \times 4 + 5 \times 2 \end{bmatrix}$

Step 2:

$= \begin{bmatrix} 2+9 & 4+6 \\ -4+15 & -8+10 \end{bmatrix}$

Step 3:

$$= \begin{bmatrix} 11 & 10 \\ 11 & 2 \end{bmatrix}$$

Hence, $BA = \begin{bmatrix} 11 & 10 \\ 11 & 2 \end{bmatrix}$

2. Compute the following:

(i) $\begin{bmatrix} a & b \\ -b & a \end{bmatrix} + \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ **[2 marks]**

(ii) $\begin{bmatrix} a^2+b^2 & b^2+c^2 \\ a^2+c^2 & a^2+b^2 \end{bmatrix} + \begin{bmatrix} 2ab & 2bc \\ -2ac & -2ab \end{bmatrix}$ **[2 marks]**

(iii) $\begin{bmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{bmatrix} + \begin{bmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{bmatrix}$ **[2 marks]**

(iv) $\begin{bmatrix} \cos^2 x & \sin^2 x \\ \sin^2 x & \cos^2 x \end{bmatrix} + \begin{bmatrix} \sin^2 x & \cos^2 x \\ \cos^2 x & \sin^2 x \end{bmatrix}$ **[2 marks]**

Solution:

(i) **Given:**

$$\begin{bmatrix} a & b \\ -b & a \end{bmatrix} + \begin{bmatrix} a & b \\ b & a \end{bmatrix}$$

Step 1:

$$= \begin{bmatrix} a+a & b+b \\ -b+b & a+a \end{bmatrix}$$

Step 2:

$$= \begin{bmatrix} 2a & 2b \\ 0 & 2a \end{bmatrix}$$

Therefore, $\begin{bmatrix} a & b \\ -b & a \end{bmatrix} + \begin{bmatrix} a & b \\ b & a \end{bmatrix} = \begin{bmatrix} 2a & 2b \\ 0 & 2a \end{bmatrix}$

(ii) **Given:**

$$\begin{bmatrix} a^2+b^2 & b^2+c^2 \\ a^2+c^2 & a^2+b^2 \end{bmatrix} + \begin{bmatrix} 2ab & 2bc \\ -2ac & -2ab \end{bmatrix}$$

Step 1:

$$= \begin{bmatrix} a^2+b^2+2ab & b^2+c^2+2bc \\ a^2+c^2-2ac & a^2+b^2-2ab \end{bmatrix}$$

Step 2:

$$= \begin{bmatrix} (a+b)^2 & (b+c)^2 \\ (a-c)^2 & (a-b)^2 \end{bmatrix}$$

Therefore, $\begin{bmatrix} a^2+b^2 & b^2+c^2 \\ a^2+c^2 & a^2+b^2 \end{bmatrix} + \begin{bmatrix} 2ab & 2bc \\ -2ac & -2ab \end{bmatrix} = \begin{bmatrix} (a+b)^2 & (b+c)^2 \\ (a-c)^2 & (a-b)^2 \end{bmatrix}$

(iii) **Given:**

$$\begin{bmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{bmatrix} + \begin{bmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{bmatrix}$$

Step 1:

$$= \begin{bmatrix} -1+12 & 4+7 & -6+6 \\ 8+8 & 5+0 & 16+5 \\ 2+3 & 8+2 & 5+4 \end{bmatrix}$$

Step 2:

$$= \begin{bmatrix} 11 & 11 & 0 \\ 16 & 5 & 21 \\ 5 & 10 & 9 \end{bmatrix}$$

Therefore, $\begin{bmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{bmatrix} + \begin{bmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 11 & 11 & 0 \\ 16 & 5 & 21 \\ 5 & 10 & 9 \end{bmatrix}$

(iv) **Given:**

$$\begin{bmatrix} cos^2x & sin^2x \\ sin^2x & cos^{2x} \end{bmatrix} + \begin{bmatrix} sin^{2x} & cos^{2x} \\ cos^2x & sin^2x \end{bmatrix}$$

Step 1:

$$= \begin{bmatrix} cos^2x + sin^{2x} & sin^2x + cos^2x \\ sin^2x + cos^{2x} & cos^2x + sin^2x \end{bmatrix}$$

Step 2:

$$= \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

Therefore, $\begin{bmatrix} cos^2x & sin^2x \\ sin^2x & cos^{2x} \end{bmatrix} + \begin{bmatrix} sin^{2x} & cos^{2x} \\ cos^2x & sin^2x \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

3. Compute the indicated products.

(i) $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ **[2 marks]**

(ii) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}[2\ 3\ 4]$ **[2 marks]**

(iii) $\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$ **[2 marks]**

(iv) $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}\begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$ **[2 marks]**

(v) $\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}$ **[2 marks]**

(vi) $\begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{bmatrix}$ **[2 marks]**

Solution:

(i) **Given:**

$\begin{bmatrix} a & b \\ -b & a \end{bmatrix}\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

Step 1:

$= \begin{bmatrix} a \times a + b \times b & a \times (-b) + b \times a \\ -b \times a + b \times b & (-b) \times (-b) + a \times a \end{bmatrix}$

Step 2:

$= \begin{bmatrix} a^2 + b^2 & -ab + ab \\ -ab + b^2 & b^2 + a^2 \end{bmatrix}$

$= \begin{bmatrix} a^2 + b^2 & 0 \\ 0 & b^2 + a^2 \end{bmatrix}$

Hence, $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}\begin{bmatrix} a & -b \\ b & a \end{bmatrix} = \begin{bmatrix} a^2 + b^2 & 0 \\ 0 & b^2 + a^2 \end{bmatrix}$

(ii) **Given:**

$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}[2\ 3\ 4]$

Step 1:

$$= \begin{bmatrix} 1\times 2 & 1\times 3 & 1\times 4 \\ 2\times 2 & 2\times 3 & 2\times 4 \\ 3\times 2 & 3\times 3 & 3\times 4 \end{bmatrix}$$

Step 2:

$$= \begin{bmatrix} 2 & 3 & 4 \\ 4 & 6 & 8 \\ 6 & 9 & 12 \end{bmatrix}$$

Hence, $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} [2\ 3\ 4] = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 6 & 8 \\ 6 & 9 & 12 \end{bmatrix}$

(iii) **Given:**

$$\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$$

Step 1:

$$= \begin{bmatrix} 1\times 1+(-2)\times 2 & 1\times 2+(-2)\times 3 & 1\times 3+(-2)\times 1 \\ 2\times 1+3\times 2 & 2\times 2+3\times 3 & 2\times 3+3\times 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-4 & 2-6 & 3-2 \\ 2+6 & 4+9 & 6+3 \end{bmatrix}$$

Step 2:

$$= \begin{bmatrix} -3 & -4 & 1 \\ 8 & 13 & 9 \end{bmatrix}$$

Hence, $\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix} = \begin{bmatrix} -3 & -4 & 1 \\ 8 & 13 & 9 \end{bmatrix}$

(iv) **Given:**

$$\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$$

Step 1:

$$= \begin{bmatrix} 2\times 1+3\times 0+4\times 3 & 2\times(-3)+3\times 2+4\times 0 & 2\times 5+3\times 4+4\times 5 \\ 3\times 1+4\times 0+5\times 3 & 3\times(-3)+4\times 2+5\times 0 & 3\times 5+4\times 4+5\times 5 \\ 4\times 1+5\times 0+6\times 3 & 4\times(-3)+5\times 2+6\times 0 & 4\times 5+5\times 4+6\times 5 \end{bmatrix}$$

Step 2:

$$=\begin{bmatrix}2+0+12 & -6+6+0 & 10+12+20\\ 3+0+15 & -9+0+0 & 15+16+25\\ 4+0+18 & -12+10+0 & 20+20+30\end{bmatrix}$$

$$=\begin{bmatrix}14 & 0 & 42\\ 18 & -1 & 56\\ 22 & -2 & 70\end{bmatrix}$$

Hence, $\begin{bmatrix}2 & 3 & 4\\ 3 & 4 & 5\\ 4 & 5 & 6\end{bmatrix}\begin{bmatrix}1 & -3 & 5\\ 0 & 2 & 4\\ 3 & 0 & 5\end{bmatrix}=\begin{bmatrix}14 & 0 & 42\\ 18 & -1 & 56\\ 22 & -2 & 70\end{bmatrix}$

(v) **Given:**

$$\begin{bmatrix}2 & 1\\ 3 & 2\\ -1 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 1\\ -1 & 2 & 1\end{bmatrix}$$

Step 1:

$$=\begin{bmatrix}2\times 1+1\times(-1) & 2\times 0+1\times 2 & 2\times 1+1\times 1\\ 3\times 1+2\times(-1) & 3\times 0+2\times 2 & 3\times 1+2\times 1\\ -1\times 1+1\times(-1) & -1\times 0+1\times 2 & -1\times 1+1\times 1\end{bmatrix}$$

Step 2:

$$=\begin{bmatrix}1 & 2 & 3\\ 1 & 4 & 5\\ -2 & 2 & 0\end{bmatrix}$$

Hence, $\begin{bmatrix}2 & 1\\ 3 & 2\\ -1 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 1\\ -1 & 2 & 1\end{bmatrix}=\begin{bmatrix}1 & 2 & 3\\ 1 & 4 & 5\\ -2 & 2 & 0\end{bmatrix}$

(vi) **Given:**

$$\begin{bmatrix}3 & -1 & 3\\ -1 & 0 & 2\end{bmatrix}\begin{bmatrix}2 & -3\\ 1 & 0\\ 3 & 1\end{bmatrix}$$

Step 1:

$$=\begin{bmatrix}3\times 2+(-1)\times 1+3\times 3 & 3\times(-3)+(-1)\times 0+3\times 1\\ -1\times 2+0\times 1+2\times 3 & -1\times(-3)+0\times 0+2\times 1\end{bmatrix}$$

Step 2:

$$= \begin{bmatrix} 6-1+9 & -9-0+3 \\ -2+0+6 & 3+0+2 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & -6 \\ 4 & 5 \end{bmatrix}$$

Hence, $\begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 14 & -6 \\ 4 & 5 \end{bmatrix}$

4. If $A = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$, then compute $(A+B)$ and $(B—C)$. Also, verify that $A + (B - C) = (A + B) - C$. **[3 marks]**

Solution:

Given:

$A = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$

Step 1:

So, $A + B = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix} + \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 1+3 & 2-1 & -3+2 \\ 5+4 & 0+2 & 2+5 \\ 1+2 & -1+0 & 1+3 \end{bmatrix}$$

$$\therefore\ A + B = \begin{bmatrix} 4 & 1 & -1 \\ 9 & 2 & 7 \\ 3 & -1 & 4 \end{bmatrix}$$

Step 2:

Now, $B - C = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 3-4 & -1-1 & 2-2 \\ 4-0 & 2-3 & 5-2 \\ 2-1 & 0-(-2) & 3-3 \end{bmatrix}$$

$$\therefore\ B - C = \begin{bmatrix} -1 & -2 & 0 \\ 4 & -1 & 3 \\ 1 & 2 & 0 \end{bmatrix}$$

Step 3:

$$LHS = A + (B - C) = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix} + \begin{bmatrix} -1 & -2 & 0 \\ 4 & -1 & 3 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -3 \\ 9 & -1 & 5 \\ 2 & 1 & 1 \end{bmatrix}$$

$$RHS = (A + B) - C = \begin{bmatrix} 4 & 1 & -1 \\ 9 & 2 & 7 \\ 3 & -1 & 4 \end{bmatrix} - \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -3 \\ 9 & -1 & 5 \\ 2 & 1 & 1 \end{bmatrix}$$

$\therefore LHS = RHS$

Hence, $A + (B - C) = (A + B) - C = \begin{bmatrix} 0 & 0 & -3 \\ 9 & -1 & 5 \\ 2 & 1 & 1 \end{bmatrix}$

5. If $A = \begin{bmatrix} \frac{2}{3} & 1 & \frac{5}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3} \\ \frac{7}{3} & 2 & \frac{2}{3} \end{bmatrix}$ and $= \begin{bmatrix} \frac{2}{5} & \frac{3}{5} & 1 \\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5} \\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5} \end{bmatrix}$, then compute $3A - 5B$. **[3 Marks]**

Solution:

Given:

$$A = \begin{bmatrix} \frac{2}{3} & 1 & \frac{5}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3} \\ \frac{7}{3} & 2 & \frac{2}{3} \end{bmatrix} \text{ and } B = \begin{bmatrix} \frac{2}{5} & \frac{3}{5} & 1 \\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5} \\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5} \end{bmatrix}$$

Step 1:

$$3A - 5B = 3\begin{bmatrix} \frac{2}{3} & 1 & \frac{5}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3} \\ \frac{7}{3} & 2 & \frac{2}{3} \end{bmatrix} - 5\begin{bmatrix} \frac{2}{5} & \frac{3}{5} & 1 \\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5} \\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5} \end{bmatrix}$$

Step 2:

$$= \begin{bmatrix} \frac{2}{3} \times 3 & 1 \times 3 & \frac{5}{3} \times 3 \\ \frac{1}{3} \times 3 & \frac{2}{3} \times 3 & \frac{4}{3} \times 3 \\ \frac{7}{3} \times 3 & 2 \times 3 & \frac{2}{3} \times 3 \end{bmatrix} - \begin{bmatrix} \frac{2}{5} \times 5 & \frac{3}{5} \times 5 & 1 \times 5 \\ \frac{1}{5} \times 5 & \frac{2}{5} \times 5 & \frac{4}{5} \times 5 \\ \frac{7}{5} \times 5 & \frac{6}{5} \times 5 & \frac{2}{5} \times 5 \end{bmatrix}$$

Step 3:

$$= \begin{bmatrix} 2 & 3 & 5 \\ 1 & 2 & 4 \\ 7 & 6 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 3 & 5 \\ 1 & 2 & 4 \\ 7 & 6 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2-2 & 3-3 & 5-5 \\ 1-1 & 2-2 & 4-4 \\ 7-7 & 6-6 & 2-2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

Therefore, $3A - 5B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

6. Simplify $\cos\theta \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta \begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$ **[3 Marks]**

Solution:

Given:

$$cos\theta \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix} + sin\theta \begin{bmatrix} sin\theta & -cos\theta \\ cos\theta & sin\theta \end{bmatrix}$$

Step 1:

$$= \begin{bmatrix} \cos\theta(\cos\theta) & \cos\theta(\sin\theta) \\ \cos\theta(-\sin\theta) & \cos\theta(\cos\theta) \end{bmatrix} + \begin{bmatrix} \sin\theta(\sin\theta) & \sin\theta(-\cos\theta) \\ \sin\theta(\cos\theta) & \sin\theta(\sin\theta) \end{bmatrix}$$

$$= \begin{bmatrix} cos^2\theta & sin\theta cos\theta \\ -sin\theta cos\theta & cos^2\theta \end{bmatrix} + \begin{bmatrix} sin^2\theta & -sin\theta cos\theta \\ sin\theta cos\theta & sin^2\theta \end{bmatrix}$$

Step 2:

$$= \begin{bmatrix} cos^2\theta + sin^2\theta & sin\theta cos\theta - sin\theta cos\theta \\ -sin\theta cos\theta + sin\theta cos\theta & cos^2\theta + sin^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta + \sin^2\theta & 0 \\ 0 & \cos^2\theta + \sin^2\theta \end{bmatrix} \quad (\because \cos^2\theta + \sin^2\theta = 1)$$

Step 3:

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Hence, $\cos\theta \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta \begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

7. Find X and Y, if

(i) $X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ and $X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ **[3 Marks]**

(ii) $2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ and $3X + 2Y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$ **[3 Marks]**

Solution:

(i) **Given:**

$X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} \cdots (1)$

and $X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \ldots. (2)$

Step 1:

Adding equation (1) and (2), we get

$X + Y + X - Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$2X = \begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix}$

$\Rightarrow X = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$ **[$1\frac{1}{2}$ Mark]**

Step 2:

Putting the value of X in equation (1), we get

$\begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix} + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$

$Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$

$\Rightarrow Y = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$

Therefore, $X = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$ and $Y = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$ **[$1\frac{1}{2}$ Mark]**

(ii) **Given:**

$2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} \ldots (1)$

$3X+2Y=\begin{bmatrix}2 & -2\\ -1 & 5\end{bmatrix}\cdots(2)$

Step 1:

Multiply equation (1) by 3 and equation (2) by 2, on subtracting, we get

$3\,(2X+3Y)-2(3X+2Y)=3\begin{bmatrix}2 & 3\\ 4 & 0\end{bmatrix}-2\begin{bmatrix}2 & -2\\ -1 & 5\end{bmatrix}$

$\Rightarrow 6X+9Y-6X-4Y=\begin{bmatrix}6 & 9\\ 12 & 0\end{bmatrix}-\begin{bmatrix}4 & -4\\ -2 & 10\end{bmatrix}$

$\Rightarrow 5Y=\begin{bmatrix}2 & 13\\ 14 & -10\end{bmatrix}$

$\Rightarrow Y=\begin{bmatrix}\frac{2}{5} & \frac{13}{5}\\ \frac{14}{5} & -2\end{bmatrix}$ **[$1\frac{1}{2}$ Mark]**

Step 2:

Putting the value of Y in equation (1), we get

$2X+3\begin{bmatrix}\frac{2}{5} & \frac{13}{5}\\ \frac{14}{5} & -2\end{bmatrix}=\begin{bmatrix}2 & 3\\ 4 & 0\end{bmatrix}$

$\Rightarrow 2X=\begin{bmatrix}2 & 3\\ 4 & 0\end{bmatrix}-\begin{bmatrix}\frac{6}{5} & \frac{39}{5}\\ \frac{42}{5} & -6\end{bmatrix}$

$=\begin{bmatrix}2-\frac{6}{5} & 3-\frac{39}{5}\\ 4-\frac{42}{5} & 0+6\end{bmatrix}=\begin{bmatrix}\frac{4}{5} & -\frac{24}{5}\\ -\frac{22}{5} & 6\end{bmatrix}$

$\Rightarrow X=\begin{bmatrix}\frac{2}{5} & -\frac{12}{5}\\ -\frac{11}{5} & 3\end{bmatrix}$

Hence, $X=\begin{bmatrix}\frac{2}{5} & -\frac{12}{5}\\ -\frac{11}{5} & 3\end{bmatrix}$ and $Y=\begin{bmatrix}\frac{2}{5} & \frac{13}{5}\\ \frac{14}{5} & -2\end{bmatrix}$ **[$1\frac{1}{2}$ Mark]**

8. Find X, if $Y=\begin{bmatrix}3 & 2\\ 1 & 4\end{bmatrix}$ and $2X+Y=\begin{bmatrix}1 & 0\\ -3 & 2\end{bmatrix}$ **[3 Marks]**

Solution:

Given:

$Y = \begin{bmatrix}3 & 2\\1 & 4\end{bmatrix}$

Step 1:

$2X + Y = \begin{bmatrix}1 & 0\\-3 & 2\end{bmatrix}$

$\Rightarrow 2X + \begin{bmatrix}3 & 2\\1 & 4\end{bmatrix} = \begin{bmatrix}1 & 0\\-3 & 2\end{bmatrix}$ $[\because Y = \begin{bmatrix}3 & 2\\1 & 4\end{bmatrix}]$

[$1\frac{1}{2}$ Mark]

Step 3:

$\Rightarrow 2X = \begin{bmatrix}1 & 0\\-3 & 2\end{bmatrix} - \begin{bmatrix}3 & 2\\1 & 4\end{bmatrix} = \begin{bmatrix}-2-2\\-4-2\end{bmatrix}$

$\Rightarrow X = \begin{bmatrix}-1-1\\-2-1\end{bmatrix}$

Hence, the value of $X = \begin{bmatrix}-1-1\\-2-1\end{bmatrix}$**[$1\frac{1}{2}$ Mark]**

9. Find x and y, if $2\begin{bmatrix}1 & 3\\0 & x\end{bmatrix} + \begin{bmatrix}y & 0\\1 & 2\end{bmatrix} = \begin{bmatrix}5 & 6\\1 & 8\end{bmatrix}$ **[3 Marks]**

Solution:

Given:

$2\begin{bmatrix}1 & 3\\0 & x\end{bmatrix} + \begin{bmatrix}y & 0\\1 & 2\end{bmatrix} = \begin{bmatrix}5 & 6\\1 & 8\end{bmatrix}$

Step 1:

$\Rightarrow \begin{bmatrix}2 & 6\\0 & 2x\end{bmatrix} + \begin{bmatrix}y & 0\\1 & 2\end{bmatrix} = \begin{bmatrix}5 & 6\\1 & 8\end{bmatrix}$

$\Rightarrow \begin{bmatrix}2+y & 6+0\\0+1 & 2x+2\end{bmatrix} = \begin{bmatrix}5 & 6\\1 & 8\end{bmatrix}$

Step 2:

If two matrices are equal, then their corresponding elements are also equal.

Comparing the corresponding elements, we get

$2 + y = 5$ and $2x + 2 = 8$

Step 3:

By solving these equations, we get

$y = 3$ and $x = 3$

Therefore, the values of $y = 3$ and $x = 3$.

10. Solve the equation for x, y, z and t, if $2\begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$ **[3 Marks]**

Solution:

Given:

$$2\begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$$

Step 1:

$$\begin{bmatrix} x \times 2 & z \times 2 \\ y \times 2 & t \times 2 \end{bmatrix} + \begin{bmatrix} 1 \times 3 & -1 \times 3 \\ 0 \times 3 & 2 \times 3 \end{bmatrix} = \begin{bmatrix} 3 \times 3 & 5 \times 3 \\ 4 \times 3 & 6 \times 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x & 2z \\ 2y & 2t \end{bmatrix} + \begin{bmatrix} 3 & -3 \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}$$

Step 2:

If two matrices are equal, then their corresponding elements are also equal.

Comparing the corresponding elements, we get

$2x + 3 = 9$

$2z - 3 = 15$

$2y = 12$

$2t + 6 = 18$

Step 3:

By solving these equations, we get

$x = 3, z = 9, y = 6$ and $t = 6$

Hence, the values of $x = 3, z = 9, y = 6$ and $t = 6$

11. If $x\begin{bmatrix}2\\3\end{bmatrix}+y\begin{bmatrix}-1\\1\end{bmatrix}=\begin{bmatrix}10\\5\end{bmatrix}$, find the values of x and y. **[3 Marks]**

Solution:

Given:

$x\begin{bmatrix}2\\3\end{bmatrix}+y\begin{bmatrix}-1\\1\end{bmatrix}=\begin{bmatrix}10\\5\end{bmatrix}$

Step 1:

$\Rightarrow \begin{bmatrix}2x\\3x\end{bmatrix}+\begin{bmatrix}-y\\y\end{bmatrix}=\begin{bmatrix}10\\5\end{bmatrix}$

$\Rightarrow \begin{bmatrix}2x-y\\3x+y\end{bmatrix}=\begin{bmatrix}10\\5\end{bmatrix}$

Step 2:

If two matrices are equal, then their corresponding elements are also equal.

Comparing the corresponding elements, we get

$2x - y = 10$

$3x + y = 5$

Step 3:

Adding both the equations, we get

$5x = 15$

$\Rightarrow x = 3$

Putting the value of x in the equation $3x + y = 5$, we get

$3(3) + y = 5$

$\Rightarrow y = -4$

Therefore, the values x and y are 3 and -4 respectively.

12. Given $3\begin{bmatrix}x & y\\z & w\end{bmatrix}=\begin{bmatrix}x & 6\\-1 & 2w\end{bmatrix}+\begin{bmatrix}4 & x+y\\z+w & 3\end{bmatrix}$, find the values of x, y, z and w. **[3 Marks]**

Solution:

Given:

$$3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$$

Step 1:

$$\begin{bmatrix} 3\times x & 3\times y \\ 3\times z & 3\times w \end{bmatrix} = \begin{bmatrix} x+4 & 6+x+y \\ -1+z+w & 2w+3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3x & 3y \\ 3z & 3w \end{bmatrix} = \begin{bmatrix} x+4 & 6+x+y \\ 1+z+w & 2w+3 \end{bmatrix}$$

Step 2:

If two matrices are equal, then their corresponding elements are also equal.

Comparing the corresponding elements, we get

$3x = x + 4$

$3y = 6 + x + y$

$3z = -1 + z + w$

$3w = 2w + 3$

Step 3:

By solving these equations, we get

$\Rightarrow x = 2, y = 4, z = 1$ and $w = 3$

Therefore, the values of $x = 2, y = 4, z = 1$ and $w = 3$

13. If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$, show that $F(x)F(y) = F(x+y)$. **[3 Marks]**

Solution:

Given:

$$F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Step 1:

For $F(y)$, replace x by y in $F(x)$

$$F(y) = \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ **[$\frac{1}{2}$ Mark]**

Step 2:

By taking LHS $= F(x)F(y)$

$$= \begin{bmatrix} cosx & -sinx & 0 \\ sinx & cosx & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos y & -\sin y\, 0 & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ **[$\frac{1}{2}$ Mark]**

Step 3:

$$= \begin{bmatrix} \text{cos}x\ \text{cos}y + (-\text{sin}x)\text{sin}y + 0 & \text{cos}x(-\text{sin}y) + (-\text{sin}x)\text{cos}y + 0 & 0 + 0 + 0 \times 1 \\ \text{sin}x\ \text{cos}y + \text{cos}x\ \text{sin}y + 0 & \text{sin}x(-\text{sin}y) + \text{cos}x\ \text{cos}y + 0 & 0 + 0 + 0 \times 1 \\ 0 \times \text{cos}y + 0 \times \text{sin}y + 0 \times 1 & 0 \times (-\text{sin}y) + 0 \times \text{cos}y + 0 & 0 + 0 + 1 \times 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(x+y) & -[\text{cos}x\ \text{sin}y + \text{sin}x\ \text{cos}y] & 0 \\ \sin(x+y) & \text{cos}x\ \text{cos}y - \text{sin}x\ \text{sin}y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\because \text{cos}x\ \text{cos}y - \text{sin}x\ \text{sin}y = \cos(x+y)$$

$$\text{sin}x\ \text{cos}y + \text{cos}x\ \text{sin}y = \sin(x+y)$$ **[1Mark]**

Step 4:

$$= \begin{bmatrix} cos(x+y) & -sin(x+y) & 0 \\ sin(x+y) & cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix} = F(x+y) = RHS$$

$LHS = RHS$

Hence it is proved $F(x)F(y) = F(x+y)$.

14. Show that

(i) $\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \neq \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$ **[3 Marks]**

(ii) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \neq \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$ **[4 Marks]**

Solution:

(i)**Step 1:**

By taking $LHS = \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$

$$= \begin{bmatrix} 5\times 2+(-1)\times 3 & 5\times 1+(-1)\times 4 \\ 6\times 2+7\times 3 & 6\times 1+7\times 4 \end{bmatrix}$$ **[$\frac{1}{2}$ Mark]**

Step 2:

$$= \begin{bmatrix} 10-3 & 5-4 \\ 12+21 & 6+28 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 1 \\ 33 & 34 \end{bmatrix}$$

Step 3:

Now,

By taking $RHS = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$

$$= \begin{bmatrix} 2\times 5+1\times 6 & 2\times(-1)+1\times 7 \\ 3\times 5+4\times 6 & 6\times 1+7\times 4 \end{bmatrix}$$ **[$\frac{1}{2}$ Mark]**

Step 4:

$$= \begin{bmatrix} 10+6 & -2+7 \\ 15+24 & -3+28 \end{bmatrix} = \begin{bmatrix} 16 & 5 \\ 39 & 25 \end{bmatrix}$$

Hence, $\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \neq \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$

(ii)**Step 1:**

By taking $LHS = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$

$$= \begin{bmatrix} 1\times(-1)+2\times 0+3\times 2 & 1\times 1+2\times(-1)+3\times 3 & 1\times 0+2\times 1+3\times 4 \\ 0\times(-1)+1\times 0+0\times 2 & 0\times 1+1\times(-1)+0\times 3 & 0\times 0+1\times 1+0\times 4 \\ 1\times(-1)+1\times 0+0\times 2 & 1\times 1+1\times(-1)+0\times 3 & 1\times 0+1\times 1+0\times 4 \end{bmatrix}$$

Step 2:

$$= \begin{bmatrix} -1+0+6 & 1-0+9 & 0+2+12 \\ 0+0+0 & 0-1+0 & 0+1+0 \\ -1+0+0 & 1-1+0 & 0+1+0 \end{bmatrix}$$

$$LHS = \begin{bmatrix} 5 & 8 & 14 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

Step 3:

Now,

By taking $RHS = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

$$= \begin{bmatrix} -1 \times 1 + 1 \times 0 + 0 \times 1 & -1 \times 2 + 1 \times 1 + 0 \times 1 & -1 \times 3 + 1 \times 0 + 0 \times 0 \\ 0 \times 1 + (-1) \times 0 + 1 \times 1 & 0 \times 2 + (-1) \times 1 + 1 \times 1 & 0 \times 3 + (-1) \times 0 + 1 \times 0 \\ 2 \times 1 + 3 \times 0 + 4 \times 1 & 2 \times 2 + 3 \times 1 + 4 \times 1 & 2 \times 3 + 3 \times 0 + 4 \times 0 \end{bmatrix}$$

Step 4:

$$= \begin{bmatrix} -1 + 0 + 0 & -2 + 1 + 0 & -3 + 0 + 0 \\ 0 + 0 + 1 & 0 - 1 + 1 & 0 + 0 + 0 \\ 2 + 0 + 4 & 4 + 3 + 4 & 6 + 0 + 0 \end{bmatrix}$$

$$RHS = \begin{bmatrix} -1 & -1 & -3 \\ 1 & 0 & 0 \\ 6 & 11 & 6 \end{bmatrix}$$

$\therefore LHS \neq RHS$

Hence, $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \neq \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 3 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

15. Find $A^2 - 5A + 6I$, if $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$ **[4 marks]**

Solution:

Given:

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

Step 1:

We know that, $A^2 = A.A$

$$\Rightarrow A^2 = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times 2+0\times 2+1\times 1 & 2\times 0+0\times 1+1\times(-1) & 2\times 1+0\times 3+1\times 0 \\ 2\times 2+1\times 2+3\times 1 & 2\times 0+1\times 1+3\times(-1) & 2\times 1+1\times 3+3\times 0 \\ 1\times 2+(-1)\times 2+0\times 1 & 1\times 0+(-1)\times 1+0\times(-1) & 1\times 1+(-1)\times 3+0\times 0 \end{bmatrix}$$

Step 2:

$$= \begin{bmatrix} 4+0+1 & 0+0-1 & 2+0+0 \\ 4+2+3 & 0+1-3 & 2+3+0 \\ 2-2+0 & 0-1+0 & 1-3+0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$$

Step 3:

Therefore, $A^2 - 5A + 6I$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - 5\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} + 6\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Step 4:

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - \begin{bmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 5-10+6 & -1-0+0 & 2-5+0 \\ 9-10+0 & -2-5+6 & 5-15+0 \\ 0-5+0 & -1+5+0 & -2-0+6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}$$

Hence, $A^2 - 5A + 6I = \begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}$

16. If $A=\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}$, prove that $A^3-6A^2+7A+2I=0$ **[5 Marks]**

Solution:

Given:

$$A=\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}$$

Step 1:

We know that, $A^2=A.A$

$$\Rightarrow A^2=\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}$$

$$=\begin{bmatrix}1\times1+0\times0+2\times2&1\times0+0\times2+2\times0&1\times2+0\times1+2\times3\\0\times1+2\times0+1\times2&0\times0+2\times2+1\times0&0\times2+2\times1+1\times3\\2\times1+0\times0+3\times2&2\times0+0\times2+3\times0&2\times2+0\times1+3\times3\end{bmatrix}$$

Step 2:

$$=\begin{bmatrix}1+0+4&0+0+0&2+0+6\\0+0+2&0+4+0&0+2+3\\2+0+6&0+0+0&4+0+9\end{bmatrix}$$

$$A^2=\begin{bmatrix}5&0&8\\2&4&5\\8&0&13\end{bmatrix}$$

Step 3:

Now,

$$A^3=A^2A=\begin{bmatrix}5&0&8\\2&4&5\\8&0&13\end{bmatrix}\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}$$

$$=\begin{bmatrix}5\times1+0\times0+8\times2&5\times0+0\times2+8\times0&5\times2+0\times1+8\times3\\2\times1+4\times0+5\times2&2\times0+4\times2+5\times0&2\times2+4\times1+5+3\\8\times1+0\times0+13\times2&8\times0+0\times2+13\times0&8\times2+0\times1+13\times3\end{bmatrix}$$

$$=\begin{bmatrix}5+0+16&0+0+0&10+0+24\\2+0+10&0+8+0&4+4+15\\8+0+26&0+0+0&16+0+39\end{bmatrix}$$

$$A^3=\begin{bmatrix}21&0&34\\12&8&23\\34&0&55\end{bmatrix}$$

Step 4:

Therefore, $A^3 - 6A^2 + 7A + 2I$

$$LHS = \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - 6\begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - \begin{bmatrix} 6(5) & 0(5) & 8(6) \\ 2(6) & 4(6) & 5(6) \\ 8(6) & 0(6) & 13(6) \end{bmatrix} + \begin{bmatrix} 1(7) & 0(7) & 2(7) \\ 0(7) & 2(7) & 1(7) \\ 2(7) & 0(7) & 3(7) \end{bmatrix} +$$

$$\begin{bmatrix} 2(1) & 2(0) & 2(0) \\ 2(0) & 1(2) & 0(2) \\ 2(0) & 0(2) & 1(2) \end{bmatrix}$$

Step 5:

$$= \begin{bmatrix} 21 - 30 + 7 + 2 & 0 - 0 + 0 + 0 & 34 - 48 + 14 + 0 \\ 12 - 12 + 0 + 0 & 8 - 24 + 14 + 2 & 23 - 30 + 7 + 0 \\ 34 - 48 + 14 + 0 & 0 + 0 + 0 + 0 & 55 - 78 + 21 + 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

$\because LHS = RHS$

Hence proved.

17. If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find k so that $A^2 = kA - 2I$ **[3 marks]**

Solution:

Given:

$A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Step 1:

$A^2 = kA - 2I$

We know that, $A^2 = A.A$

$$\Rightarrow \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = k\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Step 2:

$$\Rightarrow \begin{bmatrix} 3\times 3+(-2)\times 4 & 3\times(-2)+(-2)\times(-2) \\ 4\times 3+(-2)\times 4 & 4\times(-2)+(-2)\times(-2) \end{bmatrix} = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k-2 & -2k \\ 4k & -2k-2 \end{bmatrix}$$

Step 3:

Here, matrices are equal.

Comparing the corresponding elements, we get

$\Rightarrow 4k = 4$

$\therefore k = 1$

Hence, the value of k is 1.

18. If $A = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and I is the identity matrix of order 2, show that

$I + A = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ **[5 Marks]**

Solution:

Given:

$$A = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$$

Step 1:

By considering LHS $= I + A$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix} = \begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix}$$

Step 2:

Now,

By taking RHS $= (I - A)\begin{bmatrix} cos\alpha & -sin\alpha \\ sin\alpha & cos\alpha \end{bmatrix}$

$$= \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}\right)\begin{bmatrix} cos\alpha & -sin\alpha \\ sin\alpha & cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix}\begin{bmatrix} cos\alpha & -sin\alpha \\ sin\alpha & cos\alpha \end{bmatrix}$$

Step 3:

$$= \begin{bmatrix} \cos\alpha + \tan\frac{\alpha}{2}\sin\alpha & -\sin\alpha + \tan\frac{\alpha}{2}\cos\alpha \\ -\tan\frac{\alpha}{2}\cos\alpha + \sin\alpha & \tan\frac{\alpha}{2}\sin\alpha + \cos\alpha \end{bmatrix}$$

Step 4:

$$= \begin{bmatrix} cos\alpha + \frac{\sin\frac{\alpha}{2}}{cos\frac{\alpha}{2}} sin\alpha & -sin\alpha + \frac{sin\frac{\alpha}{2}}{cos\frac{\alpha}{2}} cos\alpha \\ -\frac{sin\frac{\alpha}{2}}{cos\frac{\alpha}{2}} cos\alpha + sin\alpha & \frac{sin\frac{\alpha}{2}}{cos\frac{\alpha}{2}} sin\alpha + cos\alpha \end{bmatrix}$$ [$\frac{1}{2}$ **Mark**]

Step 5:

$$= \begin{bmatrix} \frac{cos\alpha\ cos\frac{\alpha}{2} + sin\frac{\alpha}{2} sin\alpha}{cos\frac{\alpha}{2}} & \frac{-sin\alpha\ cos\frac{\alpha}{2} + sin\frac{\alpha}{2} cos\alpha}{cos\frac{\alpha}{2}} \\ \frac{-cos\alpha\ sin\frac{\alpha}{2} + cos\frac{\alpha}{2} sin\alpha}{cos\frac{\alpha}{2}} & \frac{sin\alpha\ sin\frac{\alpha}{2} + cos\frac{\alpha}{2} cos\alpha}{cos\frac{\alpha}{2}} \end{bmatrix}$$ [$\frac{1}{2}$ **Mark**]

Step 6:

$$= \begin{bmatrix} \frac{\cos\left(\alpha - \frac{\alpha}{2}\right)}{cos\frac{\alpha}{2}} & \frac{-\sin\left(\alpha - \frac{\alpha}{2}\right)}{cos\frac{\alpha}{2}} \\ \frac{\sin\left(\alpha - \frac{\alpha}{2}\right)}{cos\frac{\alpha}{2}} & \frac{\cos\left(\alpha - \frac{\alpha}{2}\right)}{cos\frac{\alpha}{2}} \end{bmatrix}$$ [$\frac{1}{2}$ **Mark**]

Step 7:

$$= \begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix} = LHS$$ [$\frac{1}{2}$ **Mark**]

$\because LHS = RHS$

Hence, it is proved.

19. A trust fund has ₹ 30,000 that must be invested in two different types of bonds. The first bond pays 5% interest per year, and the second bond pays 7% interest per year. Using matrix multiplication, determine how to divide ₹ 30,000 among the two types of bonds. If the trust fund must obtain an annual total interest of: **[5 Marks]**

(a) ₹1800

(b) ₹2000

Solution:

Given:

Trust fund= ₹ 30,000

First bond pays interest = 5% per year

Second bond pays interest = 7% per year

Step 1:

Let the amount invested in first bond = ₹ x

The amount invested in second bond = ₹$(30000 - x)$

(a) If the total annual interest is ₹1800, then

Investment in Bonds (in ₹)	Annual Interest Rate	Interest (in ≤]
$\begin{bmatrix} x \\ 30{,}000 - x \end{bmatrix}$	$\begin{bmatrix} 5\% \\ 7\% \end{bmatrix}$	$[1800]$

Step 2:

By solving, we get

$x \times 5\% + (30000 - x) \times 7\%$=1800

$\Rightarrow \frac{5x}{100} + \frac{7}{100}(30000 - x) = 1800$

$\Rightarrow 5x + 210000 - 7x = 180000$

$\Rightarrow -2x = -30000$

$x = 15000$

Step 3:

So, amount investment at 5% = ₹15000

The amount investment at 7% = ₹$(30000 - x)$

$= ₹(30000 - 15000) = ₹15000$

Therefore, the amount invested in first bond is ₹15000 and second bond is ₹15000 **[$\frac{1}{2}$ Mark]**

Step 4:

(b) If the total annual interest is ₹2000, then

Investment in Bonds (in ₹]	Annual Interest Rate	Interest (in₹)
$\begin{bmatrix} x \\ 30{,}000 - x \end{bmatrix}$	$\begin{bmatrix} 5\% \\ 7\% \end{bmatrix}$	$[2000]$

Step 5:

By solving, we get

$$x \times 5\% + (30000 - x) \times 7\% = 2000$$

$$\Rightarrow \frac{5x}{100} + \frac{7}{100}(30000 - x) = 2000$$

$$\Rightarrow 5x + 210000 - 7x = 200000$$

$$\Rightarrow -2x = -10000$$

$$\Rightarrow x = 5000$$

Step 6:

So, amount investment at 5% = ₹5000

The amount investment at 7% = ₹$(30000 - x)$

= ₹$(30000 - 5000)$ = ₹25000

Therefore, the amount invested in first bond is ₹ 5000 and second bond is ₹25000. **[$\frac{1}{2}$ Mark]**

20. The bookshop of a particular school has 10 dozen chemistry books, 8 dozen physics books, 10 dozen economics books. Their selling prices are ₹ 80, ₹ 60 and ₹ 40 each respectively. Find the total amount the bookshop will receive from selling all the books using matrix algebra.

[3 Marks]

Solution:

Given:

Step 1:

Number of chemistry books $= 10$ dozen $= 10 \times 12 = 120$

Number of physics books $= 8$ dozen $= 8 \times 12 = 96$

Number of economics books $= 10$ dozen $= 10 \times 12 = 120$ **[$\frac{1}{2}$ Mark]**

Step 2:

Number of books Chemistry Physics Economics	Selling Price per book	Total amount (in ₹)
$[120\ 96\ 120]$	$\begin{bmatrix}80\\60\\40\end{bmatrix}$	$[x]$

[1Mark]

Step 3:

Now,

The shopkeeper receives the total amount of $=$ Number of books $\times$ Selling price per book

$$= \begin{bmatrix}120 & 96 & 120\end{bmatrix}\begin{bmatrix}80\\60\\40\end{bmatrix}$$

$$= [180 \times 80 + 96 \times 60 + 120 \times 40]$$

$$= 120 \times 80 + 96 \times 60 + 120 \times 40$$

$$= 9600 + 5760 + 4800$$

$$= ₹\ 20160$$ **[1$\frac{1}{2}$ Marks]**

Hence, the bookshop will receive ₹20160 from selling all the books.

21. Assume X, Y, Z, W and P are matrices of order 2 × n, 3 × k, 2 × p, n × 3 and p × k, respectively.

The restriction on n, k and p so that $PY + WY$ will be defined are: **[2 Marks]**

(A) $k = 3, p = n$

(B) k is arbitrary, $p = 2$

(C) p is arbitrary, $k = 3$

(D) $k = 2, p = 3$

Solution:

Given:

The order of $P = p \times k$

The order of $Y = 3 \times k$.

Step 1:

So, PY will be defined if, $k = 3$.

Hence, the order of PY is $p \times k$.

Step 2:

Order of $W = n \times 3$ and order of $Y = 3 \times k$

According to order, WY is defined and its order is $n \times k$.

$PY + WY$ is defined, if the order of PY and WY are equal.

$\Rightarrow p \times k = n \times k$

$\Rightarrow p = n,$

Therefore, the option (A) is correct.

22. Assume X, Y, Z, W and P are matrices of order 2 × n, 3 × k, 2 × p, n × 3 and p × k, respectively.

If $n = p$, then the order of the matrix $7X - 5Z$ is: **[2 marks]**

(A) $p \times 2$

(B) $2 \times n$

(C) $n \times 3$

(D) $p \times n$

Solution:

Given:

The order of $X = 2 \times n$

The order of $Z = 2 \times p$

$n = p$

Step 1:

During the addition or subtraction, the order of matrix doesn't change.

Therefore, the order of $7X - 5Z =$ order of $X =$ order of Z

Step 2:

$\Rightarrow 2 \times n = 2 \times p$

$= 2 \times n$

Hence, the option (B) is correct

EXERCISE 3.3

1. Find the transpose of each of the following matrices:

(i) $\begin{bmatrix} 5 \\ \frac{1}{2} \\ -1 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

(iii) $\begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}$

Solution:

(i) Let $A = \begin{bmatrix} 5 \\ \frac{1}{2} \\ -1 \end{bmatrix}$

$$A' = \begin{bmatrix} 5 \\ \frac{1}{2} \\ -1 \end{bmatrix}' = \begin{bmatrix} 5 & \frac{1}{2} & -1 \end{bmatrix}$$

Hence the transpose of the given matrix is $[5\ \frac{1}{2} - 1]$

(ii) Let $B = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

$$B' = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}' = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$$

Hence the transpose of the given matrix is $\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$

(iii) Let $C = \begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}$

$$C' = \begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}' = \begin{bmatrix} -1 & \sqrt{3} & 2 \\ 5 & 5 & 3 \\ 6 & 6 & -1 \end{bmatrix}$$

Hence the transpose of the given matrix is $\begin{bmatrix} -1 & \sqrt{3} & 2 \\ 5 & 5 & 3 \\ 6 & 6 & -1 \end{bmatrix}$

2. If $A = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$, then verify that

(i) $(A+B)' = A' + B'$, **[3 Marks]**

(ii) $(A-B)' = A' - B'$ **[3 Marks]**

Solution:

Given:

$A = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$

(i) $(A+B)' = A' + B'$

Step 1:

$(A+B)$

$$=\begin{bmatrix}-1 & 2 & 3\\ 5 & 7 & 9\\ -2 & 1 & 1\end{bmatrix}+\begin{bmatrix}-4 & 1 & -5\\ 1 & 2 & 0\\ 1 & 3 & 1\end{bmatrix}=\begin{bmatrix}-1-4 & 2+1 & 3-5\\ 5+1 & 7+2 & 9+0\\ -2+1 & 1+3 & 1+1\end{bmatrix}$$

$$(A+B)=\begin{bmatrix}-5 & 3 & -2\\ 6 & 9 & 9\\ -1 & 4 & 2\end{bmatrix}$$

Step 2:

Hence $(A+B)'=\begin{bmatrix}-5 & -6 & -1\\ 3 & 9 & 4\\ -2 & 9 & 2\end{bmatrix}$ ....(1) **[$\frac{1}{2}$ Mark]**

Step 3:

Now,

$A'+B'=\begin{bmatrix}-1 & 2 & 3\\ 5 & 7 & 9\\ -2 & 1 & 1\end{bmatrix}'+\begin{bmatrix}-4 & 1 & -5\\ 1 & 2 & 0\\ 1 & 3 & 1\end{bmatrix}'$ **[$\frac{1}{2}$ Mark]**

Step 4:

$$=\begin{bmatrix}-1 & 5 & -2\\ 2 & 7 & 1\\ 3 & 9 & 1\end{bmatrix}+\begin{bmatrix}-4 & 1 & 1\\ 1 & 2 & 3\\ -5 & 0 & 1\end{bmatrix}=\begin{bmatrix}-1-4 & 5+1 & -2+1\\ 2+1 & 7+2 & 1+3\\ 3-5 & 9+0 & 1+1\end{bmatrix}=\begin{bmatrix}-5 & 6 & -1\\ 3 & 9 & 4\\ -2 & 9 & 2\end{bmatrix}\text{....(2)}$$

From the equation (1) and (2], we get

$(A+B)'=A'+B'$ **[1Mark]**

Hence it is proved.

(ii) $(A-B)'=A'-B'$

Step 1:

$(A-B)$

$$=\begin{bmatrix}-1 & 2 & 3\\ 5 & 7 & 9\\ -2 & 1 & 1\end{bmatrix}-\begin{bmatrix}-4 & 1 & -5\\ 1 & 2 & 0\\ 1 & 3 & 1\end{bmatrix}=\begin{bmatrix}-1+4 & 2-1 & 3+5\\ 5-1 & 7-2 & 9-0\\ -2-1 & 1-3 & 1-1\end{bmatrix}$$

$(A-B)=\begin{bmatrix}3 & 1 & 8\\ 4 & 5 & 9\\ -3 & -2 & 0\end{bmatrix}$ **[1Mark]**

Step 2:

Thus, $(A-B)'=\begin{bmatrix}3 & 4 & -3\\ 1 & 5 & -2\\ 8 & 9 & 0\end{bmatrix}\cdots(1)$ **[$\frac{1}{2}$ Mark]**

Step 3:

$$A' - B' = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}' - \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}'$$ **[$\frac{1}{2}$ Mark]**

Step 4:

$$= \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix} - \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1+4 & 5-1 & -2-1 \\ 2-1 & 7-2 & 1-3 \\ 3+5 & 9-0 & 1-1 \end{bmatrix} = \begin{bmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{bmatrix} \cdots (2)$$

From the equation (1) and (2), we get

$(A + B)' = A' + B'$ **[1Mark]**

Hence it is proved.

3. If $A' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, then verify that

(i) $(A + B)' = A' + B'$ **[3 Marks]**

(ii) $(A - B)' = A' - B'$ **[3 Marks]**

Solution:

Given:

$A' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$

(i) $(A + B)' = A' + B'$

Step 1:

$$A' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow A = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$ $[\because (A')' = A]$ **[$\frac{1}{2}$ Mark]**

Step 2:

$$(A + B) = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3-1 & -1+2 & 0+1 \\ 4+1 & 2+2 & 1+3 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 1 \\ 5 & 4 & 4 \end{bmatrix}$$

Step 3:

Therefore $(A+B)' = \begin{bmatrix} 2 & 5 \\ 1 & 4 \\ 1 & 4 \end{bmatrix}$ .... (1) **[$\frac{1}{2}$ Mark]**

Step 4:

$$A' + B' = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix}' + \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}'$$

$$= \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 3-1 & 4+1 \\ -1+2 & 2+2 \\ 0+1 & 1+3 \end{bmatrix}$$

$A' + B' = \begin{bmatrix} 2 & 5 \\ 1 & 4 \\ 1 & 4 \end{bmatrix} \cdots$ (2)

From the equation (1) and (2), we get

$(A+B)' = A' + B'$

Hence, it is proved.

(ii) $(A-B)' = A' - B'$

Step 1:

$$A' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow A = \begin{bmatrix} 3 & -l & 0 \\ 4 & 2 & 1 \end{bmatrix}$ $[\because (A')' = A]$ **[$\frac{1}{2}$ Mark]**

Step 2:

$$(\mathrm{A} - \mathrm{B}) = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3+1 & -1-2 & 0-1 \\ 4-1 & 2-2 & 1-3 \end{bmatrix} = \begin{bmatrix} 4 & -3 & -1 \\ 3 & 0 & -2 \end{bmatrix}$$

Step 3:

Therefore $(\mathrm{A}-\mathrm{B})' = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix} \cdots$ (1) **[$\frac{1}{2}$ Mark]**

Step 4:

$$A' - B' = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix}' - \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}'$$

$$= \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 3+1 & 4-1 \\ -1-2 & 2-2 \\ 0-1 & 1-3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix} \cdots (2)$$

From the equation (1) and (2], we get

$(A - B)' = A' - B'$

Hence, it is proved.

4. If $A = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$, then find $(A + 2B)'$ **[2 Marks]**

Solution:

Given:

$A = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$

Step 1:

$$(A + 2B) = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix} + 2\begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} -2 & 0 \\ 2 & 4 \end{bmatrix}$$

Step 2:

$$= \begin{bmatrix} -2-2 & 3+0 \\ 1+2 & 2+4 \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ 3 & 6 \end{bmatrix}$$ **[$\frac{1}{2}$ Mark]**

Step 3:

$$(A + 2B)' = \begin{bmatrix} -4 & 3 \\ 3 & 6 \end{bmatrix}' = \begin{bmatrix} -4 & 3 \\ 3 & 6 \end{bmatrix}$$ **[$\frac{1}{2}$ Mark]**

Therefore, $(A + 2B)' = \begin{bmatrix} -4 & 3 \\ 3 & 6 \end{bmatrix}$

5. For the matrices A and B, verify that $(AB)' = B'A'$, where

(i) $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}, B = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$ **[3 marks]**

(ii) $A = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 5 & 7 \end{bmatrix}$ **[3 marks]**

Solution:

(i) **Given:**

$$A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}, B = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$$

Step 1:

So, $AB = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix} [-1\ 2\ 1]$

$$= \begin{bmatrix} 1 \times (-1) & 1 \times 2 & 1 \times 1 \\ -4 \times (-1) & -4 \times 2 & -4 \times 1 \\ 3 \times (-1) & 3 \times 2 & 3 \times 1 \end{bmatrix} = \begin{bmatrix} -1 & 2 & 1 \\ 4 & -8 & -4 \\ -3 & 6 & 3 \end{bmatrix}$$

Step 2:

$$AB' = \begin{bmatrix} -1 & 2 & 1 \\ 4 & -8 & -4 \\ -3 & 6 & 3 \end{bmatrix}' = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$$

Therefore, $AB' = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$ .....(1) **[$\frac{1}{2}$ Mark]**

Step 3:

$$B' = [-1\ 2\ 1]' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$$

$A' = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix} = [1\ -4\ 3]$ **[$\frac{1}{2}$ Mark]**

Step 4:

$$B'A' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} [1\ -4\ 3] = \begin{bmatrix} -1 \times 1 & -1 \times (-4) & -1 \times 3 \\ 2 \times 1 & 2 \times (-4) & 2 \times 3 \\ 1 \times 1 & 1 \times (-4) & 1 \times 3 \end{bmatrix}$$

$$=\begin{bmatrix}-1 & 4 & -3\\ 2 & -8 & 6\\ 1 & -4 & 3\end{bmatrix}$$

Therefore, $B'A'=\begin{bmatrix}-1 & 4 & -3\\ 2 & -8 & 6\\ 1 & -4 & 3\end{bmatrix}\cdots(2)$

From the equation (1] and [2], we get

$(AB)=B'A'$

Hence proved.

(ii) **Given:**

$A=\begin{bmatrix}0\\1\\2\end{bmatrix}, B=\begin{bmatrix}1 & 5 & 7\end{bmatrix}$

Step 1:

So, AB=$\begin{bmatrix}0\\1\\2\end{bmatrix}=\begin{bmatrix}1 & 5 & 7\end{bmatrix}$

$$=\begin{bmatrix}0\times1 & 0\times5 & 0\times7\\ 1\times1 & 1\times5 & 1\times7\\ 2\times1 & 2\times5 & 2\times7\end{bmatrix}=\begin{bmatrix}0 & 0 & 0\\ 1 & 5 & 7\\ 2 & 10 & 14\end{bmatrix}$$

Step 2:

$$AB'=\begin{bmatrix}0 & 0 & 0\\ 1 & 5 & 7\\ 2 & 10 & 14\end{bmatrix}'=\begin{bmatrix}0 & 1 & 2\\ 0 & 5 & 10\\ 0 & 7 & 14\end{bmatrix}$$

Therefore $AB'=\begin{bmatrix}0 & 1 & 2\\ 0 & 5 & 10\\ 0 & 7 & 14\end{bmatrix}\cdots(1)$ **[$\frac{1}{2}$ Mark]**

Step 3:

$B'=\begin{bmatrix}1 & 5 & 7\end{bmatrix}'=\begin{bmatrix}1\\5\\7\end{bmatrix}$ and $A'=\begin{bmatrix}0\\1\\2\end{bmatrix}=\begin{bmatrix}0 & 1 & 2\end{bmatrix}$ **[$\frac{1}{2}$ Mark]**

Step 4:

So, $B'A'=\begin{bmatrix}1\\5\\7\end{bmatrix}\begin{bmatrix}0 & 1 & 2\end{bmatrix}=\begin{bmatrix}1\times0 & 1\times1 & 1\times2\\ 5\times0 & 5\times1 & 5\times2\\ 7\times0 & 7\times1 & 7\times2\end{bmatrix}$

$$= \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix} \cdots (2)$$

From the equation (1) and (2), we get

$(AB)' = B'A'$

Hence proved

6. (i) If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then verify that $A'A = I$ **[2 Marks]**

(ii) If $A = \begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}$, then verify that $A'A = I$ **[2 marks]**

Solution:

(i) **Given:**

$$A = \begin{bmatrix} cos\alpha & sin\alpha \\ -sin\alpha & cos\alpha \end{bmatrix}$$

Step 1:

$\Rightarrow A' = \begin{bmatrix} cos\alpha & -sin\alpha \\ sin\alpha & cos\alpha \end{bmatrix}$ **[$\frac{1}{2}$ Mark]**

Step 2:

Therefore $A'A = \begin{bmatrix} cos\alpha & -sin\alpha \\ sin\alpha & cos\alpha \end{bmatrix} \begin{bmatrix} cos\alpha & sin\alpha \\ -sin\alpha & cos\alpha \end{bmatrix}$

$$= -\begin{bmatrix} cos^2\alpha + sin^2\alpha & cos\alpha\, sin\alpha - sin\alpha\, cos\alpha \\ sin\alpha\, cos\alpha - cos\alpha\, sin\alpha & sin^2\alpha + cos^2\alpha \end{bmatrix}$$

Step 3:

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$\therefore\ A'A = I$ **[$\frac{1}{2}$ Mark]**

Hence proved.

(ii) **Given:**

$$A = \begin{bmatrix} sin\alpha & cos\alpha \\ -cos\alpha & sin\alpha \end{bmatrix}$$

Step 1:

$\Rightarrow A' = \begin{bmatrix} sin\alpha & -cos\alpha \\ cos\alpha & sin\alpha \end{bmatrix}$ **[$\frac{1}{2}$ Mark]**

Step 2:

Therefore $A'A = \begin{bmatrix} sin\alpha & -cos\alpha \\ cos\alpha & sin\alpha \end{bmatrix}\begin{bmatrix} sin\alpha & cos\alpha \\ -cos\alpha & sin\alpha \end{bmatrix}$

$$= \begin{bmatrix} sin^2\alpha + cos^2\alpha & sin\alpha\, cos\alpha - cos\alpha\, sin\alpha \\ cos\alpha\, sin\alpha - sin\alpha\, cos\alpha & cos^2\alpha + sin^2\alpha \end{bmatrix}$$

Step 3:

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$\therefore\ A'A = I$ **[$\frac{1}{2}$ Mark]**

Hence proved.

7. i) Show that the matrix $A = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$ is a symmetric matrix. **[2 Marks]**

(ii) Show that the matrix $A = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$ is a skew symmetric matrix. **[2 Marks]**

Solution:

(i) **Given:**

$$A = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$$

Step 1:

$$\Rightarrow A' = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}'$$

$$= \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$$

Step 2:

$\therefore A' = A$

Hence, the matrix $A = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$ is a symmetric matrix.

(ii) $A = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$

Step 1:

$$\Rightarrow A' = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}' = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}$$

$$= -\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} = -A$$

Step 2:

$\Rightarrow A' = -A$

Hence, the matrix A $= \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$ is a skew symmetric matrix.

8. For the matrix $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$, verify that

(i) $(A + A')$ is a symmetric matrix **[3 Marks]**

(ii) $(A - A')$ is a skew symmetric matrix **[3 Marks]**

(i) **Given:**

$$A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$$

Step 1:

$\Rightarrow A' = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}' = \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix}$

Step 2:

Therefore, $(A + A') = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix}$

Step 3:

$(A + A')' = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix}' = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix}$

$\Rightarrow (A + A')' = (A + A')$

Hence the matrix $(A + A')$ is a symmetric matrix.

(ii) **Given:**

$A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$

Step 1:

$\Rightarrow A' = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}' = \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix}$

Step 2:

$(A - A') = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} - \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

Step 3:

$\therefore (A - A')' = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}' = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = -(A - A')$

$\Rightarrow (A - A')' = -(A - A')$

Hence the matrix $(A - A')$ is a skew symmetric matrix.

9. Find $\frac{1}{2}(A + A')$ and $\frac{1}{2}(A - A')$, when $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$ **[3 Marks]**

Solution:

Given that: $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$

Step 1:

$$\Rightarrow A' = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}' = \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}$$

Step 2:

$$\text{So, } \frac{1}{2}(A + A') = \frac{1}{2}\left(\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} + \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}\right)$$

$$= \frac{1}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Step 3:

$$\frac{1}{2}(A - A') = \frac{1}{2}\left(\begin{bmatrix} 0 & a & b \\ -a & 0 & -c \\ -b & -c & 0 \end{bmatrix} - \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}\right)$$

$$= \frac{1}{2}\begin{bmatrix} 0 & 2a & 2b \\ -2a & 0 & 2c \\ -2b & -2c & 0 \end{bmatrix} = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & c & 0 \end{bmatrix}$$

Hence, the required matrix of $\frac{1}{2}(A + A')$ is $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ and $\frac{1}{2}(A - A')$ is $\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & c & 0 \end{bmatrix}$

10. Express the following matrices as the sum of a symmetric and a skew symmetric matrix:

(i) $\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$ **[4 Marks]**

(ii) $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ **[4 Marks]**

(iii) $\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$ **[4 Marks]**

(iv) $\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}$ **[4 Marks]**

Solution:

(i) Given: $A = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$

Step 1:

$$A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$$

Consider, $P = \frac{1}{2}(A + A')$ and $Q = \frac{1}{2}(A - A')$ **[$\frac{1}{2}$ Mark]**

Step 2:

We have $A' = \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}$ **[$\frac{1}{2}$ Mark]**

Step 3:

$$P = \frac{1}{2}(A + A') = \frac{1}{2}\left(\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}\right)$$

$= \frac{1}{2}\begin{bmatrix} 6 & 6 \\ 6 & -2 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix}$ **[$\frac{1}{2}$ Mark]**

Step 4:

$$P' = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix} = P$$

$\Rightarrow P$ is a symmetric matrix. **[$\frac{1}{2}$ Mark]**

Step 5:

$$Q = \frac{1}{2}(A - A') = \frac{1}{2}\left(\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}\right)$$

$= \frac{1}{2}\begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$ **[$\frac{1}{2}$ Mark]**

Step 6:

$$Q' = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}' = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} = -Q$$

$\Rightarrow Q$ is a skew symmetric matrix **[$\frac{1}{2}$ Mark]**

Step 7:

Hence, $A = P + Q = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$

$$A = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$$

Thus, A is a sum of symmetric and skew symmetric matrix.

(ii) Given that: $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

Step 1:

$A' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ **[$\frac{1}{2}$ Mark]**

Step 2:

Therefore, $A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$

Let, $P = \frac{1}{2}(A + A')$ and $Q = \frac{1}{2}(A - A')$ **[$\frac{1}{2}$ Mark]**

Step 3:

$$P = \frac{1}{2}(A + A') = \frac{1}{2}\left(\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}\right)$$

$= \frac{1}{2}\begin{bmatrix} 12 & -4 & 4 \\ -4 & 6 & -2 \\ 4 & -2 & 6 \end{bmatrix} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ **[$\frac{1}{2}$ Mark]**

Step 4:

$P' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} = P$ **[$\frac{1}{2}$ Mark]**

$\Rightarrow P$ is a symmetric matrix

Step 5:

$$Q = \frac{1}{2}(A - A') = \frac{1}{2}\left(\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} - \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}\right)$$

$= \frac{1}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ **[$\frac{1}{2}$ Mark]**

Step 6:

$Q' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = -Q$ **[$\frac{1}{2}$ Mark]**

$\Rightarrow Q$ is a skew symmetric matrix.

Step 7:

Hence, $A = P + Q = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

Thus, A is a sum of symmetric and skew symmetric matrix.

(iii) Given: $A = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$

Step 1:

$A' = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$ **[$\frac{1}{2}$ Mark]**

Step 2:

Therefore, $\mathrm{A} = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$

Let, $P = \frac{1}{2}(A + A')$ and $Q = \frac{1}{2}(A - A')$ **[$\frac{1}{2}$ Mark]**

Step 3:

$$P = \frac{1}{2}(A + A') = \frac{1}{2}\left(\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} + \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}\right)$$

$= -\frac{1}{2}\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix} = \begin{bmatrix} 3 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{bmatrix}$ **[$\frac{1}{2}$ Mark]**

Step 4:

$P' = \begin{bmatrix} 3 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{bmatrix}' = \begin{bmatrix} 3 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{bmatrix} = P$ **[$\frac{1}{2}$ Mark]**

$\Rightarrow P$ is a symmetric matrix.

Step 5:

$Q = \frac{1}{2}(A - A') = \frac{1}{2}\left(\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} - \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}\right)$ **[$\frac{1}{2}$ Mark]**

$$= \frac{1}{2}\begin{bmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix} = \begin{bmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ -\frac{5}{2} & 0 & 3 \\ -\frac{3}{2} & -3 & 0 \end{bmatrix}$$

Step 6:

$$Q' = \begin{bmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ -\frac{5}{2} & 0 & 3 \\ -\frac{3}{2} & -3 & 0 \end{bmatrix}' = \begin{bmatrix} 0 & -\frac{5}{2} & -\frac{3}{2} \\ \frac{5}{3} & 0 & -3 \\ \frac{3}{2} & 3 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -\frac{5}{2} & -\frac{3}{2} \\ \frac{5}{2} & 0 & -3 \\ \frac{3}{2} & 3 & 0 \end{bmatrix} = -Q$$

$\Rightarrow Q$ is a skew symmetric matrix. **[$\frac{1}{2}$ Mark]**

Step 7:

Hence, $A = P + Q = \begin{bmatrix} 3 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ -\frac{5}{2} & 0 & 3 \\ -\frac{3}{2} & -3 & 0 \end{bmatrix}$

$$A = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$$

Thus, A is a sum of symmetric and skew symmetric matrix.

(iv) Given that: $A = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}$

Step 1:

$A' = \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}$ **[$\frac{1}{2}$ Mark]**

Step 2:

Therefore, $A = -\frac{1}{2}(A + A') + \frac{1}{2}(A - A')$

Let, $P = \frac{1}{2}(A + A')$ and $Q = \frac{1}{2}(A - A')$ **[$\frac{1}{2}$ Mark]**

Step 3:

$P = \frac{1}{2}(A + A')$

$= \frac{1}{2}\left(\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}\right)$

$= \frac{1}{2}\begin{bmatrix} 2 & 4 \\ 4 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$ **[$\frac{1}{2}$ Mark]**

Step 4:

$P' = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix} = P$

$\Rightarrow P$ is a symmetric matrix **[$\frac{1}{2}$ Mark]**

Step 5:

$Q = \frac{1}{2}(A - A')$

$= \frac{1}{2}\left(\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}\right)$

$= \frac{1}{2}\begin{bmatrix} 0 & 6 \\ -6 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$ **[$\frac{1}{2}$ Mark]**

Step 6:

$Q' = \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}' = \begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix} = -Q$

$\Rightarrow Q$ is a skew symmetric matrix **[$\frac{1}{2}$ Mark]**

Step 7:

Hence, $A = P + Q = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}$

Thus, A is a sum of symmetric and skew symmetric matrix.

11. If A, B are symmetric matrices of same order, then $AB - BA$ is a **[2 Marks]**

(A) Skew symmetric matrix

(B) Symmetric matrix

(C) Zero matrix

(D) Identity matrix

Solution:

Given:

A and B are symmetric matrices of same order

Step 1:

Let, $(AB - BA)' = (AB)' - (BA)'$ $[\because (X - Y)' = X' - Y']$ **[$\frac{1}{2}$ Mark]**

Step 2:

$= B'A' - A'B'$ $[\because (XY)' = Y'X']$ **[$\frac{1}{2}$ Mark]**

Step 3:

$= BA - AB$ [$\because$Given: $A' = A, B' = B$] **[$\frac{1}{2}$ Mark]**

Step 4:

$= -(AB - BA)$

$\Rightarrow (AB - BA)' = -(AB - BA)$

Therefore, the matrix *(AB — BA)* is a skew symmetric matrix

Hence, the option (A) is correct. **[$\frac{1}{2}$ Mark]**

12. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, and $A + A' = I$, then the value of α is **[2 marks]**

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{3}$

(C) π

(D) $\frac{3\pi}{2}$

Solution:

(B)

Given that: $A + A' = I$

$$A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

Step 1:

$$A' = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$ **[$\frac{1}{2}$ Mark]**

Step 2:

$$\Rightarrow \begin{bmatrix} cos\alpha & -sin\alpha \\ sin\alpha & cos\alpha \end{bmatrix} + \begin{bmatrix} cos\alpha & sin\alpha \\ -sin\alpha & cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2cos\alpha & 0 \\ 0 & 2cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$ **[$\frac{1}{2}$ Mark]**

Step 3:

$$\Rightarrow 2\cos\alpha = 1$$

$$\Rightarrow \cos\alpha = \frac{1}{2}$$

$\cos\alpha = \cos 60^0$ $[\because \cos 60^0 = \frac{1}{2}]$ **[$\frac{1}{2}$ Mark]**

Step 4:

On comparing angles, we get

$$\alpha = 60^\circ$$
$$\alpha = 60^\circ \times \frac{\pi}{180^\circ}$$

$$\Rightarrow \alpha = \frac{\pi}{3}$$

Hence, option (B) is correct. **[$\frac{1}{2}$ Mark]**

EXERCISE 3.4

Using elementary transformations, find the inverse of each of the matrices, if it exists in Exercises 1 to 17.

1. $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ **[3 Marks]**

Solution:

Let $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$,

Step 1:

We know that, $A = IA$, where $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

$\Rightarrow \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$ **[$\frac{1}{2}$ Mark]**

Step 2:

$\Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A$ [applying $R_2 \longrightarrow R_2 - 2R_1$] **[$\frac{1}{2}$ Mark]**

Step 3:

$\Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix} A$ $\left[\text{applying } R_2 \longrightarrow \frac{1}{5} R_2\right]$ **[$\frac{1}{2}$ Mark]**

Step 4:

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix} A$ [applying $R_1 \rightarrow R_1 + R_2$] **[$\frac{1}{2}$ Mark]**

Step 5:

$\mathrm{I} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix} A$ **[$\frac{1}{2}$ Mark]**

Step 6:

$\therefore \mathrm{I} = \mathrm{A}^{-1}\mathrm{A}$

Hence, $A^{-1} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix}$ **[$\frac{1}{2}$ Mark]**

2. $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$ **[2 Marks]**

Solution:

Let, $A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$,

Step 1:

We know that, $A = IA$, where $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$ **[$\frac{1}{2}$ Mark]**

Step 2:

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A$ [applying $R_1 \rightarrow R_1 - R_2$] **[$\frac{1}{2}$ Mark]**

Step 3:

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} A$ [applying $R_2 \rightarrow R_2 - R_1$] **[$\frac{1}{2}$ Mark]**

Step 4:

$\therefore \mathrm{I} = \mathrm{A}^{-1}\mathrm{A}$

Hence, $A^{-1} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$ **[$\frac{1}{2}$ Mark]**

3. $\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$

Solution:

Let, $A = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$,

We know that, $A = IA$, where $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

$\Rightarrow \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A$ [applying $R_2 \rightarrow R_2 - 2R_1$]

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix} A$ [applying $R_1 \rightarrow R_1 - 3R_2$]

$\therefore \mathrm{I} = \mathrm{A}^{-1}\mathrm{A}$

Hence, $A^{-1} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}$

4. $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$ **[3 Marks]**

Solution:

Let, $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$,

Step 1:

We know that, $A = IA$, where $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$ **[$\frac{1}{2}$ Mark]**

Step 2:

$\Rightarrow \begin{bmatrix} 1 & 2 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 0 & 1 \end{bmatrix} A$ [applying $R_1 \to 3R_1 - R_2$] **[$\frac{1}{2}$ Mark]**

Step 3:

$\Rightarrow \begin{bmatrix} 1 & 2 \\ 0 & -3 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ -15 & 6 \end{bmatrix} A$ [applying $R_2 \to R_2 - 5R_1$] **[$\frac{1}{2}$ Mark]**

Step 4:

$\Rightarrow \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 5 & -2 \end{bmatrix} A$ $\left[\text{applying } R_2 \to -\frac{1}{3} R_2\right]$ **[$\frac{1}{2}$ Mark]**

Step 5:

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix} A$ [applying $R_1 \to R_1 - 2R_2$] **[$\frac{1}{2}$ Mark]**

Step 6:

$\therefore \mathrm{I} = \mathrm{A}^{-1}\mathrm{A}$

Hence, $A^{-1} = \begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix}$ **[$\frac{1}{2}$ Mark]**

5. $\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$ **[3 Marks]**

Solution:

Let, $A = \begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$,

Step 1:

We know that, $A = IA$, where $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$ **[$\frac{1}{2}$ Mark]**

Step 2:

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ 0 & 1 \end{bmatrix} A$ [applying $R_1 \rightarrow 4R_1 - R_2$] **[$\frac{1}{2}$ Mark]**

Step 3:

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ -28 & 8 \end{bmatrix} A$ [applying $R_2 \rightarrow R_2 - 7R_1$] **[$\frac{1}{2}$ Mark]**

Step 4:

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix} A$ $\left[\text{applying } R_2 \rightarrow \frac{1}{4}R_2\right]$ **[$\frac{1}{2}$ Mark]**

Step 5:

$\therefore \mathrm{I} = \mathrm{A}^{-1}\mathrm{A}$

Hence, $A^{-1} = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}$1`

6. $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$ **[3 marks]**

Solution:

Let, $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$,

Step 1:

We know that, $A = IA$, where $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$ **[$\frac{1}{2}$ Mark]**

Step 2:

$\Rightarrow \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A$ [applying $R_1 \rightarrow R_1 - R_2$] **[$\frac{1}{2}$ Mark]**

Step 3:

$\Rightarrow \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} A$ [applying $R_2 \rightarrow R_2 - R_1$] **[$\frac{1}{2}$ Mark]**

Step 4:

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} A$ [applying $R_1 \to R_1 - 2R_2$] **[$\frac{1}{2}$ Mark]**

Step 5:

$\therefore \mathrm{I} = \mathrm{A}^{-1}\mathrm{A}$

Hence, $A^{-1} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$

7. $\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$ **[3 Marks]**

Solution:

Let, $A = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$,

Step 1:

We know that, $A = IA$, where $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

$\Rightarrow \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$ **[$\frac{1}{2}$ Mark]**

Step 2:

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix} A$ [Applying $R_1 \to 2R_1 - R_2$] **[$\frac{1}{2}$ Mark]**

Step 3:

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -10 & 6 \end{bmatrix} A$ [Applying $R_2 \to 2R_2 - 5R_1$] **[$\frac{1}{2}$ Mark]**

Step 4:

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix} A$ $\left[\text{Applying } R_2 \to \frac{1}{2} R_2\right]$ **[$\frac{1}{2}$ Mark]**

Step 5:

$\therefore \mathrm{I} = \mathrm{A}^{-1}\mathrm{A}$

Hence, $A^{-1} = \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$

8. $\begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$ **[3 Marks]**

Solution:

Let, $A = \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$,

Step 1:

We know that, $A = IA$, where $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

$\Rightarrow \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$ **[$\frac{1}{2}$ Mark]**

Step 2:

$\Rightarrow \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A$ [applying $R_1 \to R_1 - R_2$] **[$\frac{1}{2}$ Mark]**

Step 3:

$\Rightarrow \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -3 & 4 \end{bmatrix} A$ [applying $R_2 \to R_2 - 3R_1$] **[$\frac{1}{2}$ Mark]**

Step 4:

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix} A$ [applying $\mathrm{R}_1 \to R_1 - \mathrm{R}_2$] **[$\frac{1}{2}$ Mark]**

Step 5:

$\therefore \mathrm{I} = \mathrm{A}^{-1}\mathrm{A}$

Hence, $A^{-1} = \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix}$

9. $\begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$ **[3 Marks]**

Solution:

Let, $A = \begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$,

Step 1:

We know that, $A = IA$, where $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

$\Rightarrow \begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$ **[$\frac{1}{2}$ Mark]**

Step 2:

$\Rightarrow \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A$ [applying $R_1 \to R_1 - R_2$] **[$\frac{1}{2}$ Mark]**

Step 3:

$\Rightarrow \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix} A$ [applying $R_2 \to R_2 - 2R_1$] **[$\frac{1}{2}$ Mark]**

Step 4:

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix} A$ [applying $R_1 \to R_1 - 3R_2$] **[$\frac{1}{2}$ Mark]**

Step 5:

$\therefore \mathrm{I} = \mathrm{A}^{-1}\mathrm{A}$

Hence, $A^{-1} = \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix}$

10. $\begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}$ **[3 Marks]**

Solution:

Let, $A = \begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}$,

Step 1:

We know that, $A = IA$, where $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

$\Rightarrow \begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$ **[$\frac{1}{2}$ Mark]**

Step 2:

$\Rightarrow \begin{bmatrix} 1 & 1 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix} A$ [applying $R_1 \to 3R_1 + 2R_2$] **[$\frac{1}{2}$ Mark]**

Step 3:

$\Rightarrow \begin{bmatrix} 1 & 1 \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 12 & 9 \end{bmatrix} A$ [applying $R_2 \to R_2 + 4R_1$] **[$\frac{1}{2}$ Mark]**

Step 4:

$\Rightarrow \begin{bmatrix}1 & 1\\0 & 1\end{bmatrix} = \begin{bmatrix}3 & 2\\2 & \frac{3}{2}\end{bmatrix} A$ $\left[\text{applying } R_2 \to \frac{1}{6}R_2\right]$ **[$\frac{1}{2}$ Mark]**

Step 5:

$\Rightarrow \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = \begin{bmatrix}1 & \frac{1}{2}\\2 & \frac{3}{2}\end{bmatrix} A$ [applying $R_1 \to R_1 - R_2$] **[$\frac{1}{2}$ Mark]**

Step 6:

$\therefore \mathrm{I} = \mathrm{A}^{-1}\mathrm{A}$

Hence, $A^{-1} = \begin{bmatrix}1 & \frac{1}{2}\\2 & \frac{3}{2}\end{bmatrix}$

11. $\begin{bmatrix}2 & -6\\1 & -2\end{bmatrix}$ **[3 Marks]**

Solution:

Let, $A = \begin{bmatrix}2 & -6\\1 & -2\end{bmatrix}$,

Step 1:

We know that, $A = IA$, where $I = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$.

$\Rightarrow \begin{bmatrix}2 & -6\\1 & -2\end{bmatrix} = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} A$ **[$\frac{1}{2}$ Mark]**

Step 2:

$\Rightarrow \begin{bmatrix}1 & -4\\1 & -2\end{bmatrix} = \begin{bmatrix}1 & -1\\0 & 1\end{bmatrix} A$ $\left[\text{applying } \mathrm{R}_1 \to R_1\text{-}\mathrm{R}_2\right]$ **[$\frac{1}{2}$ Mark]**

Step 3:

$\Rightarrow \begin{bmatrix}1 & -4\\0 & 2\end{bmatrix} = \begin{bmatrix}1 & -1\\-1 & 2\end{bmatrix} A$ $\left[\text{applying } \mathrm{R}_2 \to R_2\text{-}\mathrm{R}_1\right]$ **[$\frac{1}{2}$ Mark]**

Step 4:

$\Rightarrow \begin{bmatrix}1 & -4\\0 & 1\end{bmatrix} = \begin{bmatrix}1 & -1\\-\frac{1}{2} & 1\end{bmatrix} A$ $\left[\text{applying } R_2 \to \frac{1}{2}R_2\right]$ **[$\frac{1}{2}$ Mark]**

Step 5:

$\Rightarrow \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = \begin{bmatrix}-1 & 3\\-\frac{1}{2} & 1\end{bmatrix}A$ [applying$R_1 \to R_1$-4R_2] **[$\frac{1}{2}$ Mark]**

Step 6:

$\therefore \mathrm{I} = \mathrm{A}^{-1}\mathrm{A}$

Hence, $A^{-1} = \begin{bmatrix}-1 & 3\\-\frac{1}{2} & 1\end{bmatrix}$

12. $\begin{bmatrix}6 & -3\\-2 & 1\end{bmatrix}$ **[2 Marks]**

Solution:

Let, $A = \begin{bmatrix}6 & -3\\-2 & 1\end{bmatrix}$,

Step 1:

We know that, $A = IA$, where $I = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$.

$\Rightarrow \begin{bmatrix}6 & -3\\-2 & 1\end{bmatrix} = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}A$ **[$\frac{1}{2}$ Mark]**

Step 2:

$\Rightarrow \begin{bmatrix}6 & -3\\0 & 0\end{bmatrix} = \begin{bmatrix}1 & 0\\3 & 1\end{bmatrix}A$ [applying $R_2 \to R_2 + 3R_1$] **[$\frac{1}{2}$ Mark]**

Step 3:

Since, on left hand side of the matrix all the elements of second row is zero.

Hence, A^{-1} does not exist.

13. $\begin{bmatrix}2 & -3\\-1 & 2\end{bmatrix}$ **[3 marks]**

Solution:

Let, $A = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$,

Step 1:

We know that, $A = IA$ where $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

$\Rightarrow \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$ **[$\frac{1}{2}$ Mark]**

Step 2:

$\Rightarrow \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} A$ [applying $R_1 \rightarrow R_1 + R_2$] **[$\frac{1}{2}$ Mark]**

Step 3:

$\Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} A$ [applying $R_2 \rightarrow R_2 + R_1$] **[$\frac{1}{2}$ Mark]**

Step 4:

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} A$ [applying $R_1 \rightarrow R_1 + R_2$] **[$\frac{1}{2}$ Mark]**

Step 5:

$\therefore \mathrm{I} = \mathrm{A}^{-1}\mathrm{A}$

Hence, $A^{-1} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

14. $\begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$ **[2 marks]**

Solution:

Let $A = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$,

Step 1:

We know that, $A = IA$, where $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

$\Rightarrow \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$ **[$\frac{1}{2}$ Mark]**

Step 2:

$\Rightarrow \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A$ [applying $R_2 \rightarrow R_2 - 2R_1$] **[$\frac{1}{2}$ Mark]**

Step 3:

Since, on left hand side of the matrix all the elements of second row is zero.

Hence A^{-1} does not exist.

15. $\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$ **[5 Marks]**

Solution:

Let, $A = \begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$,

Step 1:

We know that, $A = IA$, where $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

$\Rightarrow \begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$ **[$\frac{1}{2}$ Mark]**

Step 2:

$\Rightarrow \begin{bmatrix} 1 & 1 & 4 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$ [applying $R_1 \rightarrow R_1 + R_2 - R_3$] **[$\frac{1}{2}$ Mark]**

Step 3:

$\Rightarrow \begin{bmatrix} 1 & 1 & 4 \\ 0 & 0 & -5 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 \\ -2 & -1 & 2 \\ 0 & 0 & 1 \end{bmatrix} A$ [applying $R_2 \rightarrow R_2 - 2R_1$] **[$\frac{1}{2}$ Mark]**

Step 4:

$\Rightarrow \begin{bmatrix} 1 & 1 & 4 \\ 0 & 0 & -5 \\ 0 & -5 & -10 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 \\ -2 & -1 & 2 \\ -3 & -3 & 4 \end{bmatrix} A$ [applying $R_3 \rightarrow R_3 - 3R_1$] **[$\frac{1}{2}$ Mark]**

Step 5:

$\Rightarrow \begin{bmatrix} 1 & 1 & 4 \\ 0 & -5 & -10 \\ 0 & 0 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 \\ -3 & -3 & 4 \\ -2 & -1 & 2 \end{bmatrix} A$ [applying $R_2 \leftrightarrow R_3$] **[$\frac{1}{2}$ Mark]**

Step 6:

$$\Rightarrow \begin{bmatrix} 1 & 1 & 4 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 \\ \frac{3}{5} & \frac{3}{5} & -\frac{4}{5} \\ \frac{2}{5} & \frac{1}{5} & -\frac{2}{5} \end{bmatrix} A \quad \left[\text{applying } R_2 \to -\frac{1}{5}R_2, R_3 \to -\frac{1}{5}R_3\right] \quad \left[\frac{1}{2}\textbf{ Mark}\right]$$

Step 7:

$$\Rightarrow \begin{bmatrix} 1 & 1 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & -\frac{2}{5} \end{bmatrix} A \quad [\text{applying } R_2 \to R_2 - 2R_3] \quad \left[\frac{1}{2}\textbf{ Mark}\right]$$

Step 8:

$$\Rightarrow \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{6}{5} & \frac{4}{5} & -1 \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & -\frac{2}{5} \end{bmatrix} A \quad [\text{applying } R_1 \to R_1 - R_2] \quad \left[\frac{1}{2}\textbf{ Mark}\right]$$

Step 9:

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{2}{5} & 0 & \frac{3}{5} \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & -\frac{2}{5} \end{bmatrix} A \quad [\text{applying } \mathrm{R}_1 \to R_1 - 4\mathrm{R}_3] \quad \left[\frac{1}{2}\textbf{ Mark}\right]$$

Step 10:

$\therefore \mathrm{I} = \mathrm{A}^{-1}\mathrm{A}$

Hence, $A^{-1} = \begin{bmatrix} -\frac{2}{5} & 0 & \frac{3}{5} \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & -\frac{2}{5} \end{bmatrix}$ **[$\frac{1}{2}$ Mark]**

16. $\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$ **[5 Marks]**

Solution:

Let, $A = \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$,

Step 1:

We know that, $A = IA$, where $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$ **[$\frac{1}{2}$ Mark]**

Step 2:

$$\Rightarrow \begin{bmatrix} 1 & 3 & -2 \\ 0 & 9 & -11 \\ 2 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \qquad [\text{applying } R_2 \to R_2 + 3R_1]$$ **[$\frac{1}{2}$ Mark]**

Step 3:

$$\Rightarrow \begin{bmatrix} 1 & 3 & -2 \\ 0 & 9 & -11 \\ 0 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} A \qquad \left[\text{applying } R_3 \to R_3\text{-}2R_1\right]$$ **[$\frac{1}{2}$ Mark]**

Step 4:

$$\Rightarrow \begin{bmatrix} 1 & 3 & -2 \\ 0 & 9 & -11 \\ 0 & 0 & 25 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -15 & 1 & 9 \end{bmatrix} A \qquad [\text{applying } R_3 \to 9R_3 + R_2]$$ **[$\frac{1}{2}$ Mark]**

Step 5:

$$\Rightarrow \begin{bmatrix} 1 & 3 & -2 \\ 0 & 9 & -11 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix} A \qquad \left[\text{applying } R_3 \to \frac{1}{25} R_3\right]$$ **[$\frac{1}{2}$ Mark]**

Step 6:

$$\Rightarrow \begin{bmatrix} 1 & 3 & -2 \\ 0 & 9 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{18}{5} & \frac{36}{25} & \frac{99}{25} \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix} A \; [\text{applying } R_2 \to R_2 + 11R_3]$$ **[$\frac{1}{2}$ Mark]**

Step 7:

$$\Rightarrow \begin{bmatrix} 1 & 3 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{2}{5} & \frac{4}{25} & \frac{11}{25} \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix} A \left[\text{applying } R_2 \to \frac{1}{9} R_2\right]$$ **[$\frac{1}{2}$ Mark]**

Step 8:

$$\Rightarrow \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{5} & \frac{2}{25} & \frac{18}{25} \\ -\frac{2}{5} & \frac{4}{25} & \frac{11}{25} \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix} A \; [\text{applying } R_1 \to R_1 + 2R_3]$$ **[$\frac{1}{2}$ Mark]**

Step 9:

$$\Rightarrow \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} = \begin{bmatrix}1 & -\frac{2}{5} & -\frac{3}{5}\\-\frac{2}{5} & \frac{4}{25} & \frac{11}{25}\\-\frac{3}{5} & \frac{1}{25} & \frac{9}{25}\end{bmatrix} A$$ [applying $R_1 \to R_1\text{-}3R_2$] **[$\frac{1}{2}$ Mark]**

Step 10:

$\therefore I = A^{-1}A$

Hence, $A^{-1} = \begin{bmatrix}1 & -\frac{2}{5} & -\frac{3}{5}\\-\frac{2}{5} & \frac{4}{25} & \frac{11}{25}\\-\frac{3}{5} & \frac{1}{25} & \frac{9}{25}\end{bmatrix}$ **[$\frac{1}{2}$ Mark]**

17. $\begin{bmatrix}2 & 0 & -1\\5 & 1 & 0\\0 & 1 & 3\end{bmatrix}$ **[5 Marks]**

Solution:

Let $A = \begin{bmatrix}2 & 0 & -1\\5 & 1 & 0\\0 & 1 & 3\end{bmatrix}$,

Step 1:

We know that, $A = IA$, where $I = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$.

$$\Rightarrow \begin{bmatrix}2 & 0 & -1\\5 & 1 & 0\\0 & 1 & 3\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} A$$ **[$\frac{1}{2}$ Mark]**

Step 2:

$$\Rightarrow \begin{bmatrix}1 & -1 & -3\\5 & 1 & 0\\0 & 1 & 3\end{bmatrix} = \begin{bmatrix}3 & -1 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} A$$ [applying $R_1 \to 3R_1\text{-}R_2$] **[$\frac{1}{2}$ Mark]**

Step 3:

$$\Rightarrow \begin{bmatrix}1 & -1 & -3\\0 & 6 & 15\\0 & 1 & 3\end{bmatrix} = \begin{bmatrix}3 & -1 & 0\\-15 & 6 & 0\\0 & 0 & 1\end{bmatrix} A$$ [applying $R_2 \to R_2\text{-}5R_1$] **[$\frac{1}{2}$ Mark]**

Step 4:

$$\Rightarrow \begin{bmatrix} 1 & -1 & -3 \\ 0 & 6 & 15 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 0 \\ -15 & 6 & 0 \\ 15 & -6 & 6 \end{bmatrix} A$$ [applying$R_3 \to 6R_3$-R_2] **[$\frac{1}{2}$ Mark]**

Step 5:

$$\Rightarrow \begin{bmatrix} 1 & -1 & -3 \\ 0 & 6 & 15 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 0 \\ -15 & 6 & 0 \\ 5 & -2 & 2 \end{bmatrix} A$$ [applying $R_3 \to \frac{1}{3}R_3$] **[$\frac{1}{2}$ Mark]**

Step 6:

$$\Rightarrow \begin{bmatrix} 1 & -1 & -3 \\ 0 & 6 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 0 \\ -90 & 36 & -30 \\ 5 & -2 & 2 \end{bmatrix} A$$ [applying $R_2 \to R_2$-15R_3] **[$\frac{1}{2}$ Mark]**

Step 7:

$$\Rightarrow \begin{bmatrix} 1 & -1 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 0 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A$$ [applying $R_2 \to \frac{1}{6}R_2$] **[$\frac{1}{2}$ Mark]**

Step 8:

$$\Rightarrow \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 18 & -7 & 6 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A$$ [applying $R_1 \to R_1$+3R_3] **[$\frac{1}{2}$ Mark]**

Step 9:

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A$$ [applying $R_1 \to R_1$+R_2] **[$\frac{1}{2}$ Mark]**

Step 10:

$\therefore \mathrm{I} = \mathrm{A}^{-1}\mathrm{A}$

Hence, $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ **[$\frac{1}{2}$ Mark]**

18. Matrices A and B will be inverse of each other only if **[2 Marks]**

(A) $AB = BA$

(B) $AB = BA = 0$

(C) $AB = 0, BA = I$

(D) $AB = BA = I$

Solution:

Given:

A and B will be inverse of each other.

$A^{-1} = B$ and $B^{-1} = A$

Step 1:

We know that,

$AA^{-1} = I$ [$\because$ $A^{-1} = B$]

$AB = I$

Step 2:

$BB^{-1} = I$ [$\because$ $B^{-1} = A$]

$BA = I$

So, $AB = BA = I$.

Hence, option (D) is correct.

Miscellaneous Exercise on Chapter 3

1. Let $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, show that $(aI + bA)^n = a^n I + na^{n-1} bA$, where I is the identity matrix of order 2 and $n \in N$. **[5 marks]**

Solution:

Given: $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

To Prove:

$(aI + bA)^n = a^n I + na^{n-1} bA$

Proof: The proof is by mathematical induction **[$\frac{1}{2}$ Mark]**

Step 1:

Consider, $P(n)$: $(aI + bA)^n = a^n I + na^{n-1} bA$ **[$\frac{1}{2}$ Mark]**

Step 2:

$\therefore$ P(1): $(aI + bA)^1 = aI + bA$

Hence, the result is true for $n = 1$. **[$\frac{1}{2}$ Mark]**

Step 3:

Let the result be true for $n = k$,

So, $P(k): (aI + bA)^k = a^k I + na^{k-1} bA$ **[$\frac{1}{2}$ Mark]**

Step 4:

We must prove that it is true for $n = k + 1$ also.

(i.e.) $P(k + 1): (aI + bA)^{k+1} = a^{k+1} I + (k + 1)a^k bA$ **[$\frac{1}{2}$ Mark]**

Step 5:

By taking LHS, we get

LHS= $(aI + bA)^{k+1}$

$= (aI + bA)^k (aI + bA)$ **[$\frac{1}{2}$ Mark]**

Step 6:

$= \left(a^k I + na^{k-1} bA\right)(aI + bA)$ $\qquad [\because (aI + bA)^{k+1} = a^k I + na^{k-1} bA]$

[$\frac{1}{2}$ Mark]

Step 7:

$= a^{k+1} I^2 + a^k IbA + na^k IbA + na^{k-1} b^2 A^2$ **[$\frac{1}{2}$ Mark]**

Step 8:

$= a^{k+1} I + (k + 1)a^k bA$ $\qquad \left[\because A^2 = A \cdot A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0\right]$

$LHS = RHS$

Therefore, the result is true for $n = k + 1$.

Hence, by the Principle of Mathematical Induction, $(aI + bA)^n = a^n I + na^{n-1} bA$ is true for all-natural numbers n.

2. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, prove that $A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}, n \in N.$ **[5 Marks]**

Solution:

Given:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

To Prove:

$$A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}, n \in N$$

Proof: The proof is by mathematical induction. **[$\frac{1}{2}$ Mark]**

Step 1:

Consider, $p(n): A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$, **[$\frac{1}{2}$ Mark]**

Step 2:

Therefore, $P(1): A^1 = \begin{bmatrix} 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = A$

Hence, the result is true for $n = 1$. **[$\frac{1}{2}$ Mark]**

Step 3:

Let the result is true for $n = k$,

So, $P(k): A^k = \begin{bmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{bmatrix}$ **[$\frac{1}{2}$ Mark]**

Step 4:

Now,

We have to prove that it is true for $n = k + 1$ also.

(i.e.) $P(k+1): A^{k+1} = \begin{bmatrix} 3^k & 3^k & 3^k \\ 3^k & 3^k & 3^k \\ 3^k & 3^k & 3^k \end{bmatrix}$ **[$\frac{1}{2}$ Mark]**

Step 5:

By taking LHS, we get

$= A^{k+1} = A^k A$

$$= \begin{bmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$ **[$\frac{1}{2}$ Mark]**

Step 6:

$$= \begin{bmatrix} 3^{k-1}+3^{k-1}+3^{k-1} & 3^{k-1}+3^{k-1}+3^{k-1} & 3^{k-1}+3^{k-1}+3^{k-1} \\ 3^{k-1}+3^{k-1}+3^{k-1} & 3^{k-1}+3^{k-1}+3^{k-1} & 3^{k-1}+3^{k-1}+3^{k-1} \\ 3^{k-1}+3^{k-1}+3^{k-1} & 3^{k-1}+3^{k-1}+3^{k-1} & 3^{k-1}+3^{k-1}+3^{k-1} \end{bmatrix}$$ **[$\frac{1}{2}$ Mark]**

Step 7:

$$= \begin{bmatrix} 3.3^{k-1} & 3.3^{k-1} & 3.3^{k-1} \\ 3.3^{k-1} & 3.3^{k-1} & 3.3^{k-1} \\ 3.3^{k-1} & 3.3^{k-1} & 3.3^{k-1} \end{bmatrix}$$ **[$\frac{1}{2}$ Mark]**

Step 8:

$$= \begin{bmatrix} 3^k & 3^k & 3^k \\ 3^k & 3^k & 3^k \\ 3^k & 3^k & 3^k \end{bmatrix}$$

$LHS = RHS$

Therefore, the result is true for $n = k + 1$.

Hence, by the Principle of Mathematical Induction, $A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$ is true for all natural numbers n.

3. If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then prove that $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$, where n is any positive integer. **[5 Marks]**

Solution:

Given:

$A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

To Prove:

$A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$,

Proof: The proof is by mathematical induction. **[$\frac{1}{2}$ Mark]**

Step 1:

Consider, $P(n): A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$ **[$\frac{1}{2}$ Mark]**

Step 2:

Therefore, $P(1): A^1 = \begin{bmatrix} 1+2(1) & -4(1) \\ 1 & 1-2(1) \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} = A$

Hence, the result is true for $n = 1$. **[$\frac{1}{2}$ Mark]**

Step 3:

Let the result is true for $n = k$, therefore,

$P(k): A^k = \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix}$ **[$\frac{1}{2}$ Mark]**

Step 4:

We have to prove that it is true for $n = k + 1$ also.

(i.e.) $P(k+1): A^{k+1} = \begin{bmatrix} 1+2(k+1) & -4(k+1) \\ k+1 & 1-2(k+1) \end{bmatrix}$ **[$\frac{1}{2}$ Mark]**

Step 5:

By taking LHS, we get

LHS= $A^{k+1} = A^k A$

$= \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix} \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ **[$\frac{1}{2}$ Mark]**

Step 6:

$= \begin{bmatrix} 3+6k-4k & -4-8k+4k \\ 3k+1-2k & -4k-1+2k \end{bmatrix}$ **[$\frac{1}{2}$ Mark]**

Step 7:

$= \begin{bmatrix} 1+(2k+2) & -4k-4 \\ k+1 & 1-(2k+2) \end{bmatrix}$ **[$\frac{1}{2}$ Mark]**

Step 8:

$= \begin{bmatrix} 1+2(k+1) & -4(k+1) \\ k+1 & 1-2(k+1) \end{bmatrix}$ =RHS

$LHS = RHS$

Therefore, the result is true for $n = k + 1$.

Hence, by the Principle of Mathematical Induction, $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$ is true for any positive integer numbers n.

4. If A and B are symmetric matrices, prove that $AB - BA$ is a skew symmetric matrix. **[2 Marks]**

Solution:

Given: A and B are symmetric matrices

$\therefore A' = A$ and $B' = B$

Step 1:

Now,

$(AB - BA)' = (AB)' - (BA)'$ $[\because (X - Y)' = X' - Y']$ **[$\frac{1}{2}$ Mark]**

Step 2:

$= B'A' - A'B'$ $[\because (AB)' = B'A']$ **[$\frac{1}{2}$ Mark]**

Step 3:

$= BA - AB$ [Given $A' = A, B' = B$] **[$\frac{1}{2}$ Mark]**

Step 4:

$= -(AB - BA)$

$\Rightarrow (AB - BA)' = -(AB - BA)$, **[$\frac{1}{2}$ Mark]**

Therefore, AB- BA is a skew symmetric matrix.

Hence proved.

5. Show that the matrix $B'AB$ is symmetric or skew symmetric according as A is symmetric or skew symmetric. **[4 Marks]**

Solution:

Step 1:

If A is a symmetric matrix, then $A' = A$

Now, $(B'AB)' = (AB)'(B')'$ $[\because (AB)' = B'A']$ **[$\frac{1}{2}$ Mark]**

Step 2:

$= (AB)'B$ $[\because (B')' = B]$ **[$\frac{1}{2}$ Mark]**

Step 3:

$= B'A'B$ $[\because (AB)' = B'A']$ **[$\frac{1}{2}$ Mark]**

Step 4:

$= B'AB$ $[\text{Given } A' = A]$

$\Rightarrow (B'AB)' = B'AB,$

Therefore, the matrix $B'AB$ is also symmetric matrix. **[$\frac{1}{2}$ Mark]**

Step 5:

If A is skew symmetric matrix, then $A' = -A$

Here, $(B'AB)' = (AB)'(B')'$ $[\because (AB)' = B'A']$ **[$\frac{1}{2}$ Mark]**

Step 6:

$= (AB)'B$ $[\because (B')' = B]$ **[$\frac{1}{2}$ Mark]**

Step 7:

$= B'A'B$ $[\because (AB)' = B'A']$ **[$\frac{1}{2}$ Mark]**

Step 8:

$= -B'AB$ $[\because \text{Given } A' = -A]$

$\Rightarrow (B'AB)' = -B'AB,$

Hence, the matrix $B'AB$ is also a skew-symmetric matrix. **[$\frac{1}{2}$ Mark]**

6. Find the values of x, y, z if the matrix $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfy the equation $A'A = I$.

[4 Marks]

Solution:

Given:

$$A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$$

$$A'A = I$$

Step 1:

$$\Rightarrow \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ **[$\frac{1}{2}$ Mark]**

Step 2:

$$\Rightarrow \begin{bmatrix} 0 + 4y^2 + z^2 & 0 + 2y^2 - z^2 & 0 - 2y^2 + z^2 \\ 0 + 2y^2 - z^2 & x^2 + y^2 + z^2 & x^2 - y^2 - z^2 \\ 0 - 2y^2 + z^2 & x^2 - y^2 - z^2 & x^2 + y^2 + z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Step 3:

If two matrices are equal, then their corresponding elements are also equal.

So, on comparing the corresponding elements, we get

$$4y^2 + z^2 = 1$$

$$2y^2 - z^2 = 0$$

$$x^2 + y^2 + z^2 = 1$$

Step 4:

On solving we get $x = \pm\frac{1}{\sqrt{2}}, y = \pm\frac{1}{\sqrt{6}}$ and $z = \pm\frac{1}{\sqrt{3}}$ **[$1\frac{1}{2}$ Marks]**

Hence, the values of $x = \pm\frac{1}{\sqrt{2}}, y = \pm\frac{1}{\sqrt{6}}$ and $z = \pm\frac{1}{\sqrt{3}}$

7. For what values of x: $\begin{bmatrix}1 & 2 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 0\\2 & 0 & 1\\1 & 0 & 2\end{bmatrix}\begin{bmatrix}0\\2\\x\end{bmatrix} = O$? **[3 Marks]**

Solution:

Given that: $\begin{bmatrix}1 & 2 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 0\\2 & 0 & 1\\1 & 0 & 2\end{bmatrix}\begin{bmatrix}0\\2\\x\end{bmatrix} = O$

Step 1:

$\Rightarrow \begin{bmatrix}1+4+1 & 2+0+0 & 0+2+2\end{bmatrix}\begin{bmatrix}0\\2\\x\end{bmatrix} = O$

Step 2:

$\Rightarrow \begin{bmatrix}6 & 2 & 4\end{bmatrix}\begin{bmatrix}0\\2\\x\end{bmatrix} = O$ **[$\frac{1}{2}$ Mark]**

Step 3:

$[6(0) + 2(2) + 4(x)] = 0$

$\Rightarrow [0 + 4 + 4x] = [0]$ **[$\frac{1}{2}$ Mark]**

Step 4:

$\Rightarrow 4 + 4x = 0$ **[$\frac{1}{2}$ Mark]**

Step 5:

Upon solving the above obtained equation, we get:

$\Rightarrow x = -1$

Hence, the required value of x is -1 **[$\frac{1}{2}$ Mark]**

8. If $A = \begin{bmatrix}3 & 1\\-1 & 2\end{bmatrix}$, show that $A^2 - 5A + 7I = 0$. **[3 Marks]**

Solution:

Given:

$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

Step 1:

By taking LHS= $A^2 - 5A + 7I$

$= \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Step 2:

$= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

Step 3:

$= \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$

$\therefore\ LHS = RHS$

$A^2 - 5A + 7I = 0$

Hence it is proved.

9. Find x, if $[x \ -5 \ -1]\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$ **[3 marks]**

Solution:

Given: $[x \ -5 \ -1]\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$

Step 1:

$[x \ -5 \ -1]\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$

$\Rightarrow [x+0-2 \quad 0-10+0 \quad 2x-5-3]\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$

$$\Rightarrow [x-2 \quad -10 \quad 2x-8]\begin{bmatrix}x\\4\\1\end{bmatrix} = O$$

Step 2:

$$\Rightarrow [x^2-2x \quad -40 \quad +2x-8] = [0]$$

Step 3:

$$\Rightarrow x^2 = 48$$

$$\Rightarrow x = \pm 4\sqrt{3}$$

Hence, the required value of x is $\pm\, 4\sqrt{3}$

10. A manufacturer produces three products x, y, z which he sells in two markets. Annual sales are indicated below:

Market		Products	
I	10,000	2,000	18,000
II	6,000	20,000	8,000

(a) If unit sale prices of x, y and z are ₹ 2.50, ₹ 1.50 and ₹ 1.00, respectively, find the total revenue in each market with the help of matrix algebra. **[3 Marks]**

(b) If the unit costs of the above three commodities are ₹ 2.00, ₹ 1.00 and 50 paise respectively. Find the gross profit. **[3 Marks]**

Solution:

(a) **Step 1:**

If unit sale prices of x, y and z are ₹2.50, ₹1.50 and ₹1.00, then

Products | Selling price

$$\begin{array}{lc} & \begin{array}{ccc} x & y & z \end{array} \\ \begin{array}{l}\text{Market I}\\ \text{Market II}\end{array} & \begin{bmatrix}10000 & 2000 & 18000\\ 6000 & 20000 & 8000\end{bmatrix} \end{array} \qquad \begin{bmatrix}₹2.50\\ ₹1.50\\ ₹1.00\end{bmatrix}$$

Step 2:

Total revenue of each market = Total product × Unit selling price

$$= \begin{bmatrix}10000 & 2000 & 18000\\6000 & 20000 & 8000\end{bmatrix}\begin{bmatrix}\text{₹}2.50\\\text{₹}1.50\\\text{₹}1.00\end{bmatrix}$$

Step 3:

$$= \begin{bmatrix}\text{₹}25000 + \text{₹}3000 + \text{₹}18000\\\text{₹}15000 + \text{₹}30000 + \text{₹}8000\end{bmatrix} = \begin{bmatrix}\text{₹}46000\\\text{₹}53000\end{bmatrix}$$

Thus, the revenue of market I is ₹46,000 and that of the market II is ₹53,000.

(b) **Step 1:**

If the unit costs of the three commodities are ₹2.00, ₹1.00 and 50 paise, then

Products Selling price

$$\begin{array}{lc} & \begin{array}{ccc} x & y & z \end{array} \\ \begin{array}{l}\text{Market I}\\ \text{Market II}\end{array} & \begin{bmatrix}10000 & 2000 & 18000\\6000 & 20000 & 8000\end{bmatrix} \end{array} \qquad \begin{bmatrix}\text{₹}2.50\\\text{₹}1.50\\\text{₹}0.00\end{bmatrix}$$

Step 2:

Gross profit from each market = Total product × Unit selling price

$$= \begin{bmatrix}10000 & 2000 & 18000\\6000 & 20000 & 8000\end{bmatrix}\begin{bmatrix}\text{₹}2.50\\\text{₹}1.50\\\text{₹}1.00\end{bmatrix}$$

$$= \begin{bmatrix}\text{₹}20000 + \text{₹}2000 + \text{₹}9000\\\text{₹}12000 + \text{₹}20000 + \text{₹}4000\end{bmatrix} = \begin{bmatrix}\text{₹}31000\\\text{₹}36000\end{bmatrix}$$

Step 3:

Hence, the total revenue from market I is ₹46,000 and that of the market II is ₹31,000.

Therefore, the gross profit of market I=Revenue – Cost

$= \text{₹}46000 - \text{₹}31000 = \text{₹}15000$

Therefore, the gross profit of market II $= \text{₹}53000 - \text{₹}36000 = \text{₹}17000$

Hence, the total gross profit from market I is ₹15,000 and that of the market II is ₹17,000.

11. ind the matrix X so that $X\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix} = \begin{bmatrix}-7 & -8 & -9\\2 & 4 & 6\end{bmatrix}$ **[5 Marks]**

Solution:

Given:

$$X\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}_{2\times3} = \begin{bmatrix}-7 & -8 & -9\\2 & 4 & 6\end{bmatrix}_{2\times3}$$

Step 1:

Let, $X = \begin{bmatrix}a & b\\c & d\end{bmatrix}$ **[$\frac{1}{2}$ Mark]**

Step 2:

Therefore, $X\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix} = \begin{bmatrix}-7 & -8 & -9\\2 & 4 & 6\end{bmatrix}$

$\Rightarrow \begin{bmatrix}a & b\\c & d\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix} = \begin{bmatrix}-7 & -8 & -9\\2 & 4 & 6\end{bmatrix}$ **[$\frac{1}{2}$ Mark]**

Step 3:

$\Rightarrow \begin{bmatrix}a+4b & 2a+5b & 3a+6b\\c+4d & 2c+5d & 3c+6d\end{bmatrix} = \begin{bmatrix}-7 & -8 & -9\\2 & 4 & 6\end{bmatrix}$

Step 4:

If two matrices are equal, then their corresponding elements are also equal.

So, on comparing the corresponding elements, we get

$a + 4b = -7$ **[$\frac{1}{2}$ Mark]**

$2a + 5b = -8$ **[$\frac{1}{2}$ Mark]**

$c + 4d = 2$ **[$\frac{1}{2}$ Mark]**

$2c + 5d = 4$ **[$\frac{1}{2}$ Mark]**

Step 5:

On solving, we get $a = 1, b = -2, c = 2$ and $d = 0$

Hence, $X = \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$

12. A and B are square matrices of the same order such that $AB = BA$, then prove by induction that $AB^n = B^nA$. Further, prove that $(AB)^n = A^nB^n$ for all $n \in N$. Choose the correct answer in the following questions: **[5 marks]**

Solution:

Given:

$AB = BA$

To prove:

$AB^n = B^nA$ and

$(AB)^n = A^nB^n$ for all $n \in N$

Proof: The proof is by mathematical induction.

Step 1:

Consider, $P(n): AB^n = B^nA$, **[$\frac{1}{2}$ Mark]**

Step 2:

Therefore, $P(1): AB = BA$

Hence the result is true for $n = 1$. **[$\frac{1}{2}$ Mark]**

Step 3:

Let it be true for $n = k$,

so, $P(k): AB^k = B^kA$ **[$\frac{1}{2}$ Mark]**

Step 4:

Now we have to prove that it is true for $n = k + 1$ also.

(i.e.) $P(k + 1): AB^{k+1} = \mathrm{B}^{k+1}A$ **[$\frac{1}{2}$ Mark]**

Step 5:

By taking LHS, we get

LHS= AB^{k+1}

$= AB^kB$

$= B^kAB \qquad [\because AB^k = B^kA]$

$= B^kBA \qquad [\because AB = BA]$

$= B^{k+1}A$ =RHS

Hence, the result is true for $n = k + 1$, **[$\frac{1}{2}$ Mark]**

Therefore, by the Principle of Mathematical Induction $AB^n = B^nA$ is true for all natural numbers n.

Step 6:

If $(AB)^n = A^nB^n$

Now,

Consider $P(n): (AB)^n = A^nB^n$ therefore, $P(1): (AB)^1 = A^1B^1$

Hence the result is true for $n = 1$. **[$\frac{1}{2}$ Mark]**

Step 7:

Let it is true for $n = k$,

so, $P(k): (AB)^k = A^kB^k$ **[$\frac{1}{2}$ Mark]**

Step 8:

Now we have to prove that it is true for $n = k + 1$ also.

(i.e.) $P(k + 1): (AB)^{k+1} = A^{k+1}B^{k+1}$ **[$\frac{1}{2}$ Mark]**

Step 9:

By taking LHS, we get

LHS = $(AB)^{k+1}$

$= (AB)^kAB$

$= A^kB^kAB \qquad \left[\because (AB)^k = A^kB^k\right]$ **[$\frac{1}{2}$ Mark]**

Step 10:

$= A^kAB^kB \qquad [\because AB = BA]$

$= A^{k+1}B^{k+1}$ =RHS

Hence, the result is true for $n = k + 1$.

Therefore, by the Principle of Mathematical Induction $(AB)^n = A^n B^n$ is true for all natural numbers n. **[$\frac{1}{2}$ Mark]**

13. If $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is such that $A^2 = I$, then **[2 Marks]**

(A) $1 + \alpha^2 + \beta\gamma = 0$

(B) $1 - \alpha^2 + \beta\gamma = 0$

(C) $1 - \alpha^2 - \beta\gamma = 0$

(D) $1 + \alpha^2 - \beta\gamma = 0$

Solution:

Given that: $A^2 = I$

Step 1:

$\Rightarrow \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} \alpha^2 + \beta\gamma & \alpha\beta - \beta\alpha \\ \alpha\gamma - \alpha\gamma & \beta\gamma + \alpha^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Step 2:

If two matrices are equal, then their corresponding elements are also equal.

So, on comparing the corresponding elements, we get

$\alpha^2 + \beta\gamma = 1$

$1 - \alpha^2 - \beta\gamma = 0$

Hence the option (C) is correct.

14. If the matrix A is both symmetric and skew symmetric, then **[2 Marks]**

(A) A is a diagonal matrix

(B) A is a zero matrix

(C) A is a square matrix

(D) None of these

Solution:

Given:

Matrix A is both symmetric and skew symmetric

Step 1:

We know that only a zero matrix is always both symmetric and skew symmetric.

Proof:

$\therefore\ A' = A$ and $A' = -A$

Step 2:

On comparing both the equations, we get

$\Rightarrow A = -A$

$A + A = O$

$2A = O$

$A = O$

Hence, the option (B) is correct.

15. If A is square matrix such that $A^2 = A$, then $(I + A)^3 - 7\,A$ is equal to **[2 Marks]**

(A) A

(B) $I - A$

(C) I

(D) $3A$

Solution:

Given:

$A^2 = A$

Step 1:

$(I + A)^3 - 7A$

$= I^3 + A^3 + 3I^2A + 3IA^2 - 7A$ **[$\frac{1}{2}$ Mark]**

Step 2:

$= -I + A^2A + 31A + 31A^2 - 7A$ $[\because I^3 = I^2 = I]$ **[$\frac{1}{2}$ Mark]**

Step 3:

$= I + AA + 3A + 3IA - 7A$ $[\because A^2 = A]$ **[$\frac{1}{2}$ Mark]**

Step 4:

$= I + A + 3A + 3A - 7A$

$= I + 7A - 7A$

$= I$ $[\because IA = A]$ **[$\frac{1}{2}$ Mark]**

Hence, the option (C) is correct