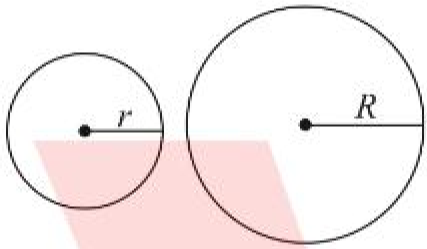
CBSE NCERT Solutions for Class 10 mathematics Chapter 6

Exercise 6.1

Q.1. All circles are _____. (congruent, similar)
similar

Solution: All circles have the same shape i.e. they are round. But the size of a circle may vary.

Thus circles are similar. Each circle has a different radius so the size of the circle may vary.



Q.2. All squares are _____. (similar, congruent)
similar

Solution: We know that,

All the sides of a square are equal.

Since the ratios of the lengths of their corresponding sides are equal.
Hence, all squares are similar since size of squares may be different, but the shape will be always same.

Q.3. All _____ triangles are similar. (Isosceles, equilateral)
equilateral

Solution: We know that, all the sides of an equilateral triangle are equal.

All equilateral triangles are similar because of their same shape.

Q.4. Two polygons of the same number of sides are similar, if their corresponding angles are _____ and their corresponding sides are proportional. [proportional / equal]
equal

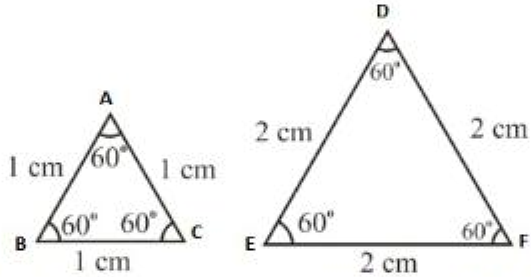
Solution: Two polygons of same number of sides are similar, if their corresponding angles are equal and their corresponding sides are proportional.

For example, if two triangles with angles 30°,60° and 90°are similar, then the ratio Hypotenuse of 1st circleHypotenuse of 2st circle will be the same.

Q.5. Give two different examples of pair of similar figures.

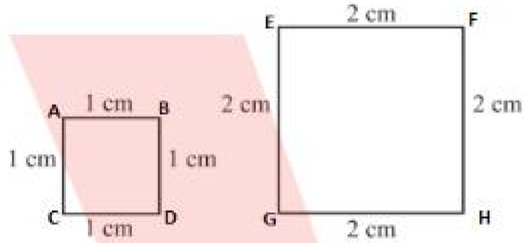
Solution: Two figures are said to be similar if the ratio of corresponding sides are equal.

Two equilateral triangles with sides 1 cm and 2 cm.



Ratio of the corresponding sides are: ABDE=12ACDF=12BCEF=12 Here ratio of the equilateral triangles are same. Therefore, the above figures are similar.

Two squares with sides 1 cm and 2 cm.

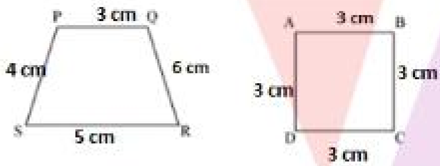


Ratio of the corresponding sides are: ABEF=12ACEG=12BDFH=12CDGH=12 Here also ratios of the corresponding sides are equal. Hence, the above two figures are similar.

Q.6. Give two different examples of a pair of non-similar figures.

Solution: Two figures are said to be non-similar if the ratio of the corresponding sides are not equal.

Consider a Trapezium and a square.

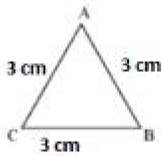


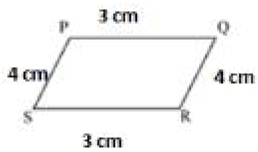
Ratio of the corresponding sides are: PQAB=3 cm3 cm=1 PSAD=4 3 QRBC=6 cm3 cm=2 SRDC=53 Thus, the ratio of the corresponding sides are not equal. Therefore, figures are not similar.

Consider a triangle and a parallelogram

Ratio of the corresponding sides are:

ACPS=34 BCSR=33=1





Hence, the above two figures are non-similar.

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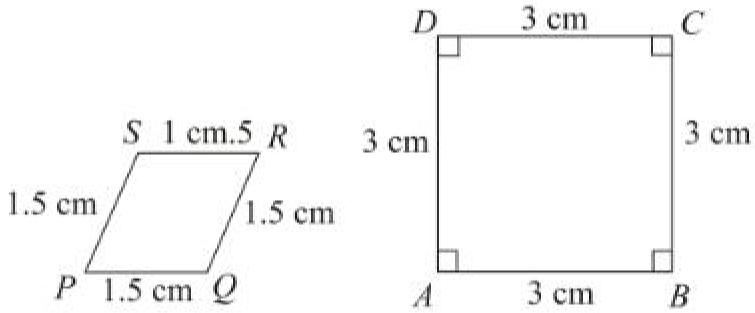
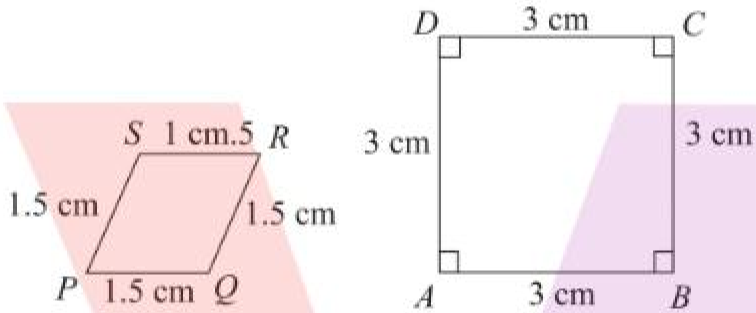
Q.7. State whether the following quadrilaterals are similar or not.



Solution:

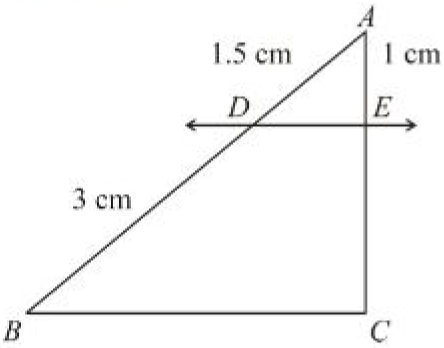


To check whether the given quadrilaterals are similar or not we need to check the ratio of the corresponding sides and angles.

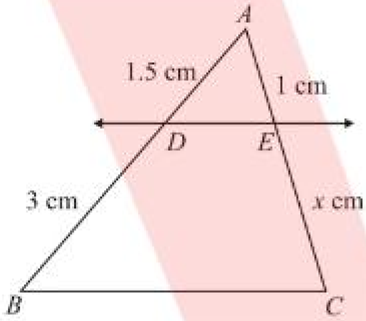
Corresponding sides of two quadrilaterals are proportional i.e., 1:2 but their corresponding angles are not equal. Hence, quadrilaterals PQRS and ABCD are not similar.

Exercise 6.2

Q.1. In the figure given below, DE∥BC. Find EC.

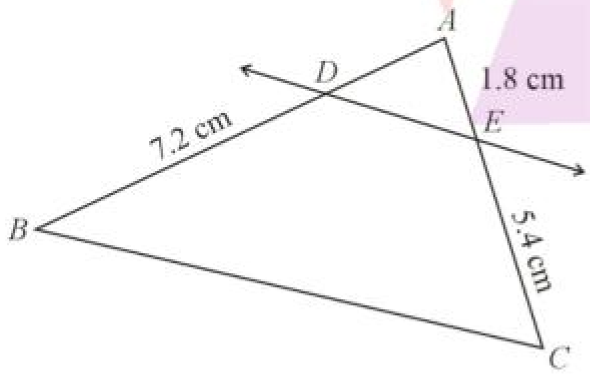


Solution:

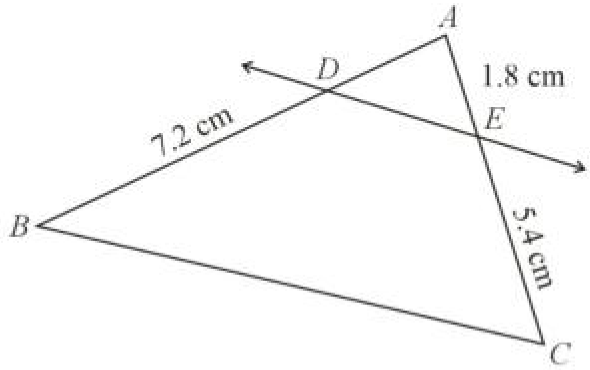


Let EC=x cm
Since DE∥BC
So, using basic proportionality theorem we get:
ADDB=AEEC
⇒1.53=1x
⇒x=3×11.5
⇒x=2
Hence, EC=2 cm.

Q.2. In Fig. DE∥BC. Find AD



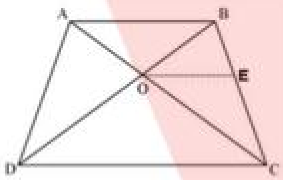
Solution:



Let AD=x cm
Since DE∥BC
Hence, using Basic proportionality theorem,
ADDB=AEEC
⇒x7.2=1.85.4
⇒x=2.4 cm
Hence, AD=2.4 cm

Q.3. The diagonals of a quadrilateral ABCD intersect each other at the point O such that AOBO=CODO. Show that ABCD is a trapezium.

Solution:

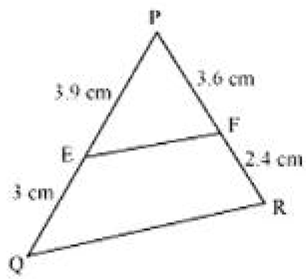


Draw a line segment OE∥AB
In △ABC
Since, OE∥AB.
Hence, AOOC=BEEC.

But by the given relation, we have:
AOBO=CODO
⇒AOOC=OBOD
Hence, OBOD=BEEC
So, using converse of basic proportionality theorem, EO∥DC.
Therefore, AB∥OE∥DC
⇒AB∥CD
Therefore, ABCD is a trapezium.

Q.4. E and F are points on the sides PQ and PR respectively of a △PQR. State whether EF∥QR where PE=3.9 cm, EQ=3 cm, PF=3.6 cm and FR=2.4 cm.

Solution:

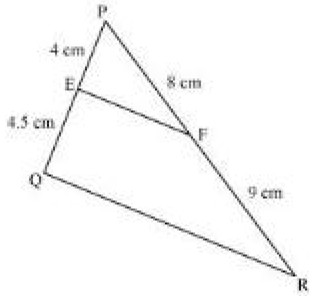


Given:
PE=3.9 cm, EQ=3 cm, PF=3.6 cm and FR=2.4 cm
Now,
PEEQ=3.93=1.3

PFFR=3.62.4=1.5 Since, PEEQ≠PFFR Hence, EF is not parallel to QR.

Q.5. E and F are points on the sides PQ and PR respectively of a ΔPQR. For each of the following cases, state whether EF∥QR:
PE=4 cm, QE=4.5 cm, PF=8 cm and RF=9 cm

Solution:



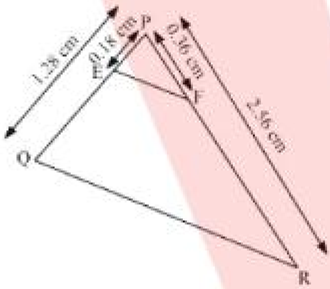
Given, PE=4 cm, QE=4.5 cm, PF=8 cm, RF=9 cm
PEEQ=44.5=89

PFFR=89 Since PEEQ=PFFR
Hence, EF∥QR (using Basic proportionality theorem)

Q.6. E and F are points on the sides PQ and PR respectively of a ΔPQR. For each of the following cases, state whether EF∥QR:

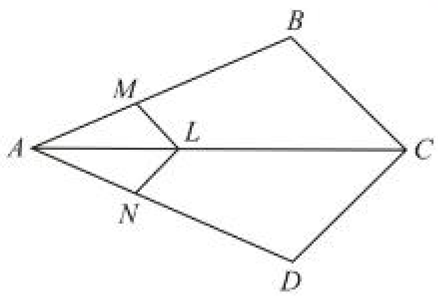
PQ=1.28 cm, PR=2.56 cm, PE=0.18 cm and PF=0.36 cm

Solution:

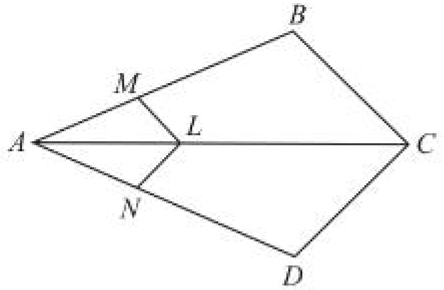


Given, PQ=1.28 cm, PR=2.56 cm, PE=0.18 cm and PF=0.36 cm EQ=PQ-PE=1.28-0.18=1.1 cm and FR=PR-PF=2.56-0.36=2.2 cm PEEQ=0.181.1=18110=955 PFFR=0.362.2=955 Since, PEEQ=PFFR Hence, EF∥QR (using basic proportionality thorem)

Q.7. In the figure, if LM∥CB and LN∥CD, prove that AMAB=ANAD



Solution:



In the given figure, LM∥CB.

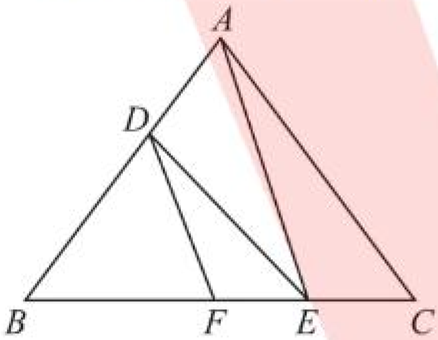
Hence, using basic proportionality theorem, AMMB=ALLC …(i) Since, LN∥CD Hence, using basic proportionality theorem, ANND=ALLC …(ii)

From i and (ii)

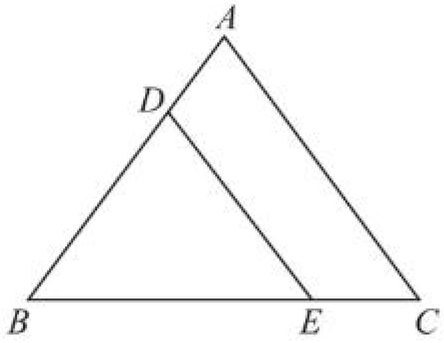
AMMB=ANND

⇒ MBAM=NDAN ⇒ MBAM+1=NDAN+1 ⇒MB+AMAM=ND+ANAN ⇒ABAM=ADAN ⇒AMAB=ANAD

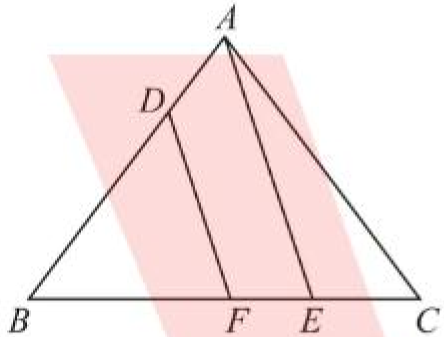
Q.8. In figure given below DE∥AC and DF∥AE. Prove that BFFE=BEEC.



Solution:

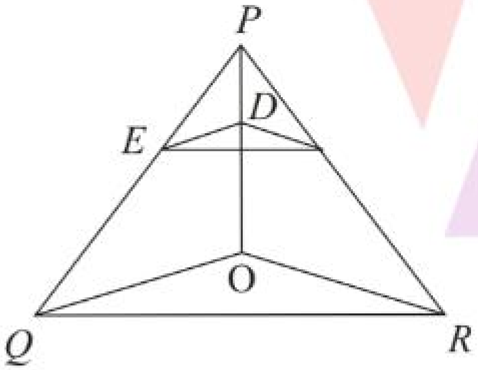


In ΔABC
Since DE∥AC
Hence, BDDA=BEEC …(i) (using basic proportionality theorem)

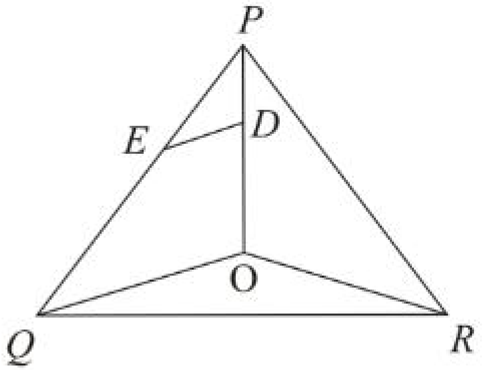


In ΔBAE,
Since DF∥AE
Hence, BDDA=BFFE…(ii) (using basic proportionality theorem)
From i and (ii), we get:
BEEC=BFFE

Q.9. In the figure, DE∥OQ and DF∥OR. Show that EF∥QR.

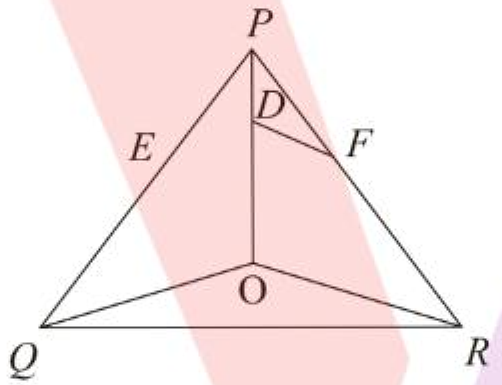


Solution:



In ΔPOQ, since DE$\parallel$OQ,

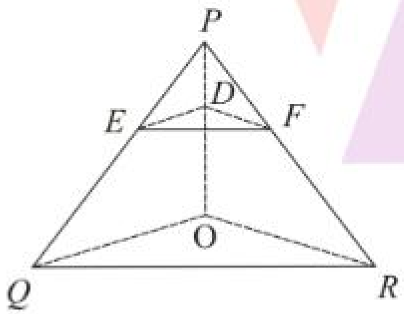
PEEQ=PDDO ...(i) [Using basic proportionality theorem]



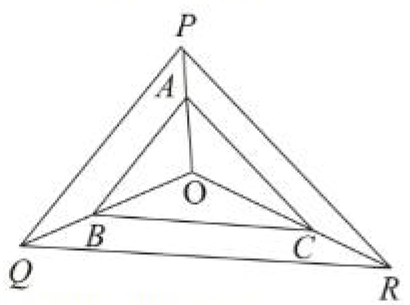
In ΔPOR, since DF$\parallel$OR,

PFFR=PDDO ...(ii) [Using basic proportionality theorem]

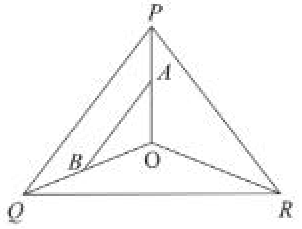
From i and (ii), we get, PEEQ=PFFR Using converse of basic proportionality theorem EF$\parallel$QR



Q.10. In the figure, A,B and C are points on OP,OQ and OR respectively such that AB$\parallel$PQ and AC$\parallel$PR. Show that BC$\parallel$QR.

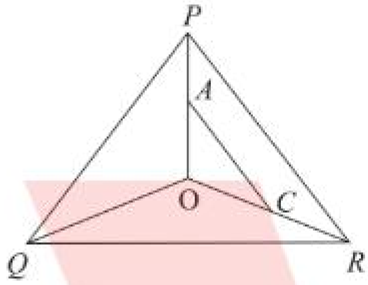


Solution:



In ΔPOQ

Since, AB∥PQ, Hence, OAAP=OBBQ …(i) [Using basic proportionality theorem]



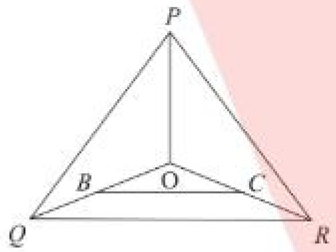
In ΔPOR

Since, AC∥PR Hence, OAAP=OCCR …(ii) [Using basic proportionality theorem]

From i and (ii)

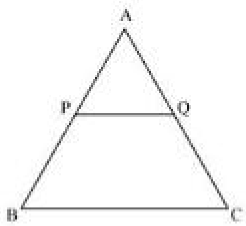
OBBQ=OCCR

Hence, BC∥QR (Using converse of basic proportionality theorem)



Q.11. Using Theorem 6.1, prove that a line drawn through the midpoint of one side of a triangle parallel to another side bisects the third side. (Recall that you have proved it in Class IX).

Solution:

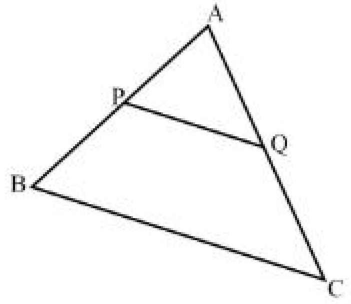


Let in the given figure PQ is a line segment drawn through mid-point P of line AB such that PQ∥BC
Hence, AP=PB

Now, using basic proportionality theorem
AQQC=APPB
⇒AQQC=APAP
⇒AQQC=1
⇒AQ=QC
Hence, Q is the mid-point of AC.

Q.12. Prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side.

Solution:

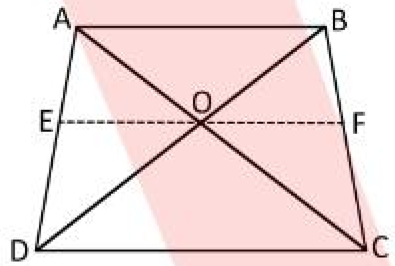


Let in the given figure PQ is a line segment joining mid-points Pand Q of line AB and AC respectively.

Hence, AP=PB and AQ=QC Now, since APPB=APAP=1 and AQQC=AQAQ=1 Hence, APPB=AQQC Now, using converse of basic proportionality theorem, we get, PQ∥BC.

Q.13. ABCD is a trapezium in which AB∥DC and its diagonals intersect each other at the point O. Show that AOBO=CODO

Solution:



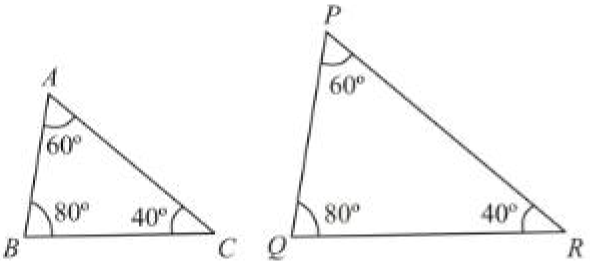
Let a line segment EF is drawn through point O such that EF∥CD

In ΔABC and in ΔBDC, FO∥AB and FO∥CD ∵EF∥CD, AB∥CD

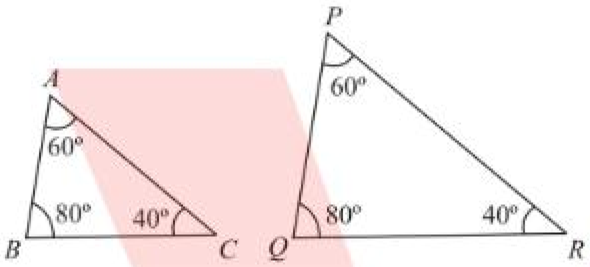
So, using basic proportionality theorem
BFFC=AOOC …(1) and BFFC=BOOD ...(2) Now, from equation 1 and (2), we get, AOOC=BOOD
⇒AOBO=OCOD

Exercise 6.3

Q.1. State which pairs of triangles in the figure are similar. Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form:



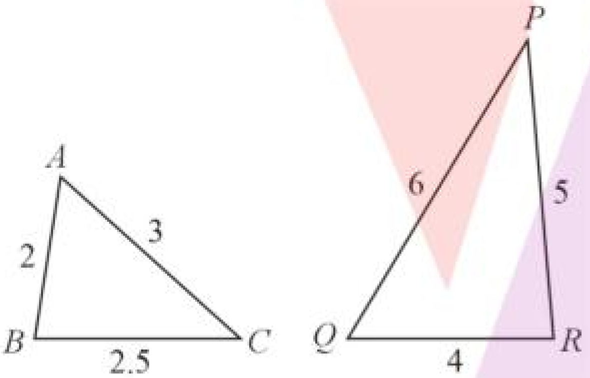
Solution: Given figures are



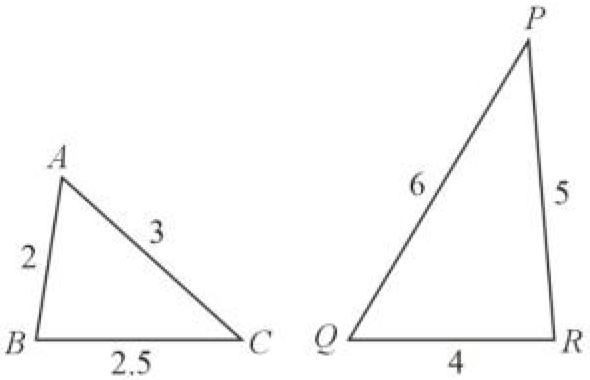
∠A=∠P=60°

∠B=∠Q=80° ∠C=∠R=40° Hence by AAA rule ΔABC~ΔPQR.

Q.2. Are the pairs of triangles in the figure similar? Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form:



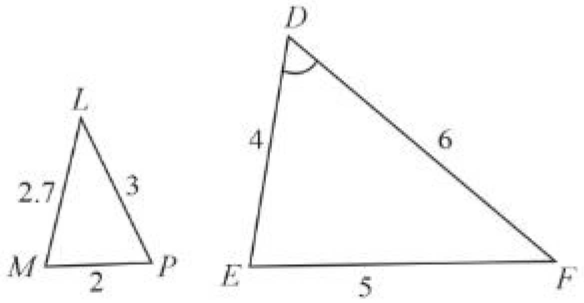
Solution:



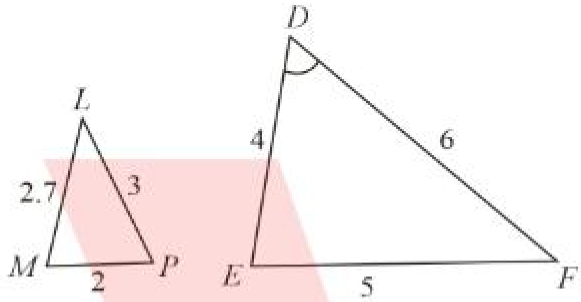
ABQR=24=12

BCRP=2.55=12 CAPQ=36=12 Since, ABQR=BCRP=CAPQ Hence, by SSS rule. ΔABC~ΔQRP.

Q.3. State whether the following pair of triangles is similar or not.

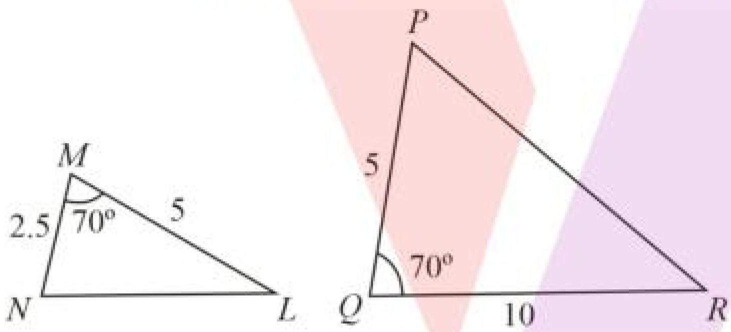


Solution: Given figure is:

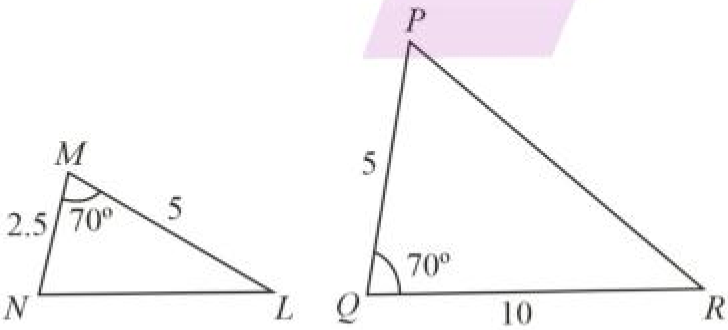


We know that two triangles are similar if they have, all their angles equal and corresponding sides are in the same ratio. Here, the corresponding sides are not proportional. Hence, the given triangles are not similar.

Q.4. State if the following pairs of triangles in the figure are similar or not. Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form:

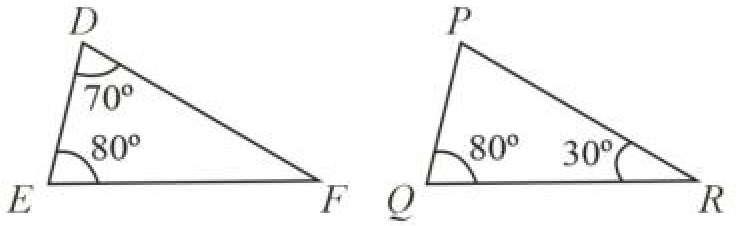


Solution:

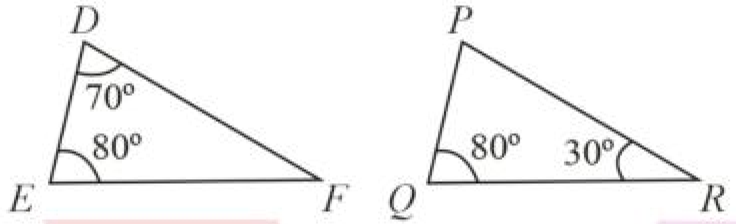


MNPQ=2.55=12
MLQR=510=12
∠M=∠Q=70o
Hence, by SAS rule
ΔMNL~ΔQPR

Q.5. State whether the following pair of triangles is similar or not.

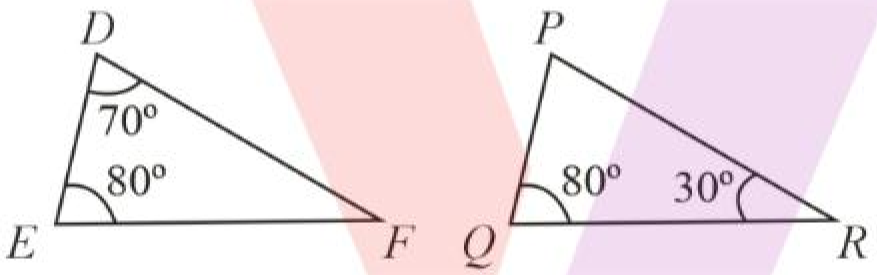


Solution: Given figure is:

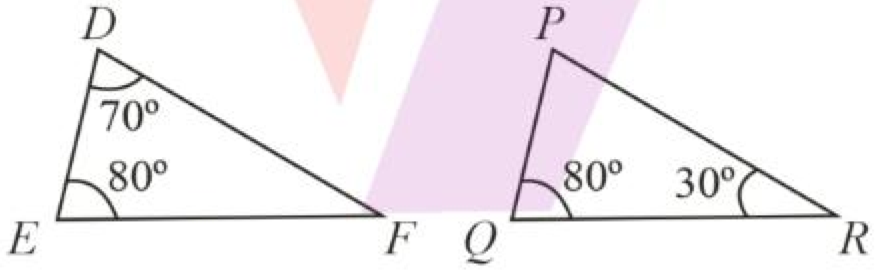


We know that two triangles are said to be similar if their corresponding angles are congruent and the corresponding sides are in proportion . In other words, similar triangles are the same shape, but not necessarily the same size. Here, as the corresponding sides are not in proportional. Hence, the given triangles is not similar.

Q.6. State whether the pair triangles are similar or not. Write the similarity criterion used by you for answering the question and also write the pair of similar triangles in the symbolic form.



Solution:

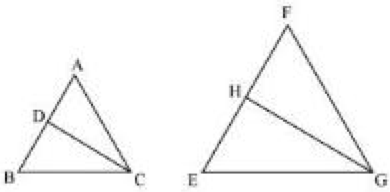


In ΔDEF,
∠D+∠E+∠F=180o(Sum of angles of a triangle is 180o.)
⇒70o+80o+∠F=180o
⇒∠F=30o
Similarly in ΔPQR,
∠P+∠Q+∠R=180o (Sum of angles of a triangle is 180o.)
⇒∠P+80o+30o=180o
⇒∠P=70o

Now, In ΔDEF and ΔPQR
since,
∠D=∠P=70o
∠E=∠Q=80o
∠F=∠R=30o
Hence, by AAA rule
ΔDEF~ΔPQR

Q.7. CD and GH are respectively the bisectors of ∠ACB and ∠EGF such that D and H lie on sides AB and FE of △ABC and △EFG respectively. If △ABC~△FEG, show that CDGH=ACFG

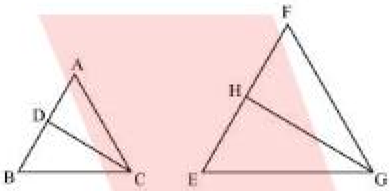
Solution:



In △ACD and △FGH,
∠A=∠F (∵△ABC~△EFG)
∠ACD=∠FGH (angle bisector)
∠ADC=∠FHG (remaining angle)
Hence, by AAA rule we have:
△ACD~△FGH
So, CDGH=ACFG (corresponding sides are proportional)

Q.8. CD and GH are respectively the bisectors of ∠ACB and ∠EGF such that D and H lie on sides AB and FE of △ABC and △EFG respectively. If △ABC~△FEG, show that △DCB~△HGE.

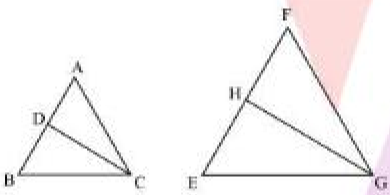
Solution:



Since △ABC~△FEG
Hence, ∠A=∠F
∠B=∠E
∠ACB=∠FGE
⇒∠ACB2=∠FGE2
And ∠DCB=∠HGE (angle bisector)
∠BDC=∠EHG (remaining Angle)
Hence, by AAA rule we have:
△DCB~△HGE

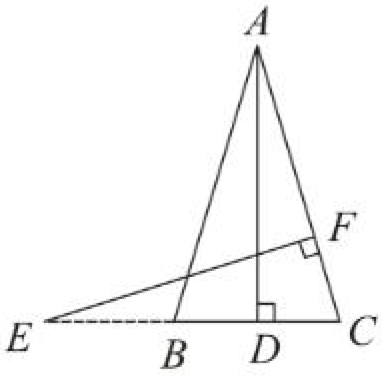
Q.9. CD and GH are respectively the bisectors of ∠ACB and ∠EGF such that D and H lie on sides AB and FE of △ABC and △EFG respectively. If △ABC~△FEG, show that △DCA~△HGF

Solution:

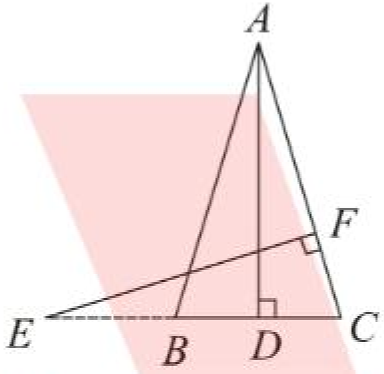


Since △ABC~△FEG
Hence, ∠A=∠F
∠B=∠E
∠ACB=∠FGE
⇒∠ACB2=∠FGE2
⇒∠ACD=∠FGH (angle bisector)
∠CDA=∠GHF (remaining angle)
Hence, by AAA rule we have:
△DCA~△HGF

Q.10. In the figure given below, E is a point on side CB produced of an isosceles triangle ABC [illegible] prove that $\triangle ABD \sim \triangle ECF$.

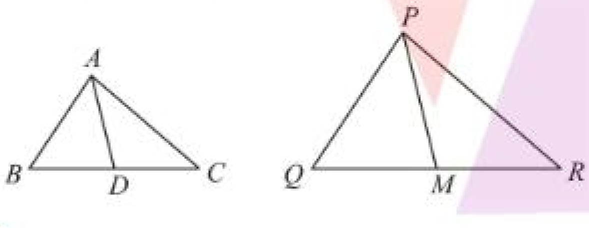


Solution:

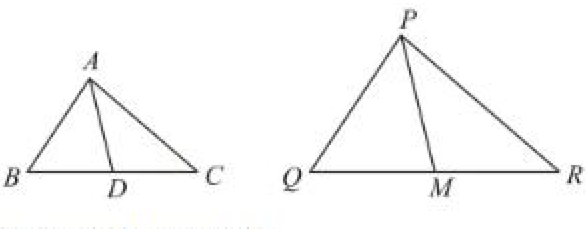


In $\triangle ABD$ and $\triangle ECF$,
Since, AB=AC (isosceles triangles)
So, $\angle ABD = \angle ECF$ (angles opposite to equal sides)
$\angle ADB = \angle EFC = 90°$
$\angle BAD = \angle CEF$ (remaining angle)
Hence, by AAA rule we have:
$\triangle ABD \sim \triangle ECF$

Q.11. Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of $\triangle PQR$ in the given figure. Show that $\triangle ABC \sim \triangle PQR$.

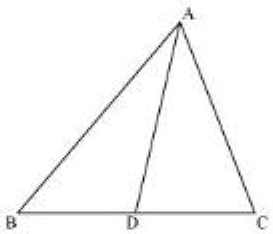


Solution:

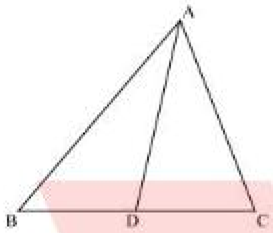


Median divides opposite side.
So, BD=BC2 and QM=QR2
Given that,
ABPQ=BCQR=ADPM
So, ABPQ=BDQM=ADPM($\because$BC=2×BD, QR=2×QM).
Hence, $\triangle ABD \sim \triangle PQM$.
So, $\angle ABD = \angle PQM = \angle PQR$ (corresponding angles of similar triangles)
And ABPQ=BCQR
Hence, $\triangle ABC \sim \triangle PQR$.

Q.12. D is a point on the side BC of a triangle ABC such that ∠ADC=∠BAC. Show that CA2=CB×CD.

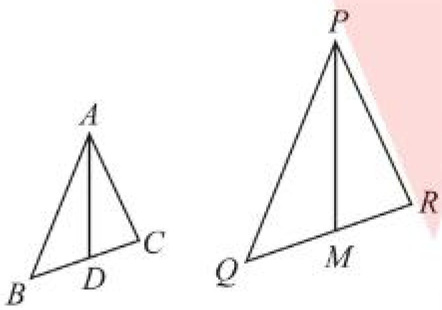


Solution:

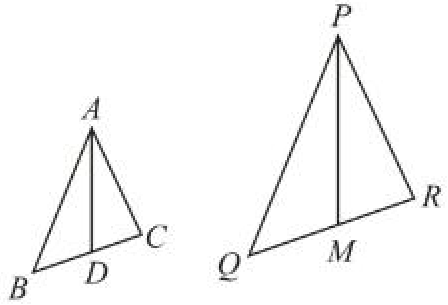


In △ACD and △BAC
It is given that ∠ADC=∠BAC
∠ACD=∠BCA (common angle)
∠CAD=∠CBA (remaining angle)
Hence, by AAA rule we have:
△ADC~△BAC
So, by corresponding sides of similar triangles will be proportional to each other.
CACB=CDCA
Hence, CA2=CB×CD.

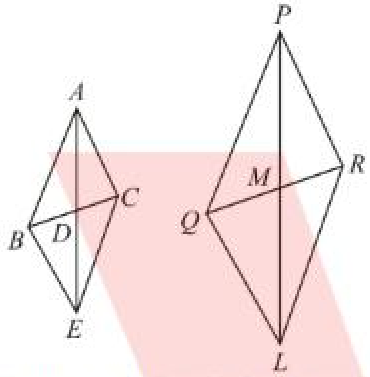
Q.13. Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that ΔABC~ΔPQR.



Solution:



Given that,
ABPQ=ACPR=ADPM



Let us extend AD and PM up to point E and L respectively such that AD=DE and PM=ML. Now join B to E, C to E, Q to L and R to L. We know that medians divide opposite sides. So, BD=DC and QM=MR Also, AD=DE (by construction) And PM=ML(By construction) So, in quadrilateral ABEC, diagonals AE and BC bisects each other at point D. Also, in quadrilateral PQLR, diagonals PL and QR bisects each other at point M. So, quadrilaterals ABED and PQLR are parallelograms. AC=BE and AB=EC (Since it is a parallelogram, opposite sides will be equal) Also PR=QL and PQ=LR (Since it is a parallelogram, opposite sides will be equal)

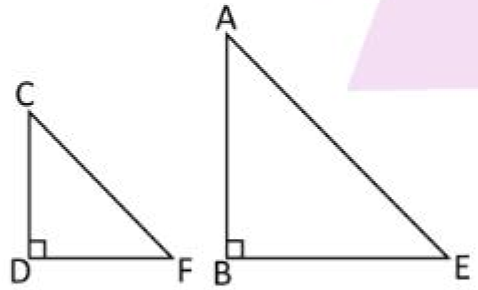
In ΔABE and ΔPQL,

ABPQ=BEQL=AEPL (ACPR=BEQL and ADPM=2AD2PM=AEPL)

Hence, by SSS rule, ΔABE~ΔPQL Similarly, ΔAEC~ΔPLR Hence, ∠BAE=∠QPL and ∠EAC=∠LPR Hence, ∠BAC=∠QPR Now, **in** ΔABC and ΔPQR, ABPQ=ACPR and ∠BAC=∠QPR Hence, by SAS rule, ΔABC~ΔPQR

Q.14. A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

Solution:

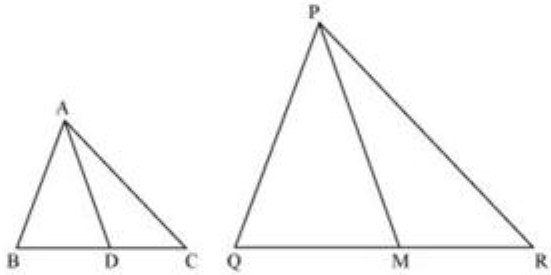


Let AB be a tower and CD be a pole

Shadow of AB is BE. Shadow of CD is DF. The sun ray will fall on tower and pole at same angle. ∴ ∠DCF=∠BAE **and** ∠DFC=∠BEA ∠CDF=∠ABE=90o(Tower and pole are vertical to ground) Hence, by AAA rule, ΔABE~ΔCDF Therefore ABCD=BEDF ⇒AB6=284 ⇒AB=42 Hence, the height of the tower =42 meters.

Q.15. If AD and PM are medians of triangles ABC and PQR, respectively where ΔABC~ΔPQR, prove that ABPQ=ADPM

Solution:



Since ΔABC~ΔPQR

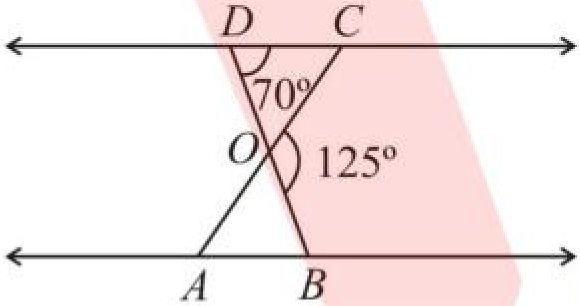
Thus, their respective sides will be in proportion Or, ABPQ=ACPR=BCQR …(1) Also, ∠A=∠P,∠B=∠Q,∠C=∠R…(2) Since, AD and PM are medians, they will divide their opposite sides equally. Hence, BD=BC2 and QM=QR2…(3)

From equation 1 and (3)

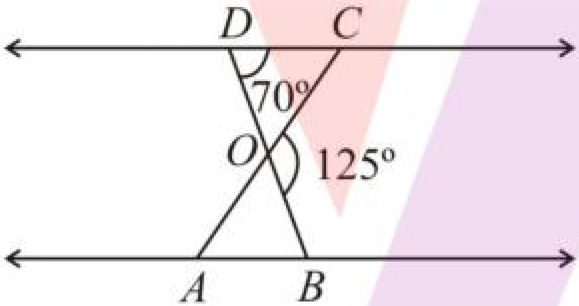
ABPQ=BDQM

∠B=∠Q (From equation 2) Hence, by SAS rule, ΔABD~ΔPQM Hence, ABPQ=ADPM (Corresponding sides are proportional)

Q.16. In the following figure, ΔODC~ΔOBA, ∠BOC=125∘ and ∠CDO=70∘. Find ∠DOC, ∠DCO and ∠OAB.



Solution:



Since DOB is a straight line

Hence, ∠DOC+∠COB=180° linear pair ⇒∠DOC=180°-125°=55°

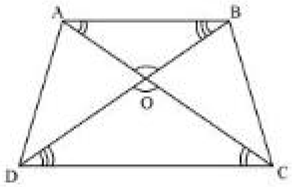
In ΔDOC,

∠DCO+∠CDO+∠DOC=180° (Angle sum property)

⇒∠DCO+70°+55°=180°. ⇒∠DCO=55° Since, ΔODC~ΔOBA. Thus, ∠OCD=∠OAB Corresponding angles equal in similar triangles Hence, ∠OAB=55°.

Q.17. Diagonals AC and BD of a trapezium ABCD with AB∥DC intersect each other at the point O. Using a similarity criterion for two triangles, show that OAOC=OBOD

Solution:

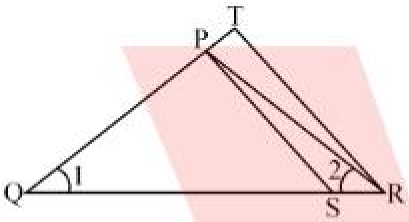


In ΔDOC and ΔBOA

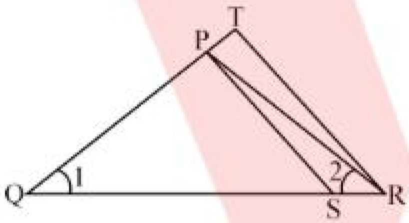
AB||CD

Hence, ∠CDO=∠ABO [Alternate interior angles] ∠DCO=∠BAO [Alternate interior angles] ∠DOC=∠BOA [Vertically opposite angles] Hence, ΔDOC~ΔBOA Using AAA rule ⇒DOBO=OCOA [Corresponding sides are proportional] ⇒OAOC=OBOD

Q.18. In the figure given below QRQS=QTPR and ∠1=∠2. Show that △PQS~△TQR.



Solution:

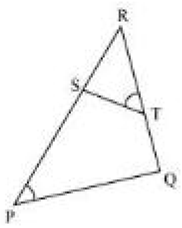


In △PQR,

∠PQR=∠PRQ Hence, PQ=PR…(i) QRQS=QTPR ....(given) Using (i), we get: QRQS=QTPQ…(ii) Also, ∠RQT=∠PQS=∠1. Hence, by SAS rule ΔPQS~ΔTQR.

Q.19. S and T are points on sides PR and QR of ΔPQR such that ∠P=∠RTS. Show that ΔRPQ~ΔRTS.

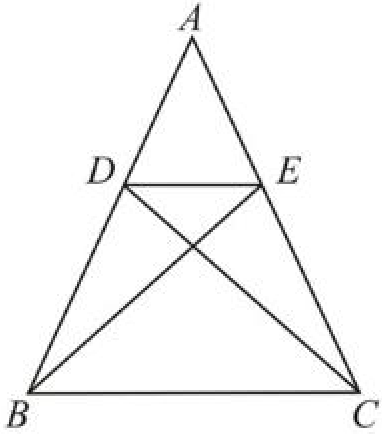
Solution:



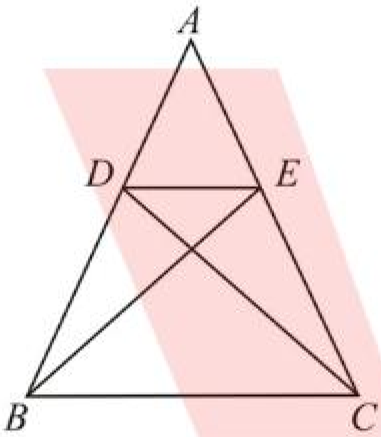
In ΔRPQ and ΔRTS

∠QPR=∠RTS [Given] ∠R=∠R [Common angle] ∠RQP=∠RST [Remaining angle] Hence, ΔRPQ~ΔRTS [by AAA rule]

Q.20. In Fig. if ΔABE≅ΔACD, show that ΔADE~ΔABC.



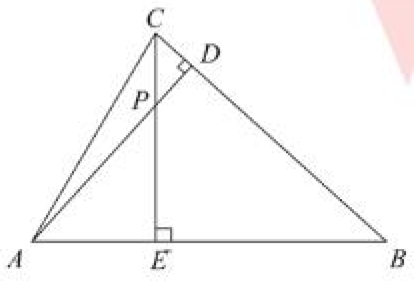
Solution:



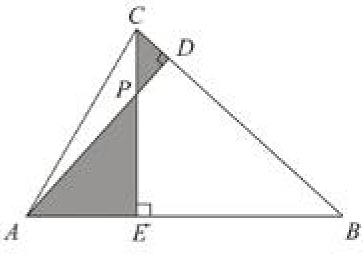
Since ΔABE≅ΔACD

Therefore, AB=AC…(1) ⇒AD=AE…(2) Now, in ΔADE and ΔABC, Dividing equation 2by (1) ADAB=AEAC ∠A=∠A [Common angle] Hence, ΔADE~ΔABC [by SAS rule]

Q.21. In the following figure altitudes AD and CE of ΔABC intersect each other at the point P. Show that ΔAEP~ΔCDP.

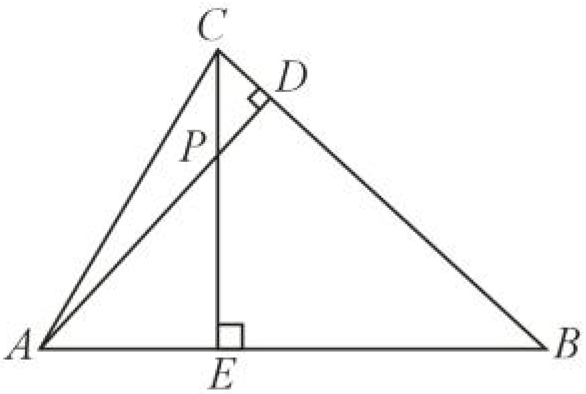


Solution:

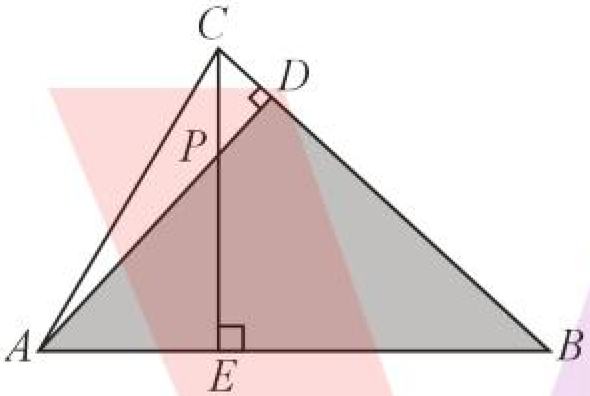


In ΔAEP and ΔCDP
∠CDP=∠AEP=90°
∠CPD=∠APE (vertically opposite angles)
∠PCD=∠PAE (remaining angle)
Hence, by AAA rule we have:
ΔAEP~ΔCDP

Q.22. In the figure, altitudes AD and CE of ΔABC intersect each other at the point P. Show that ΔABD~ΔCBE.

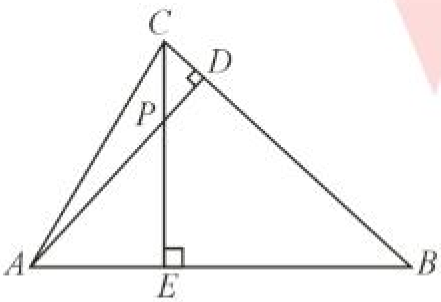


Solution:

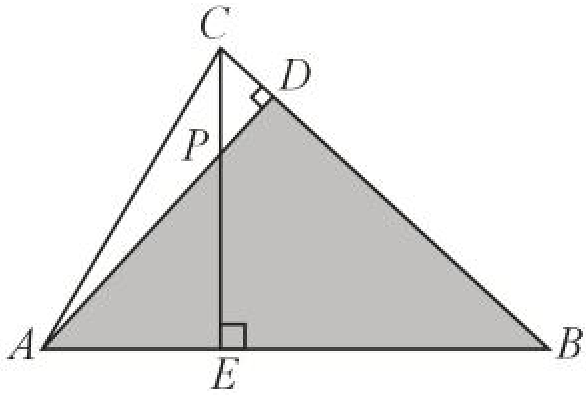


In ΔABD and ΔCBE
∠ADB=∠CEB=90o
∠ABD=∠CBE (Common angle)
∠DAB=∠ECB(Remaining angle)
Hence, by AAA rule,
ΔABD~ΔCBE

Q.23. In Fig. altitudes AD and CE of ΔABC intersect each other at the point P. Show that ΔAEP~ΔADB.

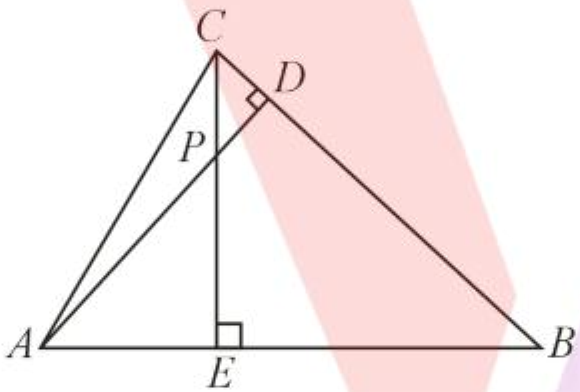


Solution:

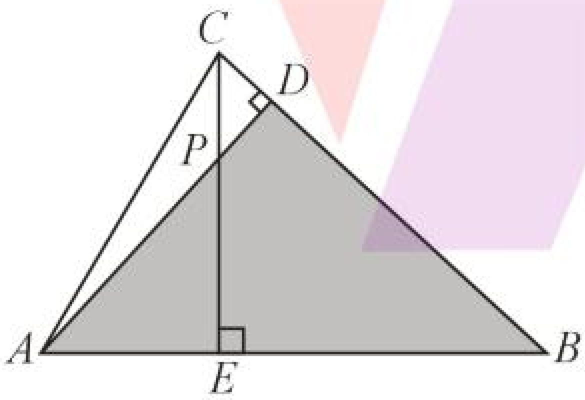


In ΔAEPand ΔADB
∠AEP=∠ADB=90o
∠PAE=∠DAB (Common angle)
∠APE=∠ABD (Remaining angle)
Hence, by AAA rule,
ΔAEP~ΔADB

Q.24. In the figure, altitudes AD and CE of ΔABC intersect each other at the point P. Show that ΔPDC~ΔBEC.



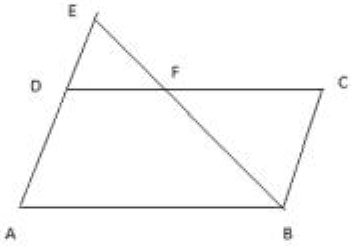
Solution:



In ΔPDC and ΔBEC
∠PDC=∠BEC=90o
∠BCE=∠PCD (Common angle)
∠CPD=∠CBE (Remaining angle)
Hence, by AAA rule,
ΔPDC~ΔBEC

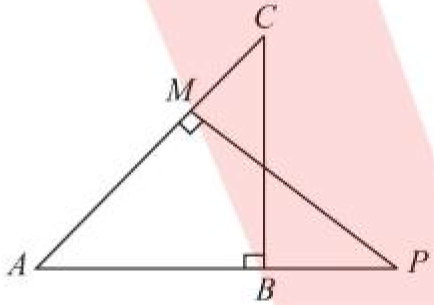
Q.25. E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that △ABE~△CFB.

Solution:

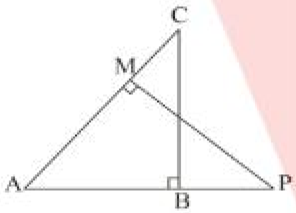


In $\triangle ABE$ and $\triangle CFB$,
$\angle A = \angle C$ (Opposite angles of parallelogram)
$\angle AEB = \angle CBF$ (alternate interior angles as $AE \parallel BC$)
$\angle ABE = \angle CFB$ (alternate interior angles as $AB \parallel DC$)
Hence, by AAA rule we have:
$\triangle ABE \sim \triangle CFB$

Q.26. In the figure, ABC and AMP are two right triangles, right-angled at B and M respectively. Prove that $\Delta ABC \sim \Delta AMP$



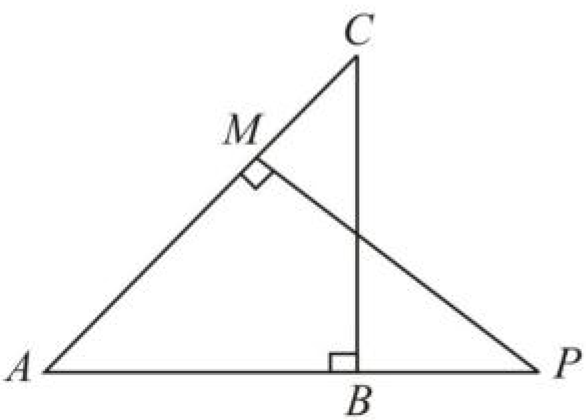
Solution:



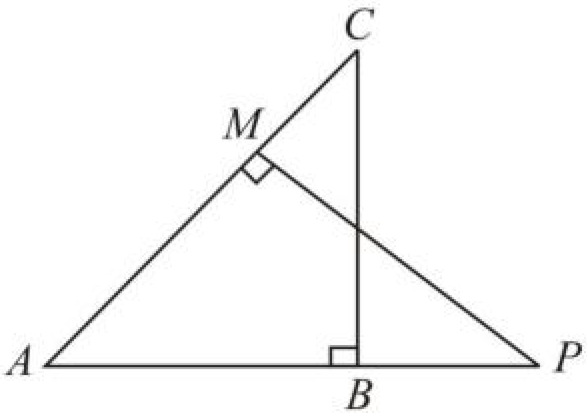
In ΔABC and ΔAMP

$\angle ABC = \angle AMP = 90o$ $\angle A = \angle A$ (Common angle) $\angle ACB = \angle APM$ (Remaining angle) Hence, by AAA rule, $\Delta ABC \sim \Delta AMP$

Q.27. In the figure, ABC and AMP are two right triangles, right-angled at B and M respectively. Prove that: CAPA=BCMP



Solution: Given figure is,



In ΔABC and ΔAMP
∠ABC=∠AMP=90o
∠A=∠A (Common angle)
∠ACB=∠APM (Remaining angle) Hence, by AAA rule,
ΔABC~ΔAMP Hence, CAPA=BCMP (Corresponding sides are proportional)

Exercise 6.4

Q.1. Let △ABC~△DEF and their areas be 64 cm2 and 121cm2 respectively. If EF=15.4 cm, find BC.

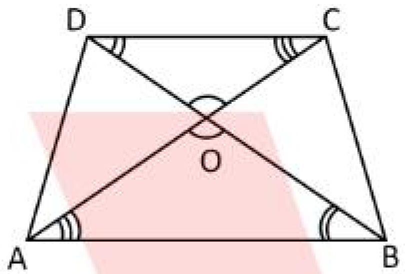
Solution: Given, △ABC~△DEF

We have,

area(△ABC)area(△DEF)=ABDE2=BCEF2=ACDF2 Since EF=15.4, area△ABC=64, area△DEF=121. Hence, 64121=BC215.42 ⇒BC15.4=811 ⇒BC=8×15.411=8×1.4=11.2 cm. Thus, BC=11.2 cm.

Q.2. Diagonals of a trapezium ABCD with AB∥DC intersect each other at the point O. If AB=2CD, find the ratio of the areas of triangles AOB and COD.

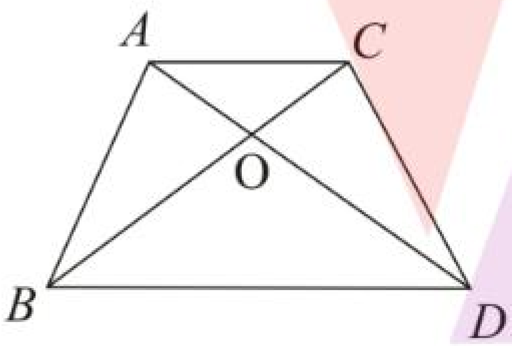
Solution:



Since AB∥CD,

∠OAB=∠OCD (Alternate interior angles) ∠OBA=∠ODC (Alternate interior angles) ∠AOB=∠COD (Vertically opposite angles) Hence, by AAA rule, ΔAOB~ΔCOD ⇒area(ΔAOB)area(ΔCOD)=ABCD2 Since AB=2CD, area(ΔAOB)area(ΔCOD)=41=4:1 Hence, the ratio of the areas of triangles AOB and COD is 4:1.

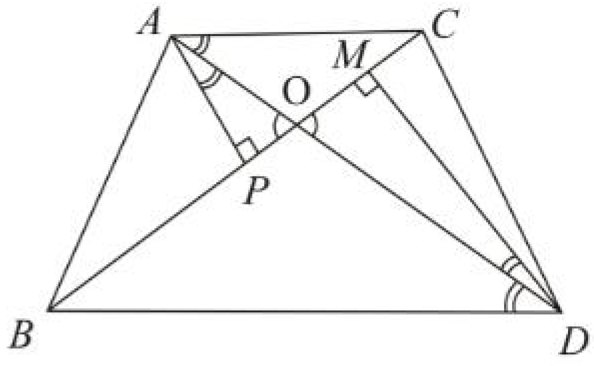
Q.3. In the figure, ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, show that arABCarDBC=AODO.



Solution: We know that the area of a triangle =12×Base×height
Since ΔABC and ΔDBC are on same base,

Hence, ratio of their areas will be same as ratio of their heights.

Let us draw two perpendiculars AP and DM on BC.



In ΔAPO and ΔDMO

∠APO=∠DMO=90o, ∠AOP=∠DOM (Vertically opposite angles)

∠OAP=∠ODM (Remaining angle)

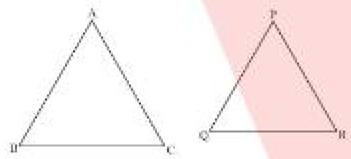
Hence, by AAA rule

ΔAPO ~ ΔDMO

Hence, APDM=AODO Hence, area(ΔABC)area(ΔDBC)=APDM=AODO

Q.4. If the areas of two similar triangles are equal, prove that they are congruent.

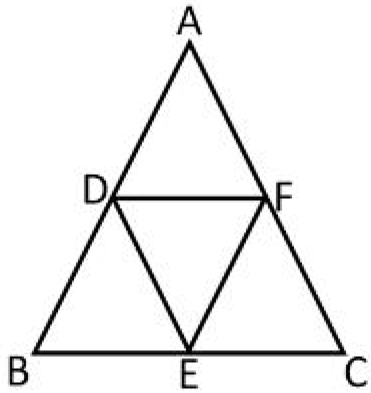
Solution:



Let us assume that △ABC ~ △PQR.
Now, area(△ABC)area(△PQR)=ABPQ2=BCQR2=ACPR2
Since, area △ABC= area (△PQR)
Hence, AB=PQ
BC=QR
AC=PR
Since, corresponding sides of two similar triangles are of same length.
Hence, △ABC≅△PQR (by SSS rule)

Q.5. D, E and F are respectively the mid-points of sides AB, BC and CA of ΔABC. Find the ratio of the areas of ΔDEF and ΔABC.

Solution:



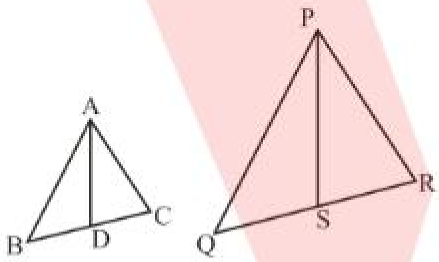
Since D and E are mid-points of AB and BC of ΔABC
Hence, DE‖AC and DE=12AC (by mid-point theorem)
Similarly, EF=12AB and DF=12BC
Now in ΔABC and ΔEFD
ABEF=BCFD=CADE=2

Therefore, by SSS rule,ΔABC~ΔEFD

Hence, area(ΔABC)area(ΔDEF)=ACDE2=4 ⇒area(ΔDEF)area(ΔABC)=14=1:4

Q.6. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

Solution:



Let us assume that ΔABC~ΔPQR. Let AD and PS be the medians of these triangles.

So, ABPQ=BCQR=ACPR…1

∠A=∠P,∠B=∠Q,∠C=∠R

Since, AD and PS are medians

So, BD=DC=BC2 and QS=SR=QR2 So, equation 1 becomes ABPQ=BDQS=ACPR Now in ΔABD and ΔPQS ∠B=∠Q and, ABPQ=BDQS

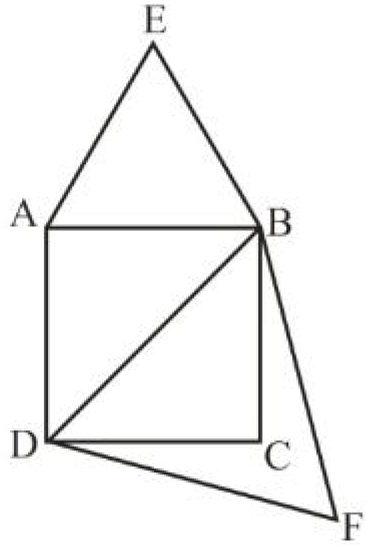
Hence, ΔABD~ΔPQS

Hence, ABPQ=BDQS=ADPS…2

Since, area(ΔABC)areaΔPQR=ABPQ2 ⇒areaΔABCareaΔPQR=ADPS2 [from equation (2)]

Q.7. Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonals.

Solution:



Let ABCD be a square of side a. Therefore, it's diagonal =2a.

Let ΔABE and ΔDBF are two equilateral triangles. Hence, AB=AE=BE=a and DB=DF=BF=2a. We know that all angles of equilateral triangles are 60o.

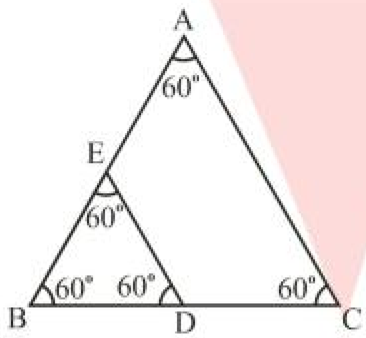
Hence, all equilateral triangles are similar to each other.

Hence, ratio of areas of these triangles will be equal to the square of the ratio between sides of these triangles.

area of ΔABEarea of ΔDBF=a2a2=12 Hence, area of ΔABE=12(area of ΔDBF).

Q.8. ABC and BDE are two equilateral triangles such that D is the midpoint of BC. Ratio of the areas of triangles ABC and BDE is 2:11:24:1

Solution:



Since, all angle of equilateral triangles are 60o, all equilateral triangles are similar to each other.

Therefore, ratio between areas of these triangles will be equal to the square of the ratio between sides of these triangles.

Let side of ΔABC=a Therefore, side of ΔBDE=a2 Hence, areaΔABCareaΔBDE=aa22=41=4:1

1:4

Q.9. Sides of two similar triangles are in the ratio 4:9. Areas of these triangles are in the ratio 2:34:981:1616:81

Solution: We know that,

If two triangles are similar to each other, the ratio between areas of these triangles will be equal to the square of the ratio between sides of these triangles.

Given that sides are in the ratio 4:9. Hence, ratio between areas of these triangles =492=1681=16:81.

Exercise 6.5

Q.1. Sides of a triangle are given below. Determine if it is a right triangle. In case of a right triangle, write the length of its hypotenuse.
7 cm, 24 cm, 25 cm

Solution: Given that sides are 7 cm, 24 cm and 25 cm.

Squaring the lengths of these sides we get 49, 576 and 625.

Clearly, 49+576=625 or 72+242=252. The given triangle satisfies Pythagoras theorem. So, it is a right triangle. We know that the longest side in a right triangle is the hypotenuse. Hence, length of hypotenuse =25 cm.

Q.2. Sides of triangles are given below. Determine which of them are right triangles. In case of a right triangle, write the length of its hypotenuse.
3 cm, 8 cm, 6 cm

Solution: Given that sides are 3 cm, 8 cm and 6 cm.

Here, 64≠36+9

Clearly, sum of squares of lengths of two sides is not equal to square of length of third side. Hence, given triangle is not satisfying Pythagoras theorem. So, it is not a right triangle.

Q.3. Sides of triangles are given below. Determine which of them are right triangles. In case of a right triangle, write the length of its hypotenuse.
50 cm, 80 cm, 100 cm

Solution: Given that sides are 50 cm, 80 cm and 100 cm.

Here, 10000≠6400+2500

Clearly, sum of squares of lengths of two sides is not equal to square of length of third side. Therefore, given triangle is not satisfying Pythagoras theorem. So, it is not a right triangle.

Q.4. The sides of a triangle are given below. Determine whether it is a right triangle. In case of a right triangle, write the length of its hypotenuse.
13 cm, 12 cm, 5 cm.

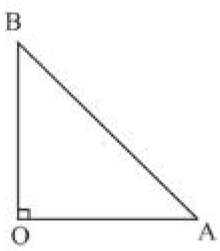
Solution: Given that sides are 13 cm, 12 cm and 5 cm.

Squaring the lengths of these sides we may get 169, 144 and 25.

We know that, 144+25=169 or 122+52=132. So, by converse of Pythagoras theorem, it is a right triangle. As we know that the longest side in a right triangle is the hypotenuse. Hence, length of hypotenuse =13 cm.

Q.5. A guy wire attached to a vertical pole of height 18 m is 24 m long and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taut?

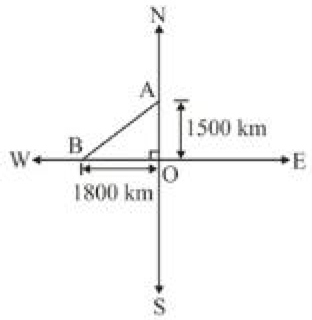
Solution:



Let OB be the pole and AB be the wire.
Therefore, by Pythagoras theorem we have:
AB2=OB2+OA2
⇒242=182+OA2
⇒OA2=576-324
⇒OA=252=6×6×7=67
Therefore, distance from base =67 m

Q.6. An aeroplane leaves an airport and flies due north at a speed of 1000 km per hour. At the same time another aeroplane leaves the same airport and flies due west at a speed of 1200 km per hour. How far apart will be the two planes after 112 hours?

Solution:



Distance traveled by the plane flying towards north in 112 hrs

=1,000×112=1,500 km

Distance traveled by the plane flying towards west in 112 hrs =1,200×112=1,800 km

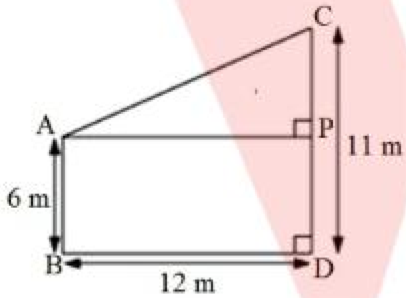
Let these distances are represented by OA and OB respectively.

Now applying Pythagoras theorem

Distance between these planes after 112 hrs, AB=OA2+OB2 =1,5002+1,8002=2250000+3240000 =5490000=9×610000=30061 So, distance between these planes will be 30061 km. after 112 hrs.

Q.7. Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between the feet of the poles is 12 m, find the distance between their tops.

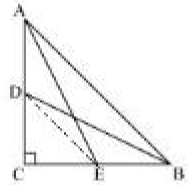
Solution:



Let CD and AB be the poles of height 11 m and 6 m.
Therefore, CP=11-6=5 m
From the figure we may observe that AP=12 m
In △APC, by applying Pythagoras theorem we get:
AP2+PC2=AC2
⇒122+52=AC2
⇒AC2=144+25=169
⇒AC=13
Therefore, the distance between their tops =13 m.

Q.8. D and E are points on the sides CA and CB respectively of a triangle ABC right-angled at C. Prove that AE2+BD2=AB2+DE2.

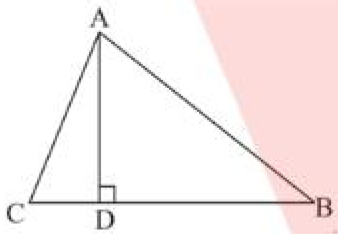
Solution:



In △ACE,
AC2+CE2=AE2…i
In △BCD,
BC2+CD2=BD2…ii
Adding i and (ii) we get:
AC2+CE2+BC2+CD2=AE2+BD2…(iii)
⇒CD2+CE2+AC2+BC2=AE2+BD2

In △CDE,
DE2=CD2+CE2
In △ABC,
AB2=AC2+CB2
Adding both equations and comparing with equation (iii), we get:
DE2+AB2=AE2+BD2

Q.9. The perpendicular from A on side BC of a △ABC intersects BC at D such that DB=3CD. Prove that 2AB2=2AC2+BC2.

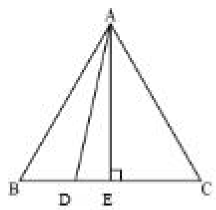


Solution: Given that, 3DC=DB.
DC=BC4DB:DC=3:1…1
and DB=3BC4…2
In △ACD,
AC2=AD2+DC2
AD2=AC2-DC2…3

In △ABD,
AB2=AD2+DB2
AD2=AB2-DB2…4
From equation (3) and 4
AC2-DC2=AB2-DB2
Since, given that 3DC=DB
AC2-BC42=AB2-3BC42 (from 1and 2)
⇒AC2-BC216=AB2-9BC216
⇒16AC2-BC2=16AB2-9BC2
⇒16AB2-16AC2=8BC2
⇒2AB2=2AC2+BC2

Q.10. In an equilateral triangle ABC, D is a point on side BC such that BD=13BC. Prove that 9AD2=7AB2.

Solution:



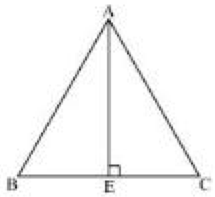
Let side of equilateral triangle be a and AE be the altitude of ΔABC

So, BE=EC=BC2=a2

and, AE=a32 Given that BD=13BC=a3 So, DE=BE-BD=a2-a3=a6 Now, in ΔADE, by applying Pythagoras theorem AD2=AE2+DE2 ⇒AD2=a322+a62 =3a24+a236=28a236 or, 9AD2=7AB2

[illegible] ve that three times the [illegible] de is equal to four times [illegible]

Solution:

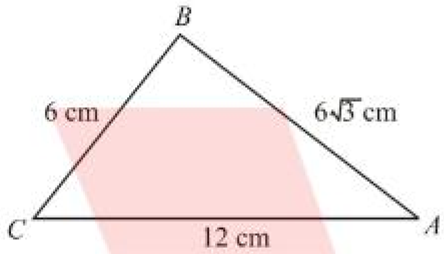


Let side of equilateral triangle be a. And AE be the altitude of ΔABC

So, BE=EC=BC2=a2 Now in ΔABE by applying Pythagoras theorem AB2=AE2+BE2 ⇒a2=AE2+a22 ⇒AE2=a2-a24 ⇒AE2=3a24 ⇒4AE2=3a2 or, 4AE2=3×square of one side.

Q.12. In ΔABC, AB=63 cm, AC=12 cm and BC=6 cm. The angle B is 120o60o90o

Solution:



Given that AB=63 cm, AC=12 cm and BC=6 cm

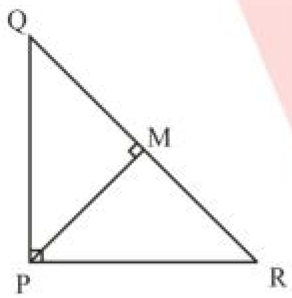
We may observe that

AB2=108, AC2=144 and BC2=36, AB2+BC2=AC2 Thus, the given ΔABC is satisfying Pythagoras theorem. Therefore, the triangle is a right angle triangle right-angled at B Therefore, ∠B=90°.

45o

Q.13. PQR is a triangle right-angled at P and M is a point on QR such that PM⊥QR. Show that PM2=QM.MR.

Solution:



Let ∠MPR=x

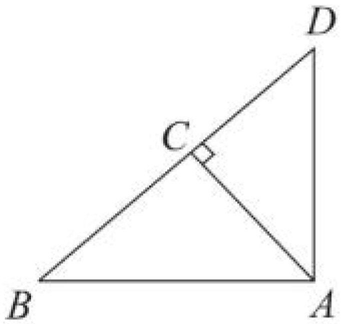
In ΔMPR ∠MRP=180o-90o-x ⇒∠MRP=90o-x Similarly in ΔMPQ ∠MPQ=90o-∠MPR=90o-x ∠MQP=180o-90o-90o-x ⇒∠MQP=x

Now in ΔMPQ and ΔMRP, we may observe that

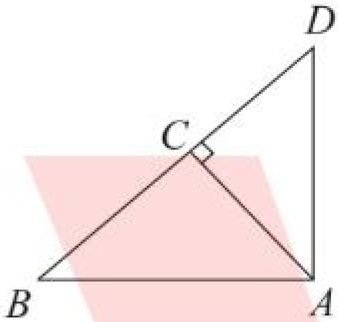
∠MPQ=∠MRP

∠PMQ=∠RMP ∠MQP=∠MPR Hence, by AAA rule, ΔMPQ~ΔMRP Hence, QMPM=MPMR ⇒PM2=QM.MR

Q.14. In the figure given below, ABD is a triangle right-angled at A and AC⊥BD. Show that AB2=BC·BD

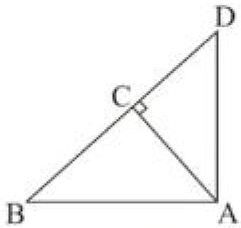


Solution:



In △ABC and △ABD,
∠CBA=∠DBA (common angles)
∠BCA=∠BAD=90°
∠BAC=∠BDA (remaining angle)
Therefore, △ABC∼△ABD (by AAA)
∴ ABBD=BCAB
⇒ AB2=BC·BD

Q.15. In the figure, ABD is a triangle right-angled at A and AC⊥BD. Show that AC2=BC·DC



Solution: Let ∠CAB=x

In ΔCBA

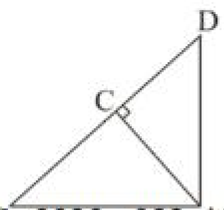
∠CBA=180o-90o-x ∠CBA=90o-x Similarly in ΔCAD ∠CAD=90o-∠CAB=90o-x ∠CDA=180o-90o-(90o-x) ∠CDA=x.

Now in ΔCBA and ΔCAD,we may observe that

∠CBA=∠CAD

∠CAB=∠CDA ∠ACB=∠DCA=90o Therefore ΔCBA∼ΔCAD (by AAA rule) Therefore, ACDC=BCAC ⇒AC2=DC×BC.

Q.16. In Fig. ABD is a triangle right-angled at A and AC⊥BD. Show that AD2=BD·CD

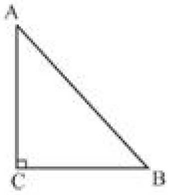


Solution:

In ΔDCA & ΔDAB
∠DCA=∠DAB=90°
∠CDA=∠ADB (Common angle)
∠DAC=∠DBA (remaining angle)
ΔDCA~ΔDAB (by AAA property)
Therefore, DCDA=DADB
⇒AD2=BD×CD

Q.17. ABC is an isosceles triangle right-angled at C. Prove that AB2=2AC2.

Solution:

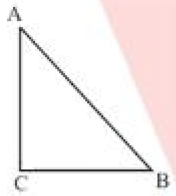


Given that ΔABC is an isosceles triangle.

Therefore, AC=CB Applying Pythagoras theorem in ΔABC (i.e. right-angled at point C) AC2+CB2=AB2
⇒2AC2=AB2 (as AC=CB)

Q.18. ABC is an isosceles triangle with AC=BC. If AB2=2AC2, prove that ABC is a right triangle.

Solution:

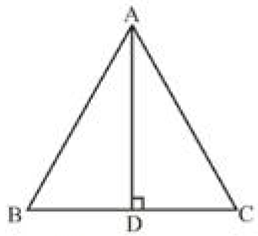


Given that AB2=2AC2

⇒AB2=AC2+AC2 ⇒AB2=AC2+BC2 (as AC=BC) Therefore, by converse of Pythagoras theorem, given triangle is a right-angled triangle.

Q.19. ABC is an equilateral triangle of side 2a. Find each of its altitudes.

Solution:



Let AD be the altitude in given equilateral ΔABC.

We know that altitude bisects the opposite side. So, BD=DC=a in ΔADB ∠ADB=90o

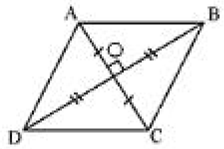
Now applying Pythagoras theorem

AD2+BD2=AB2

⇒AD2+a2=2a2 ⇒AD2+a2=4a2 ⇒AD2=3a2 ⇒AD=a3 Since in an equilateral triangle, all the altitudes are equal in length. So, length of each altitude will be 3a

Q.20. Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.

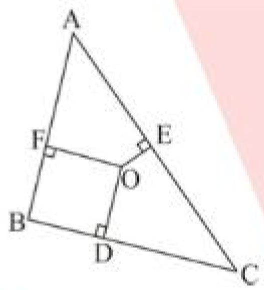
Solution:



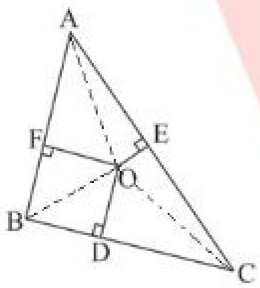
In ΔAOB,ΔBOC,ΔCOD,ΔAOD
Applying Pythagoras theorem
AB2=AO2+OB2
BC2=BO2+OC2
CD2=CO2+OD2
AD2=AO2+OD2

Adding all these equations,
AB2+BC2+CD2+AD2=2AO2+OB2+OC2+OD2
=2AC22+BD22+AC22+BD22 (diagonals bisect each other.)
=2AC22+BD22
=AC2+BD2

Q.21. In Fig. 6.54, O is a point in the interior of a triangle ABC, OD⊥BC, OE⊥AC and OF⊥AB. Show that OA2+OB2+OC2-OD2-OE2-OF2=AF2+BD2+CE2,



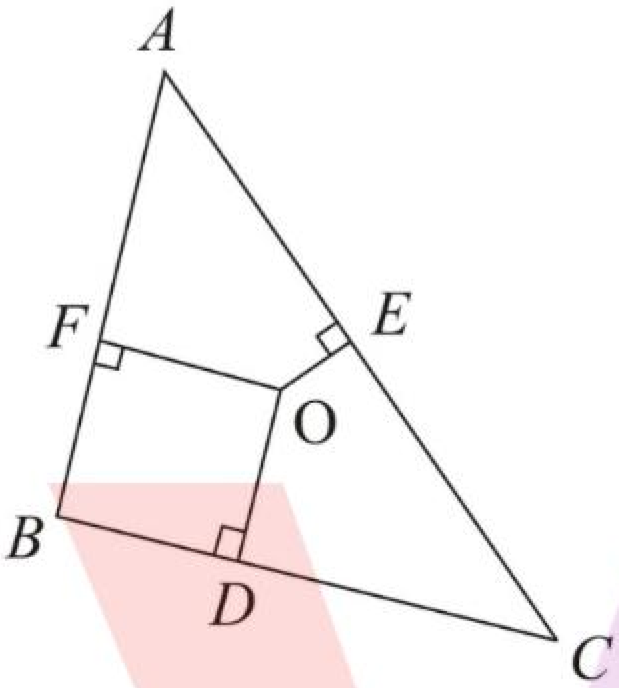
Solution:



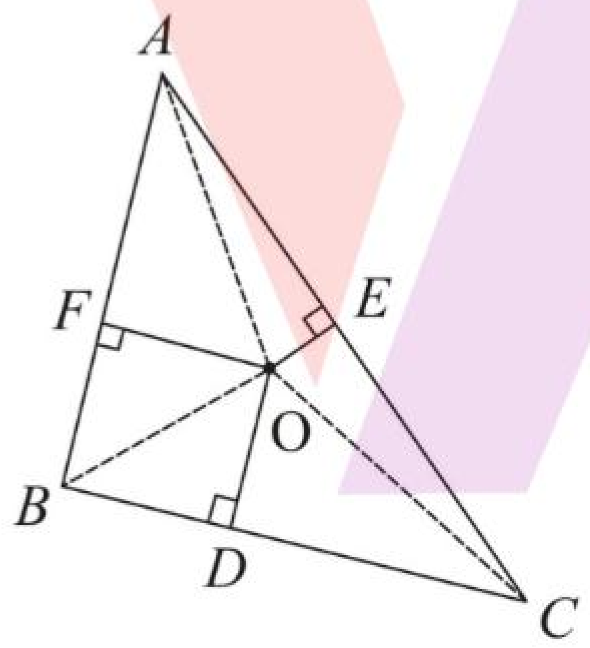
In ΔAOF
Applying Pythagoras theorem
OA2=OF2+AF2
Similarly in ΔBOD
OB2=OD2+BD2
similarly in ΔCOE
OC2=OE2+EC2
Adding these equations
OA2+OB2+OC2=OF2+AF2+OD2+BD2+OE2+EC2
⇒OA2+OB2+OC2-OD2-OE2-OF2=AF2+BD2+EC2

Q.22. In Fig. O is a point in the interior of a triangle ABC, OD⊥BC, OE⊥AC and OF⊥AB. Show that

AF2+BD2+CE2=AE2+CD2+BF2.



Solution:

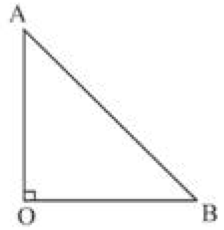


In ΔAOF
Applying Pythagoras theorem
OA2=OF2+AF2
Similarly in ΔBOD
OB2=OD2+BD2
similarly in ΔCOE
OC2=OE2+EC2
Adding these equations
OA2+OB2+OC2=OF2+AF2+OD2+BD2+OE2+EC2
⇒OA2+OB2+OC2-OD2-OE2-OF2=AF2+BD2+EC2

From above result
AF2+BD2+EC2=OA2-OE2+OC2-OD2+OB2-OF2
Therefore, AF2+BD2+EC2=AE2+CD2+BF2

Q.23. A ladder 10 m long reaches a window 8 m above the ground. Find the distance of the foot of the ladder from base of the wall.

Solution:



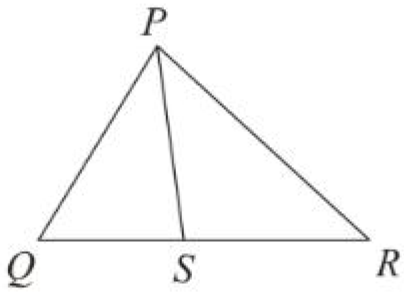
Let OA be the wall and AB be the ladder.

Therefore by Pythagoras theorem,
$AB2=OA2+BO2$
$\Rightarrow 102=82+OB2$
$\Rightarrow 100=64+OB2$
$\Rightarrow OB2=36$
$\Rightarrow OB=6$

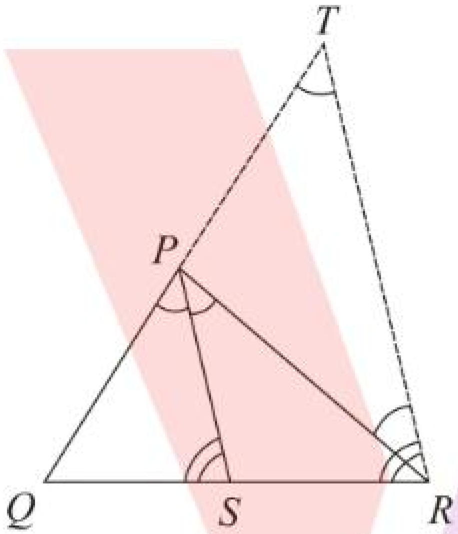
Therefore, distance of foot of ladder from of the wall =6 m

Exercise 6.6

Q.1. In the given figure, PS is the bisector of ∠QPR of ΔPQR. Prove that QSSR=PQPR.



Solution:



Given that, PS is angle bisector of ∠QPR.

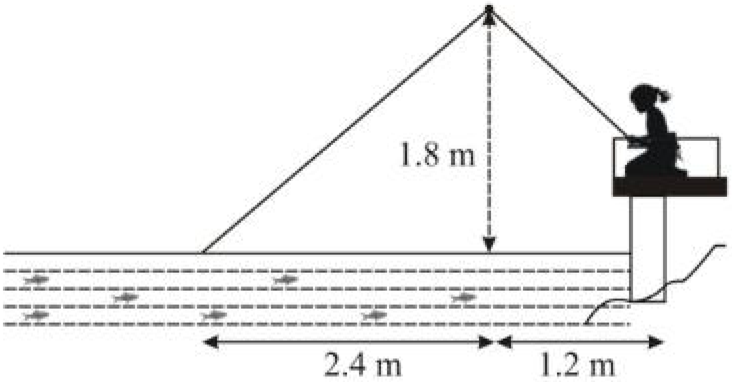
Construct a line RT parallel to SP which meets QP produced at T. ∠QPS=∠SPR ......(1) ∠SPR=∠PRT (As PS||TR, alternate interior angles) ....(2) ∠QPS=∠QTR (As PS||TR, corresponding angles) .....(3) Using these equations, we may find ∠PRT=∠QTR from (2) and (3) So, PT=PR (Since ΔPTR is isosceles triangle)

Now in ΔQPS and ΔQTR, ∠QSP=∠QRT (As PS||TR)

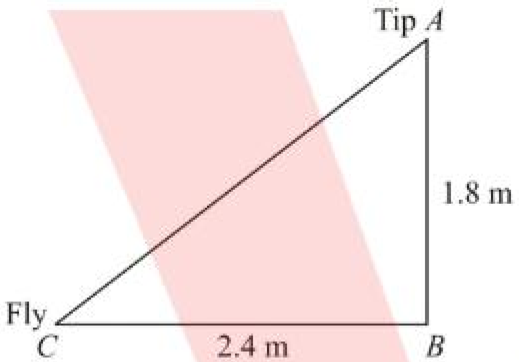
∠QPS=∠QTR (As PS || TR)

∠Q is common. ΔQPS~ΔQTR (by AAA property) So, QRQS=QTQP ⇒QRQS-1=QTQP-1 ⇒SRQS=PTQP ⇒QSSR=QPPT ⇒QSSR=PQPR

Q.2. Nazima is fly fishing in a stream. The tip of her fishing rod is 1.8 m above the surface of the water and the fly at the end of the string rests on the water 3.6 m away and 2.4 m from a point directly under the tip of the rod. Assuming that her string (from the tip of her rod to the fly) is taut, how much string does she have out (see the given figure)? If she pulls in the string at the rate of 5 cm per second, what will be the horizontal distance of the fly from her after12 seconds?



Solution:

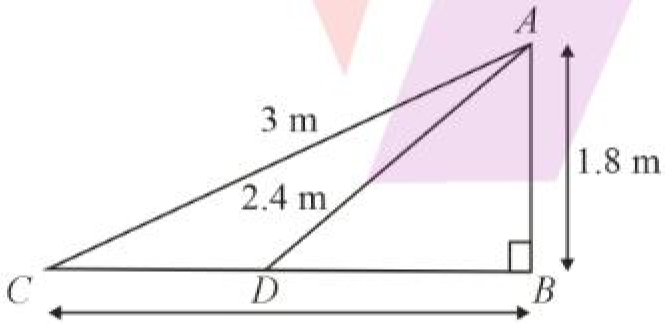


Let AB be the height of tip of fishing rod from water surface and BC be the horizontal distance of fly from the tip of fishing rod.

Then, AC is the length of string. AC can be found by applying Pythagoras theorem in ΔABC. $AC^2=AB^2+BC^2$ $AC^2=1.8^2+2.4^2$ $AC^2=3.24+5.76$ $AC^2=9.00$ Thus, length of string out is 3 m.

Now, she pulls string at rate of 5 cm per second.

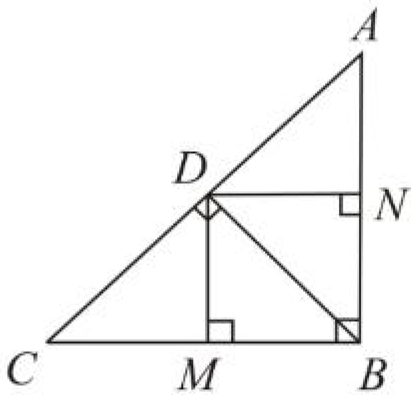
So, string pulled in 12 second=12×5=60 cm=0.6 m.



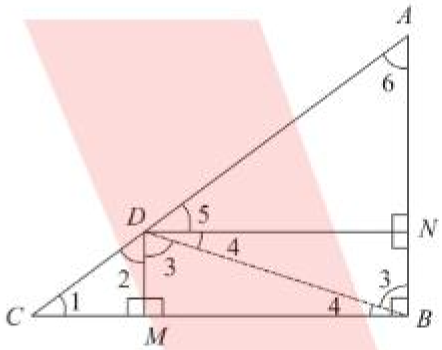
After 12 seconds, let us assume the fly to be at point D.

Length of string out after 12 second is AD. AD=AC- string pulled by Nazima in 12 second =3.00-0.6 =2.4 m In ΔADB, $AB^2+BD^2=AD^2 \Rightarrow 1.8^2+BD^2=2.4^2 \Rightarrow BD^2=5.76-3.24=2.52 \Rightarrow BD=1.587$ m Horizontal distance of fly =BD+1.2 =1.587+1.2 =2.787 =2.79 m

Q.3. In the figure, D is a point on hypotenuse AC of ΔABC, such that BD⊥AC, DM⊥BC and DN⊥AB. Prove that: DM2=DN.MC



Solution: Let us join DB.



DN||CB, DM||AB

Therefore, DNBM is a parallelogram.

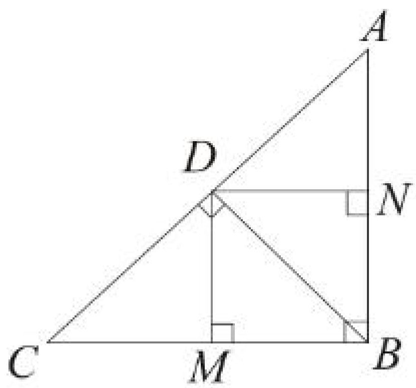
Since, ∠B is 90°, therefore, DNBM is a rectangle. Hence, DN=MB, DM=NB and ∠CDB=∠ADB=90° ∠2+∠3=90o…(1) In ΔCDM ∠1+∠2+∠DMC=180° ∠1+∠2=90°…(2) In ΔDMB ∠3+∠DMB+∠4=180° ∠3+∠4=90°…(3)

From equation (1) and (2)

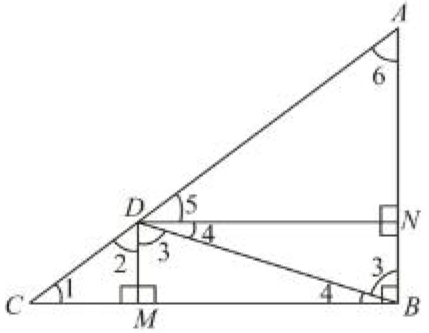
∠1=∠3

From equation (1) and (3) ∠2=∠4 So, ΔBDM~ΔDCM BMDM=DMMC ⇒DNDM=DMMC ⇒DM2=DN.MC Hence, proved.

Q.4. In the figure, D is a point on hypotenuse AC of ΔABC, such that BD⊥AC,DM⊥BC and DN⊥AB. Prove that: DN2=DM·AN



Solution: Let us join DB.



DN||CB, DM||AB

Therefore, DNBM is a parallelogram.

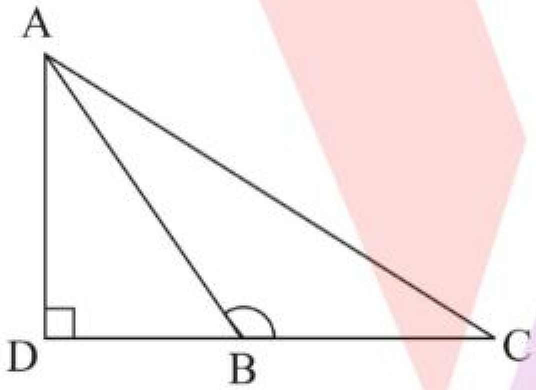
Since, ∠B is 90°. Therefore, DNBM is a rectangle. So, DN=MB, DM=NB and ∠CDB=∠ADB=90° ∠4+∠5=90o…(1) In Δ ADN ∠5+∠6=90°…(2)

From equation (1) and (2)

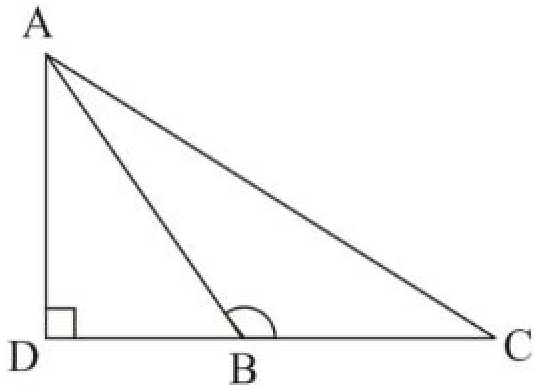
∠4=∠6

and ∠DNA=∠DNB=90° So, ΔADN~ΔBDN DNBN=ANDN ⇒DNDM=ANDN (As BN=DM) ⇒DN2=DM×AN Hence, proved.

Q.5. ABC is a triangle in which ∠ABC>90o and AD⊥CB produced. Prove that AC2=AB2+BC2+2BC·BD.



Solution:



In ΔADB, applying Pythagoras theorem
$AB2=AD2+DB2$…(1)

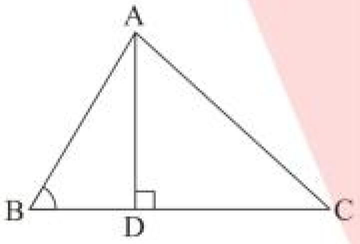
In ΔACD, applying Pythagoras theorem
$AC2=AD2+DC2$

$\Rightarrow AC2=AD2+DB+BC2$

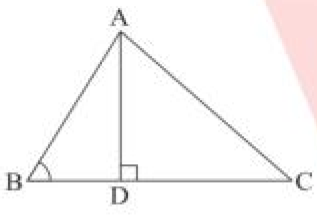
$\Rightarrow AC2=AD2+DB2+BC2+2DB\times BC$

Now using equation (1)
$AC2=AB2+BC2+2BC.BD$

Q.6. In the figure given below ABC is a triangle in which $\angle ABC<90o$ and $AD\perp BC$. Prove that $AC2=AB2+BC2-2BC.BD$.



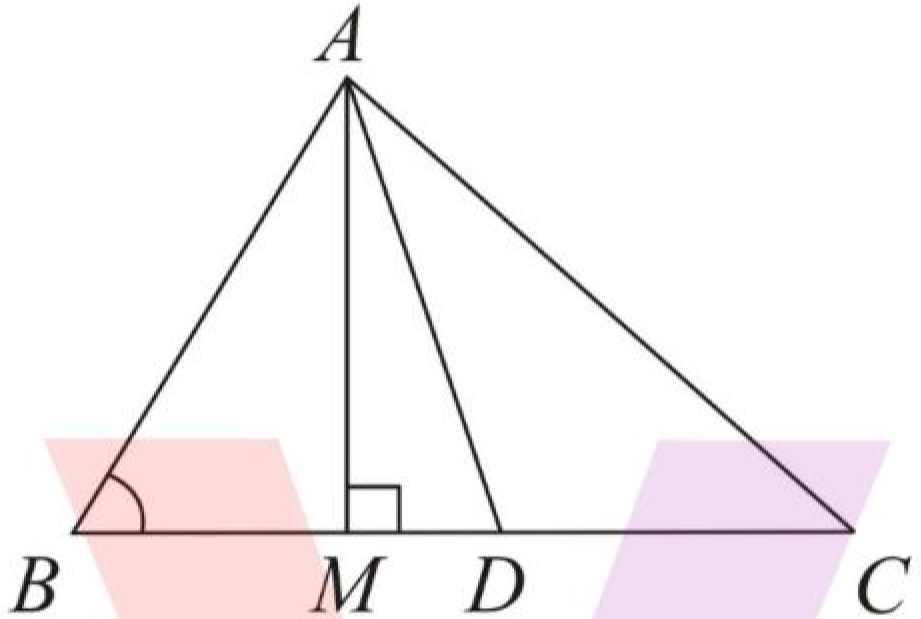
Solution:



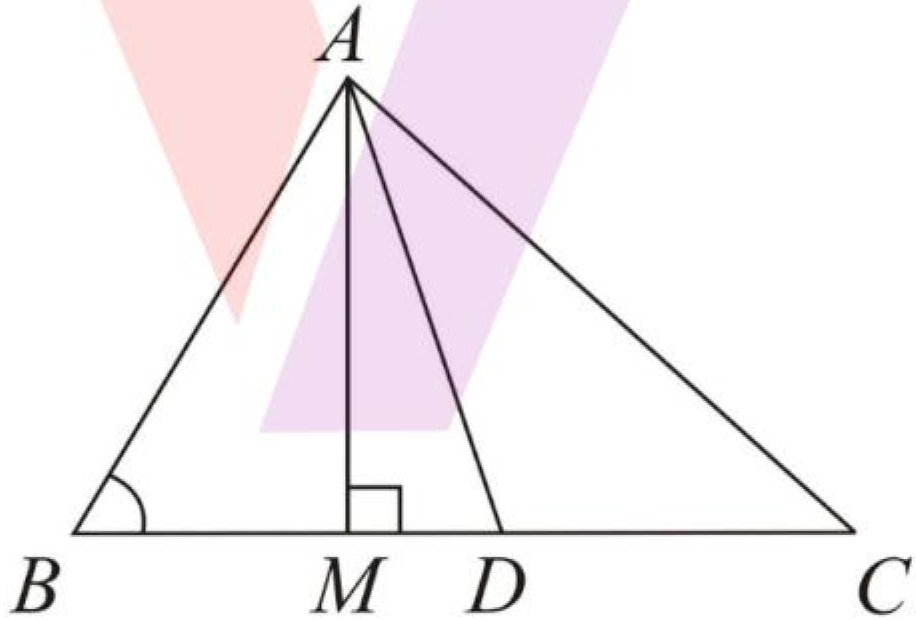
In $\triangle ADB$, applying Pythagoras theorem we get:
$AD2+DB2=AB2$
$\Rightarrow AD2=AB2-DB2$…(1)
In $\triangle ADC$, applying Pythagoras theorem we get:
$AD2+DC2=AC2$ ....(2)

Now using equation (1), we get:
$AB2-BD2+DC2=AC2$
$\Rightarrow AB2-BD2+BC-BD2=AC2$
$\Rightarrow AC2=AB2-BD2+BC2+BD2-2BC.BD$
Hence, $AC2=AB2+BC2-2BC.BD$.

Q.7. In the figure, AD is a median of a triangle ABC and AM⊥BC. Prove that :
AC2=AD2+BC·DM+BC22



Solution:



In ΔAMD, by using Pythagoras theorem,

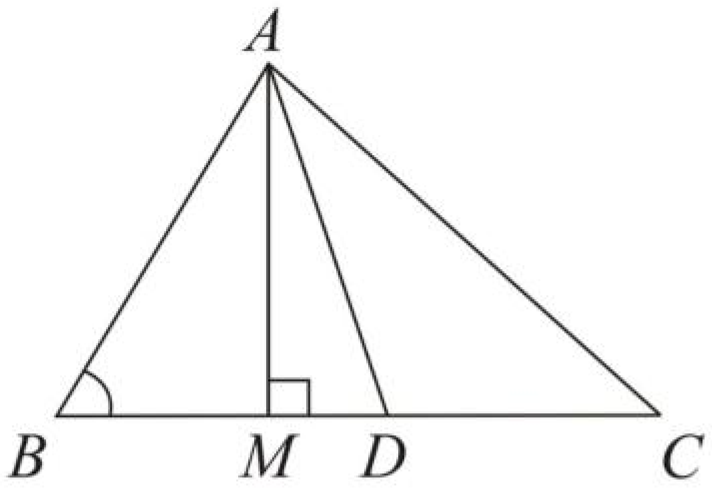
AM2+MD2=AD2…(1) In ΔAMC AM2+MC2=AC2…(2) ⇒AM2+MD+DC2=AC2
⇒AM2+MD2+DC2+2MD.DC=AC2 Using equation (1) we get, AD2+DC2+2MD.DC=AC2

Now using the result, DC=BC2

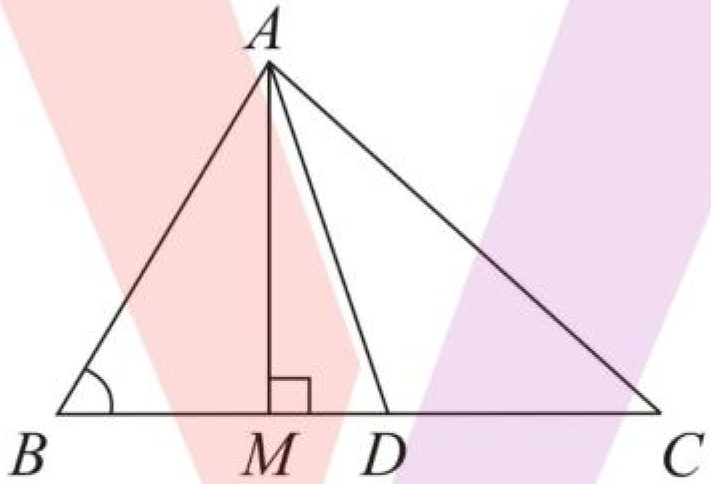
AD2+BC22+2MD.BC2=AC2

⇒AD2+BC22+MD×BC=AC2 Hence, AC2=AD2+BC·DM+BC22

Q.8. In the given figure, AD is a median of a triangle ABC and AM⊥BC. Prove that AB2=AD2-BC·MD+BC22



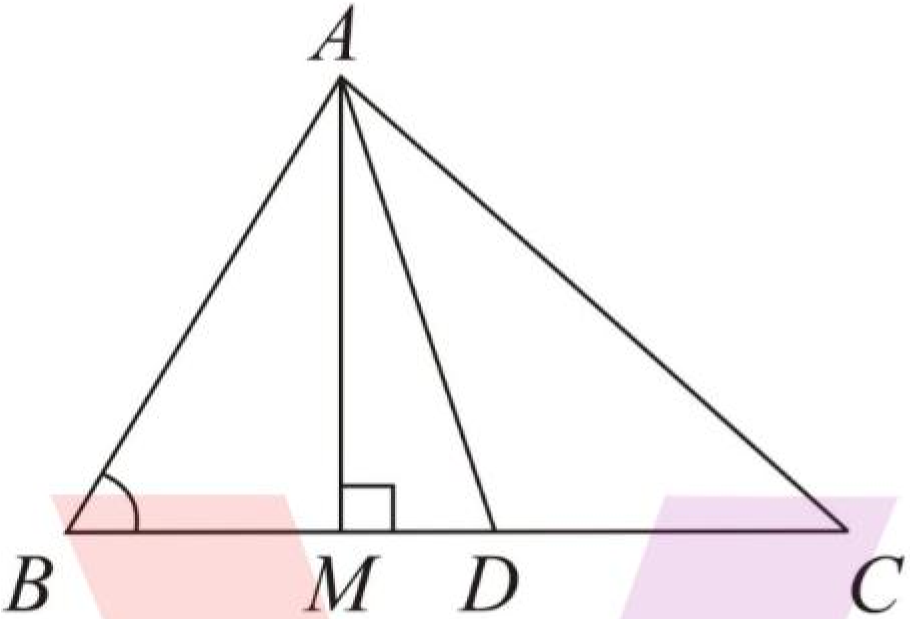
Solution:



In ΔABM, applying Pythagoras theorem
AB2=AM2+MB2

=AD2-DM2+MB2 =AD2-DM2+BD-MD2 =AD2-DM2+BD2+MD2-2BD.MD
=AD2+BD2-2BD.MD =AD2+BC22-2BC2×MD
AB2=AD2+BC22-BC×MD Hence, AB2=AD2-BC·DM+BC22

Q.9. In the figure, AD is the median of triangle ABC and AM⊥BC. Prove that:
AC2+AB2=2AD2+12BC2



Solution: In ΔAMB, by Pythagoras theorem,

AM2+MB2=AB2…(1)

In ΔAMC
AM2+MC2=AC2…(2)

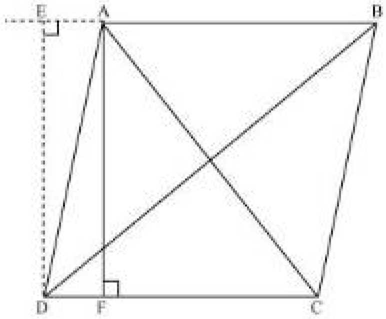
Adding equations (1) and (2)
2AM2+MB2+MC2=AB2+AC2

⇒2AM2+BD-DM2+MD+DC2=AB2+AC2

⇒2AM2+BD2+DM2-2BD.DM+MD2+DC2+2MD.DC=AB2+AC2
⇒2AM2+2MD2+BD2+DC2+2MD-BD+DC=AB2+AC2 ⇒2AM2+MD2+BC22+BC22+2MD-BC2+BC2=AB2+AC2
⇒2AD2+BC22=AB2+AC2 Hence, AC2+AB2=2AD2+12BC2

Q.10. Prove that the sum of the squares of the diagonals of parallelogram is equal to the sum of the squares of its sides.

Solution:



Let ABCD be a parallelogram

Let us draw perpendicular DE on extended side BA and AF on side DC. In ΔDEA DE2+EA2=DA2…i In ΔDEB DE2+EB2=DB2 ⇒DE2+EA+AB2=DB2 ⇒DE2+EA2+AB2+2EA.AB=DB2 ⇒DA2+AB2+2EA.AB=DB2…(ii)

In ΔADF

AD2=AF2+FD2

In ΔAFC AC2=AF2+FC2 =AF2+DC-FD2 =AF2+DC2+FD2-2DC.FD =AF2+FD2+DC2-2DC.FD ⇒AC2=AD2+DC2-2DC.FD…(iii)

Since ABCD is a parallelogram

AB=CD and BC=AD

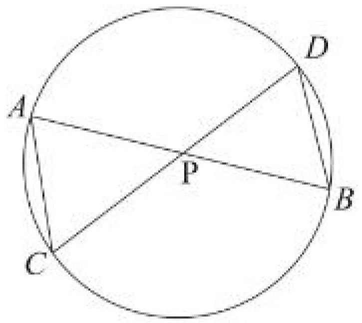
In ΔDEA and ΔADF ∠DEA=∠AFD ∠EAD=∠FDAEA∥DF ∠EDA=∠FADAF∥ED AD is common in both triangles. Since, respective angles are same and respective sides are same ΔDEA≅ΔAFD So, EA=DF

Adding equation (ii) and (iii)

DA2+AB2+2EA.AB+AD2+DC2-2DC.FD=DB2+AC2

⇒DA2+AB2+AD2+DC2+2EA.AB-2DC.FD=DB2+AC2 ⇒BC2+AB2+AD2+DC2+2EA.AB-2AB.EA=DB2+AC2 ⇒AB2+BC2+CD2+DA2=AC2+BD2

Q.11. In Fig. 6.61, two chords AB and CD intersect each other at the point P. Prove that △APC~△DPB



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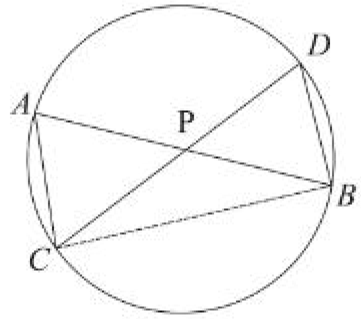
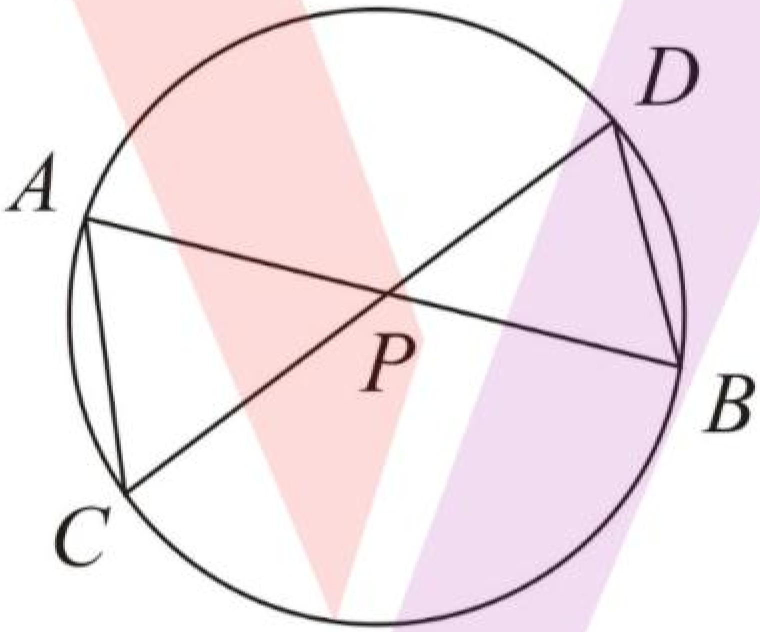


In $\triangle APC$ and $\triangle DPB$

$\angle A=\angle D$ and $\angle C=\angle B$ (Angle on same segment)

Therefore, $\triangle APC \sim \triangle DPB$ (AA criteria)

Q.12. In figure two chords AB and CD intersect each other at the point P. Prove that $AP \cdot PB = CP \cdot DP$.



Solution:

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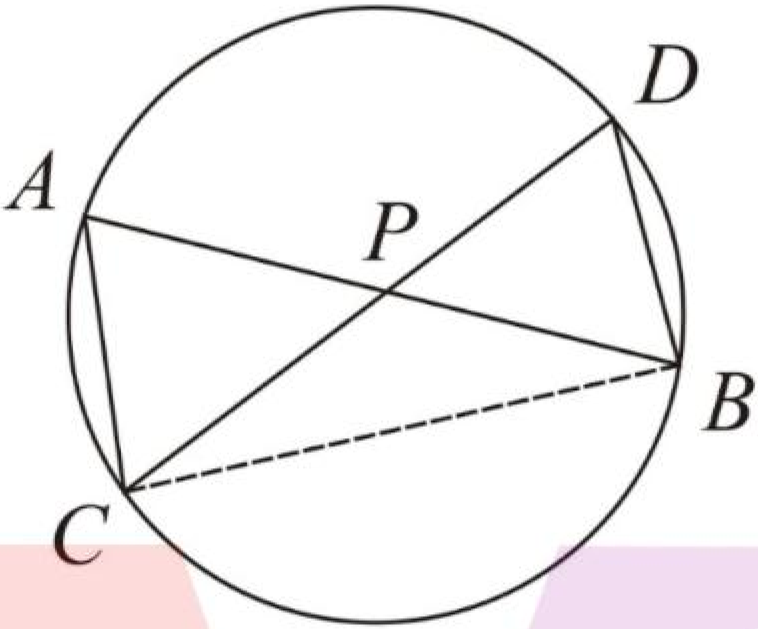
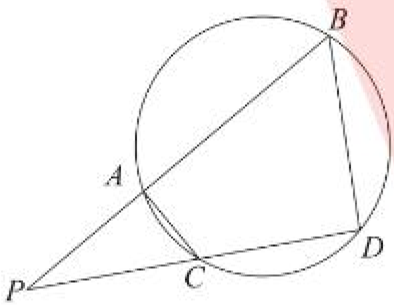


In △APC and △DPB

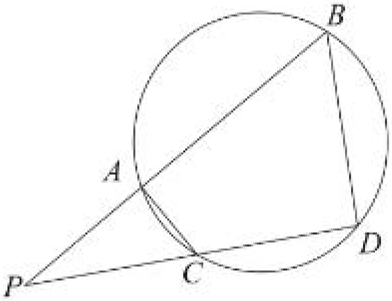
∠A=∠D and ∠C=∠B (Angle on same segment)

Therefore, △APC~△DPB (AA criteria)
We know that corresponding sides of similar triangles are proportional
∴APDP=PCPB=CABD
⇒APDP=PCPB
∴ AP.PB=PC.DP

Q.13. In the figure, two chords AB and CD of a circle intersect each other at the point P (when produced) outside the circle. Prove that △PAC~△PDB.



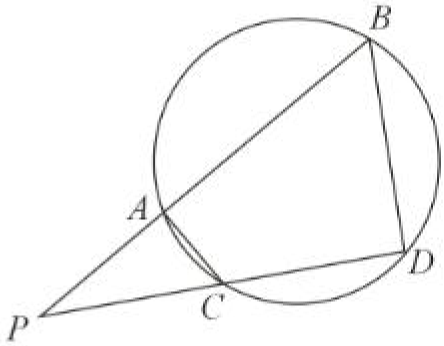
Solution:



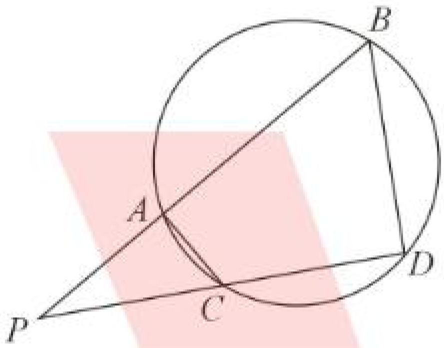
In △PAC and △PDB

∠APC=∠DPB (Common angle) ∠ACP=∠DBP (Exterior angle of a cyclic quadrilateral is equal to the opposite interior angle.) Therefore, △PAC~△PDB (AA criteria)

Q.14. In the figure, two chords AB and CD of a circle intersect each other at the point P (when p[illegible]
PA.PB=PC.PD



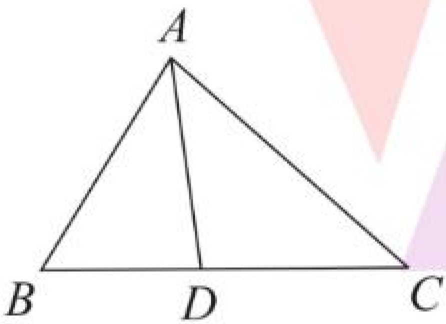
Solution:



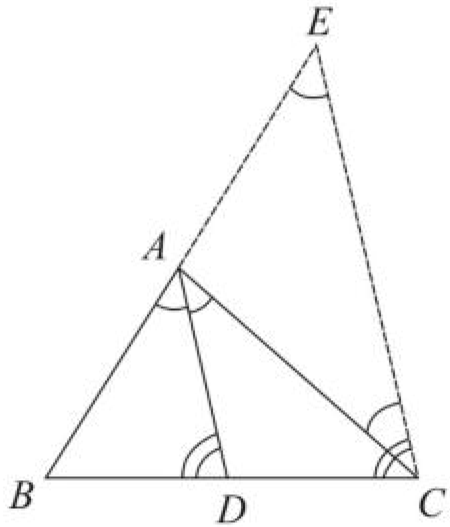
In △APC and △DPB

∠APC=∠DPB (Common angle) ∠ACP=∠DBP (Exterior angles of cyclic quadrilateral)
Therefore, △APC~△DPB (AA criteria) We know that corresponding sides of similar triangles are proportional.
PAPD=ACDB=PCPB
⇒PAPD=PCPB ∴PA.PB=PC.PD

Q.15. D is a point on side BC of ΔABC such that BDCD=ABAC. Prove that AD is the bisector of ∠BAC.



Solution:



Construct a line CE parallel to DA which meets BA produced at E.

Therefore, ∠BAD=∠BEC (Corresponding angles)......(1) ∠DAC=∠ACE (Alternate angles)......(2) In ΔDBA and ΔCBE, BDCD=ABAC (Given) ......(3) BDCD=BAAE (Basic proportionality theorem) ......(4) From (3) and (4), AE=AC Therefore, ∠ACE=∠BEC......(5) So, from (1), (2) and (5) ⇒∠BAD=∠DAC Therefore, AD is angle bisector of ∠BAC.