CBSE NCERT Solutions for Class 10 Mathematics Chapter 5

Back of Chapter Questions

1. In which of the following situations, does the list of numbers involved make an arithmetic progression, and why?

(i) The taxi fare after each km when the fare is ₹ 15 for the first km and ₹ 8 for each additional km.

(ii) The amount of air present in a cylinder when a vacuum pump removes $\frac{1}{4}$ of the air remaining in the cylinder at a time.

(iii) The cost of digging a well after every metre of digging, when it costs ₹ 150 for the first metre and rises by ₹ 50 for each subsequent metre.

(iv) The amount of money in the account every year, when ₹ 10000 is deposited at compound interest at 8 % per annum.

Solution:

(i) From the given data we see that

Taxi fare for 1^{st} km $= 15$

Taxi fare for first 2 kms $= 15 + 8 = 23$

Taxi fare for first 3 kms $= 23 + 8 = 31$

Taxi fare for first 4 kms $= 31 + 8 = 39$

Clearly $15, 23, 31, 39 \ldots \ldots$ forms an AP since every term is 8 more than the preceding term.

i.e., $(23 - 15) = (31 - 23) = (39 - 31) = 8$

(ii) Let the initial volume of air in a cylinder be V. In each stroke, the vacuum pump removes $\frac{1}{4}$ of air remaining in the cylinder at a time. In other words, after every stroke, only $1 - \frac{1}{4} = \frac{3}{4}$ the part of air will remain.

Air left in the cylinder after first removal = $\left(\frac{3}{4}\right) V$

Air again removed = $\frac{1}{4}\left(\frac{3}{4}\right) V$

Air left in the cylinder after second removal = $\left(\frac{3}{4}\right) V - \frac{1}{4}\left(\frac{3}{4}\right) V = \left(\frac{3}{4}\right)^2 V$

Air again removed $= \frac{1}{4}\left(\frac{3}{4}\right)^2 V$

Air left in the cylinder after third removal = $\left(\frac{3}{4}\right)^2 V - \frac{1}{4}\left(\frac{3}{4}\right)^2 V = \left(\frac{3}{4}\right)^3 V$

Therefore, volume of air left in cylinder follows the sequence:
$V, \left(\frac{3}{4}\right) V, \left(\frac{3}{4}\right)^2 V, \left(\frac{3}{4}\right)^3 V \ldots$

Clearly, the consecutive terms of this sequence do not have constant different between them, Thus, this is not an AP

(iii) Cost of digging for first metre = ₹ 150

Cost of digging for 2m metre = $150 + 50 =$ ₹ 200

Cost of digging for 3m metre = $200 + 50 =$ ₹ 250

Cost of digging for 4m metre = $250 + 50 =$ ₹ 300

Clearly, $150, 200, 250, 300$ .... forms an AP because every term is 50 more than its preceding term.

(iv) We know that if Principal amount (P) of ₹ 10000 is deposited at rate (r) at 8 % compound interest per annum for n years, our money (in ₹) will be

$$10000\left(1+\frac{8}{100}\right), 10000\left(1+\frac{8}{100}\right)^2, 10000\left(1+\frac{8}{100}\right)^3,$$

$$10000\left(1+\frac{8}{100}\right)^4, \ldots$$

Clearly, adjacent terms of this sequence, do not have the same difference between them. Therefore, this is not an AP.

2. Write first four terms of the AP, when the first term a and the common difference d are given as follows:

(i) $a = 10, d = 10$

(ii) $a = -2, d = 0$

(iii) $a = 4, d = -3$

(iv) $a = -1, d = \frac{1}{2}$

(v) $a = -1.25, d = -0.25$

Solution:

(i) Given: $a = 10, d = 10$

First term = $a_1 = a = 10$

Second term = $a_2 = a_1 + d = 10 + 10 = 20$

Third term = $a_3 = a_2 + d = 20 + 10 = 30$

Fourth term = $a_4 = a_3 + d = 30 + 10 = 40$

Hence, the first four terms of this AP are $10, 20, 30$, and 40.

(ii) Given: $a = -2, d = 0$

First term = $a_1 = a = -2$

Second term = $a_2 = a_1 + d = -2 + 0 = -2$

Third term = $a_3 = a_2 + d = -2 + 0 = -2$

Fourth term = $a_4 = a_3 + d = -2 + 0 = -2$

Hence, the first four terms of this AP will be $-2, -2, -2$, and -2.

(iii) Given: $a = 4, d = -3$

First term = $a_1 = a = 4$

Second term = $a_2 = a_1 + d = 4 - 3 = 1$

Third term = $a_3 = a_2 + d = 1 - 3 = -2$

Fourth term = $a_4 = a_3 + d = -2 - 3 = -5$

Hence, the first four terms of this AP will be $4, 1, -2$, and -5.

(iv) Given: $a = -1, d = \frac{1}{2}$

First term = $a_1 = a = -1$

Second term = $a_2 = a_1 + d = -1 + \frac{1}{2} = -\frac{1}{2}$

Third term = $a_3 = a_2 + d = -\frac{1}{2} + \frac{1}{2} = 0$

Fourth term = $a_4 = a_3 + d = 0 + \frac{1}{2} = \frac{1}{2}$

Hence, the first four terms of this AP will be $-1, -\frac{1}{2}, 0$, and $\frac{1}{2}$.

(v) Given: $a = -1.25, d = -0.25$

First term = $a_1 = a = -1.25$

Second term = $a_2 = a_1 + d = -1.25 - 0.25 = -1.50$

Third term = $a_3 = a_2 + d = -1.50 - 0.25 = -1.75$

Fourth term = $a_4 = a_3 + d = -1.75 - 0.25 = -2.00$

Hence, the first four terms of this AP will be $-1.25, -1.50, -1.75$, and -2.00.

3. For the following APs, write the first term and the common difference:

(i) $3, 1, -1, -3, \ldots$

(ii) $-5, -1, 3, 7, \ldots$

(iii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}, \ldots$

(iv) $0.6, 1.7, 2.8, 3.9, \ldots$

Solution:

(i) $3, 1, -1, -3 \ldots$

Here, first term, $a = 3$

Common difference, $d =$ Second term$-$First term

$= 1 - 3 = -2$

Thus, $a = 3$ *and* $d = -2$

(ii) $-5, -1, 3, 7 \ldots$

Here, first term, $a = -5$

Common difference, $d =$ Second term$-$First term

$= (-1) - (-5) = -1 + 5 = 4$

Thus, $a = -5$ *and* $d = 4$

(iii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}, \ldots$

Here, first term, $a = \frac{1}{3}$

Common difference, $d =$ Second term$-$First term

$$= \frac{5}{3} - \frac{1}{3} = \frac{4}{3}$$

Thus, $a = \frac{1}{3}$ *and* $d = \frac{4}{3}$

(iv) $0.6, 1.7, 2.8, 3.9 \ldots$

Here, first term, $a = 0.6$

Common difference, $d =$ Second term$-$First term

$= 1.7 - 0.6$

$= 1.1$

Thus, $a = 0.6$ *and* $d = 1.1$

4. Which of the following are APs ? If they form an AP, find the common difference d and write three more terms.

(i) $2, 4, 8, 16, \ldots$

(ii) $2, \frac{5}{2}, 3, \frac{7}{2}, \ldots$

(iii) $-1.2, -3.2, -5.2, -7.2, \ldots$

(iv) $-10, -6, -2, 2, \ldots$

(v) $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}, \ldots$

(vi) $0.2, 0.22, 0.222, 0.2222, \ldots$

(vii) $0, -4, -8, -12, \ldots$

(viii) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \ldots$

(ix) $1, 3, 9, 27, \ldots$

(x) $a, 2a, 3a, 4a, \ldots$

(xi) $a, a^2, a^3, a^4, \ldots$

(xii) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \ldots$

(xiii) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, \ldots$

(xiv) $1^2, 3^2, 5^2, 7^2, \ldots$

(xv) $1^2, 5^2, 7^2, 73, \ldots$

Solution:

(i) $2, 4, 8, 16 \ldots$

Here,

$a_2 - a_1 = 4 - 2 = 2$

$a_3 - a_2 = 8 - 4 = 4$

Since $2 \neq 4$

Hence, the given terms are not in AP.

(ii) $2, \frac{5}{2}, 3, \frac{7}{2} \ldots$

Here,

$$a_2 - a_1 = \frac{5}{2} - 2 = \frac{1}{2}$$

$a_3 - a_2 = 3 - \frac{5}{2} = \frac{1}{2}$

$a_4 - a_3 = \frac{7}{2} - 3 = \frac{1}{2}$

$i.e.,$ difference is the same every time.

Hence, $d = \frac{1}{2}$ and the given terms are in AP.

The next three more terms are

$a_5 = \frac{7}{2} + \frac{1}{2} = 4$

$a_6 = 4 + \frac{1}{2} = \frac{9}{2}$

$a_7 = \frac{9}{2} + \frac{1}{2} = 5$

(iii) $-1.2, -3.2, -5.2, -7.2 \ldots$

Here,

$a_2 - a_1 = (-3.2) - (-1.2) = -2$

$a_3 - a_2 = (-5.2) - (-3.2) = -2$

$a_4 - a_3 = (-7.2) - (-5.2) = -2$

$i.e.,$ difference is the same every time.

Hence, $d = -2$ and the given terms are in AP.

The next three more terms are

$a_5 = -7.2 - 2 = -9.2$

$a_6 = -9.2 - 2 = -11.2$

$a_7 = -11.2 - 2 = -13.2$

(iv) $-10, -6, -2, 2 \ldots$

Here,

$a_2 - a_1 = (-6) - (-10) = 4$

$a_3 - a_2 = (-2) - (-6) = 4$

$a_4 - a_3 = (2) - (-2) = 4$

$i.e.,$ difference is the same every time.

Hence, $d = 4$ and the given terms are in AP.

The next three more terms are

$a_5 = 2 + 4 = 6$

$a_6 = 6 + 4 = 10$

$a_7 = 10 + 4 = 14$

(v) $3, 3 + \sqrt{2}, 3 + 2\sqrt{2}, 3 + 3\sqrt{2} \ldots$

Here,

$a_2 - a_1 = 3 + \sqrt{2} - 3 = \sqrt{2}$

$a_3 - a_2 = 3 + 2\sqrt{2} - 3 - \sqrt{2} = \sqrt{2}$

$a_4 - a_3 = 3 + 3\sqrt{2} - 3 - 2\sqrt{2} = \sqrt{2}$

$i.e.,$ difference is the same every time.

Hence, $d = \sqrt{2}$ and the given terms are in AP.

The next three more terms are

$a_5 = 3 + 3\sqrt{2} + \sqrt{2} = 3 + 4\sqrt{2}$

$a_6 = 3 + 4\sqrt{2} + \sqrt{2} = 3 + 5\sqrt{2}$

$a_7 = 3 + 5\sqrt{2} + \sqrt{2} = 3 + 6\sqrt{2}$

(vi) 0.2, 0.22, 0.222, 0.2222....

Here,

$a_2 - a_1 = 0.22 - 0.2 = 0.02$

$a_3 - a_2 = 0.222 - 0.22 = 0.002$

$a_4 - a_3 = 0.2222 - 0.222 = 0.0002$

Since, $0.02 \neq 0.002$

Hence, the given terms are not in AP

(vii) $0, -4, -8, -12 \ldots$

Here,

$a_2 - a_1 = (-4) - 0 = -4$

$a_3 - a_2 = (-8) - (-4) = -4$

$a_4 - a_3 = (-12) - (-8) = -4$

$i.e.,$ difference is the same every time.

Hence, $d = -4$ and the given terms are in AP.

The next three more terms are

$a_5 = -12 - 4 = -16$

$a_6 = -16 - 4 = -20$

$a_7 = -20 - 4 = -24$

(viii) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2} \ldots .$

Here,

$a_2 - a_1 = \left(-\frac{1}{2}\right) - \left(-\frac{1}{2}\right) = 0$

$a_3 - a_2 = \left(-\frac{1}{2}\right) - \left(-\frac{1}{2}\right) = 0$

$a_4 - a_3 = \left(-\frac{1}{2}\right) - \left(-\frac{1}{2}\right) = 0$

$i.e.,$ difference is the same every time.

Hence, $d = 0$ and the given terms are in AP.

The next three more terms are

$a_5 = -\frac{1}{2} - 0 = -\frac{1}{2}$

$a_6 = -\frac{1}{2} - 0 = -\frac{1}{2}$

$a_7 = -\frac{1}{2} - 0 = -\frac{1}{2}$

(ix) $1, 3, 9, 27$

Here,

$a_2 - a_1 = 3 - 1 = 2$

$a_3 - a_2 = 9 - 3 = 6$

Since $2 \neq 6$

Hence, the given terms are not in AP.

(x) $a, 2a, 3a, 4a, \ldots .$

Here,

$a_2 - a_1 = 2a - a = a$

$a_3 - a_2 = 3a - 2a = a$

$a_4 - a_3 = 4a - 3a = a$

$i.e.,$ difference is the same every time.

Hence, $d = a$ and the given terms are in AP.

The next three more terms are

$a_5 = 4a + a = 5a$

$a_6 = 5a + a = 6a$

$a_7 = 6a + a = 7a$

(xi) $a, a^2, a^3, a^4 \ldots$

$a_2 - a_1 = a^2 - a = a(a - 1)$

$a_3 - a_2 = a^3 - a^2 = a^2(a - 1)$

$a_4 - a_3 = a^4 - a^3 = a^3(a - 1)$

$i.e., a_{x+1} - a_x$ is not the same every time.

Hence, the given terms are not in AP.

(xii) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32} \ldots$

Here,

$a_2 - a_1 = \sqrt{8} - \sqrt{2} = 2\sqrt{2} - \sqrt{2} = \sqrt{2}$

$a_3 - a_2 = \sqrt{18} - \sqrt{8} = 3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$

$a_4 - a_3 = \sqrt{32} - \sqrt{18} = 4\sqrt{2} - 3\sqrt{2} = \sqrt{2}$

$i.e.,$ difference is the same every time.

Hence, $d = \sqrt{2}$ and the given terms are in AP.

The next three more terms are

$a_5 = \sqrt{32} + \sqrt{2} = 4\sqrt{2} + \sqrt{2} = 5\sqrt{2} = \sqrt{50}$

$a_6 = 5\sqrt{2} + \sqrt{2} = 6\sqrt{2} = \sqrt{72}$

$a_7 = 6\sqrt{2} + \sqrt{2} = 7\sqrt{2} = \sqrt{98}$

(xiii) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12} \ldots$

Here,

$a_2 - a_1 = \sqrt{6} - \sqrt{3} = \sqrt{3 \times 2} - \sqrt{3} = \sqrt{3}\,(\sqrt{2} - 1)$

$a_3 - a_2 = \sqrt{9} - \sqrt{6} = 3 - \sqrt{6} = \sqrt{3}\,(\sqrt{3} - \sqrt{2})$

$a_4 - a_3 = \sqrt{12} - \sqrt{9} = 2\sqrt{3} - \sqrt{3 \times 3} = \sqrt{3}\,(2 - \sqrt{3})$

$i.e., a_{x+1} - a_x$ is not the same every time.

Hence, the given terms are not in AP.

(xiv) $1^2, 3^2, 5^2, 7^2 \ldots$

Or, $1, 9, 25, 49$ ...

Here,

$a_2 - a_1 = 9 - 1 = 8$

$a_3 - a_2 = 25 - 9 = 16$

Since, $8 \neq 16$

Hence, the given terms are not in AP.

(xv) $1^2, 5^2, 7^2, 73 \ldots$

Or, $1, 25, 49, 73$ ...

Here,

$a_2 - a_1 = 25 - 1 = 24$

$a_3 - a_2 = 49 - 25 = 24$

$a_4 - a_3 = 73 - 49 = 24$

$i.e.,$ difference is the same every time.

Hence, $d = 24$ and the given terms are in AP.

The next three more terms are

$a_5 = 73 + 24 = 97$

$a_6 = 97 + 24 = 121$

$a_7 = 121 + 24 = 145$

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EXERCISE 5.2

1. Fill in the blanks in the following table, given that a is the first term, d the common difference and a_n the nth term of the AP:

	a	d	n	a_n

(i)	7	3	8	…
(ii)	−18	…	10	0
(iii)	…	−3	18	−5
(iv)	−18.9	2.5	…	3.6
(v)	3.5	0	105	…

Solution:

(i) Given: $a = 7, d = 3, n = 8, a_n = ?$

We Know that,

For an AP : $a_n = a + (n-1)d$

Substituting the values,

$= 7 + (8-1) \times 3$

$= 7 + (7) \times 3$

$= 7 + 21 = 28$

Thus, $a_n = 28$

(ii) Given:

$a = -18, n = 10, a_n = 0, d = ?$

We Know that,

$a_n = a + (n-1)d$

Substituting the values,

$\Rightarrow 0 = -18 + (10-1) \times d$

$\Rightarrow 18 = 9 \times d$

$\Rightarrow d = \frac{18}{9} = 2$

Thus, common difference, $d = 2$

(iii) Given:

$d = -3, n = 18, a_n = -5, a = ?$

We Know that,

$a_n = a + (n-1)d$

Substituting the values,

$\Rightarrow -5 = a + (18 - 1) \times (-3)$

$\Rightarrow -5 = a + (17) \times (-3)$

$\Rightarrow -5 = a - 51$

$\Rightarrow a = 51 - 5 = 46$

Thus, $a = 46$

(iv) Given: $a = -18.9, d = 2.5, a_n = 3.6, n = ?$

We Know that,

$a_n = a + (n - 1)d$

Substituting the values,

$\Rightarrow 3.6 = -18.9 + (n - 1) \times 2.5$

$\Rightarrow 3.6 + 18.9 = (n - 1) \times 2.5$

$\Rightarrow 22.5 = (n - 1) \times 2.5$

$\Rightarrow (n - 1) = \frac{22.5}{2.5}$

$\Rightarrow n - 1 = 9$

$\Rightarrow n = 10$

Thus, $n = 10$

(v) Given: $a = 3.5, d = 0, n = 105, a_n = ?$

We Know that,

$a_n = a + (n - 1)d$

$\Rightarrow a_n = 3.5 + (105 - 1)0$

$\Rightarrow a_n = 3.5 + 104 \times 0$

$\Rightarrow a_n = 3.5$

Thus, $a_n = 3.5$

2. Choose the correct choice in the following and justify:

(i) 30th term of the AP: $10, 7, 4, \ldots,$ is

(A) 97

(B) 77

(C) -77

(D) -87

(ii) 11th term of the AP: $-3, -\frac{1}{2}, 2, \ldots$ is

(A) 28

(B) 22

(C) -38

(D) $-48\frac{1}{2}$

Solution:

(i) Given:

AP: $10, 7, 4 \ldots$

First term, $a = 10$

Common difference, $d = a_2 - a_1 = 7 - 10 = -3$

We know that, $a_n = a + (n-1)d$

Substituting $n = 30, a = 10$ and $d = -3$, we get

$a_{30} = 10 + (30 - 1) \times (-3)$

$\Rightarrow a_{30} = 10 + (29) \times (-3)$

$\Rightarrow a_{30} = 10 - 87 = -77$

Therefore, the correct answer is C.

(ii) Given:

AP: $-3, -\frac{1}{2}, 2 \ldots$

First term, $a = -3$

Common difference, $d = a_2 - a_1$

$= -\frac{1}{2} - (-3)$

$= -\frac{1}{2} + 3 = \frac{5}{2}$

We know that, $a_n = a + (n-1)d$

Substituting $n = 11, a = -3$ and $d = \frac{5}{2}$, we get

$a_{11} = -3 + (11 - 1) \times \left(\frac{5}{2}\right)$

$\Rightarrow a_{11} = -3 + (10) \times \left(\frac{5}{2}\right)$

$\Rightarrow a_{11} = -3 + 25$

$\Rightarrow a_{11} = 22$

Therefore, the correct answer is B.

3. In the following APs, find the missing terms in the boxes:

(i) 2, ☐, 26

(ii) ☐, 13, ☐, 3

(iii) 5, ☐, ☐, $9\frac{1}{2}$

(iv) – 4, ☐, ☐, ☐, ☐, 6

(v) ☐, 38, ☐, ☐, ☐, – 22

Solution:

(i) 2, ☐, 26

For the given AP,

$a = 2,\ a_3 = 26$

We know that, $a_n = a + (n - 1)d$

Substituting the values,

$a_3 = 2 + (3 - 1)d$

$\Rightarrow 26 = 2 + 2d$

$\Rightarrow 24 = 2d$

$\Rightarrow d = 12$

$a_2 = 2 + (2 - 1)12 = 14$

Hence, the missing term is 14.

(ii) ☐, 13, ☐, 3

For the given AP,

$a_2 = 13$ and $a_4 = 3$

We know that, $a_n = a + (n-1)d$

Substituting the values,

$a_2 = a + (2-1)d$

$\Rightarrow 13 = a + d \quad ...(i)$

$a_4 = a + (4-1)d$

$\Rightarrow 3 = a + 3d \quad ...(ii)$

On subtracting (ii) from (i), we obtain

$13 - 3 = (-2)d$

$\Rightarrow 10 = -2d$

$\Rightarrow d = -5$

Substituting the value of d in (i),

$a = 18$

$a_3 = 18 + (3-1)(-5)$

$= 18 + 2(-5) = 18 - 10 = 8$

Hence, the missing terms are 18 and 8 respectively.

(iii) $5, \square, \square, 9\frac{1}{2}$

For the given AP,

$a = 5, a_4 = 9\frac{1}{2} = \frac{19}{2}$

We know that,

$a_n = a + (n-1)d$

Substituting the values,

$a_4 = a + (4-1)d$

$\Rightarrow \frac{19}{2} = 5 + 3d$

$\Rightarrow \frac{19}{2} - 5 = 3d$

$\Rightarrow \frac{9}{2} = 3d$

$\Rightarrow d = \frac{3}{2}$

$a_2 = a + d = 5 + \frac{3}{2} = \frac{13}{2}$

$a_3 = a + 2d = 5 + 2\left(\frac{3}{2}\right) = 8$

Hence, the missing terms are $\frac{13}{2}$ and 8 respectively.

(iv) – 4, ☐, ☐, ☐, ☐, 6

For the given AP,

$a = -4$ and $a_6 = 6$

We know that,

$a_n = a + (n-1)d$

Substituting the values,

$a_6 = a + (6-1)d$

$\Rightarrow 6 = -4 + 5d$

$\Rightarrow 10 = 5d$

$\Rightarrow d = 2.$

$a_2 = a + d = -4 + 2 = -2$

$a_3 = a + 2d = -4 + 2(2) = 0$

$a_4 = a + 3d = -4 + 3(2) = 2$

$a_5 = a + 4d = -4 + 4(2) = 4$

Therefore, the missing terms are $-2, 0, 2$ and 4 respectively.

(v) ☐, 38, ☐, ☐, ☐, -22

For the given AP,

$a_2 = 38$

$a_6 = -22$

$\Rightarrow 38 = a + d \quad \ldots \ (1)$

$a_6 = a + (6-1)d$

$\Rightarrow -22 = a + 5d \quad \ldots \ (2)$

On subtracting equation (1) from (2), we obtain

$-22 - 38 = 4d$

$\Rightarrow -60 = 4d$

$\Rightarrow d = -15$

$a = a_2 - d = 38 - (-15)$ [From (1)]

$\Rightarrow a = 53$

$a_3 = a + 2d = 53 + 2(-15) = 23$

$a_4 = a + 3d = 53 + 3(-15) = 8$

$a_5 = a + 4d = 53 + 4(-15) = -7$

Therefore, the missing terms are $53, 23, 8$ and -7 respectively.

4. Which term of the AP : $3, 8, 13, 18, \ldots$, is 78?

Solution:

$3, 8, 13, 18, \ldots$

For the given AP,

$a = 3$

$d = a_2 - a_1 = 8 - 3 = 5$

Let n^{th} term of this AP be 78.

$a_n = a + (n-1)d$

Substituting the values,

$78 = 3 + (n-1)5$

$\Rightarrow 75 = (n-1)5$

$\Rightarrow (n-1) = 15$

$\Rightarrow n = 16$

Therefore, 16^{th} term of the given AP is 78.

5. Find the number of terms in each of the following APs:

(i) $7, 13, 19, \ldots, 205$

(ii) $18, 15\frac{1}{2}, 13, \ldots, -47$

Solution:

(i) $7, 13, 19, \ldots, 205$

For the given AP,

$a = 7$

$d = a_2 - a_1 = 13 - 7 = 6$

Let there be n number of terms in this AP

$a_n = 205$

We know that, $a_n = a + (n-1)d$

So, $205 = 7 + (n-1)6$

$\Rightarrow 198 = (n-1)6$

$\Rightarrow 33 = (n-1)$

$\Rightarrow n = 34$

Hence, this given AP has 34 terms in it.

(ii) $18, 15\frac{1}{2}, 13, \dots, -47$

For the given AP,

$a = 18$

$$d = a_2 - a_1 = 15\frac{1}{2} - 18$$

$$d = \frac{31-36}{2} = -\frac{5}{2}$$

Let there be n number of terms in this AP

$a_n = -47$

We know that, $a_n = a + (n-1)d$

So, $-47 = 18 + (n-1)\left(-\frac{5}{2}\right)$

$$\Rightarrow -47 - 18 = (n-1)\left(-\frac{5}{2}\right)$$

$$\Rightarrow -65 = (n-1)\left(-\frac{5}{2}\right)$$

$$\Rightarrow (n-1) = \frac{-130}{-5}$$

$\Rightarrow (n-1) = 26$

$\Rightarrow n = 27$

Hence, this given AP has 27 terms in it.

6. Check whether – 150 is a term of the AP : 11, 8, 5, 2 ...

Solution:

For the given AP,

$a = 11$

$d = a_2 - a_1 = 8 - 11 = -3$

Let -150 be the n^{th} term of this AP

We know that,

$a_n = a + (n-1)d$

$\Rightarrow -150 = 11 + (n-1)(-3)$

$\Rightarrow -150 = 11 - 3n + 3$

$\Rightarrow -164 = -3\text{n}$

$\Rightarrow n = \frac{164}{3}$

Clearly, n is not an integer.

Hence, -150 is not a term of this given AP.

Note: n is the number of terms in a sequence and can take only integral value.

7. Find the 31st term of an AP whose 11th term is 38 and the 16th term is 73.

Solution:

According to the question, it is given that,

$a_{11} = 38$

$a_{16} = 73$

We know that,

$a_n = a + (n-1)d$

Substituting the values,

$a_{11} = a + (11-1)d$

$\Rightarrow 38 = a + 10d \quad \ldots(1)$

Similarly,

Substituting the values,

$a_{16} = a + (16 - 1)d$

$\Rightarrow 73 = a + 15d \quad ...(2)$

On subtracting (1) from (2), we obtain

$35 = 5d$

$\Rightarrow d = 7$

From equation (1),

$38 = a + 10 \times (7)$

$\Rightarrow 38 - 70 = a$

$\Rightarrow a = -32$

$a_{31} = a + (31 - 1)d$

$= -32 + 30 \times (7)$

$= -32 + 210$

$= 178$

Therefore, the 31^{st} term in given AP is 178.

8. An AP consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term.

Solution:

According to the question it is given that,

$a_3 = 12$

$a_{50} = 106$

We know that,

$a_n = a + (n - 1)d$

Substituting the values,

$a_3 = a + (3 - 1)d$

$\Rightarrow 12 = a + 2d \quad ...(I)$

Similarly, $a_{50} = a + (50 - 1)d$

$\Rightarrow 106 = a + 49d \quad ...(II)$

On subtracting (I) from (II), we get

$94 = 47d$

$\Rightarrow d = 2$

From equation (I), we get

$12 = a + 2(2)$

$\Rightarrow a = 12 - 4 = 8$

$a_{29} = a + (29 - 1)d$

$\Rightarrow a_{29} = 8 + (28)2$

$\Rightarrow a_{29} = 8 + 56 = 64$

Hence, the 29th term in given AP is 64.

9. If the 3rd and the 9th terms of an AP are 4 and -8 respectively, which term of this AP is zero?

Solution:

As per the question, it is given that,

$a_3 = 4$

$a_9 = -8$

We know that,

$a_n = a(n - 1)d$

Substituting the values,

$a_3 = a + (3 - 1)d$

$\Rightarrow 4 = a + 2d \quad \text{... (I)}$

$a_9 = a + (9 - 1)d$

$\Rightarrow -8 = a + 8d \quad \text{... (II)}$

On subtracting equation (I) from (II), we get

$-12 = 6d$

$\Rightarrow d = -2$

From equation (I), we get

$4 = a + 2(-2)$

$\Rightarrow 4 = a - 4$

$\Rightarrow a = 8$

Let n^{th} term of the given AP be zero,

$a_n = a + (n - 1)d$

Substituting the values,

$0 = 8 + (n - 1)(-2)$

$\Rightarrow 0 = 8 - 2n + 2$

$\Rightarrow 2n = 10$

$\Rightarrow n = 5$

Therefore, the 5^{th} term of the given AP is 0.

10. The 17th term of an AP exceeds its 10th term by 7. Find the common difference.

Solutions:

For the given AP, we know that: $a_n = a + (n - 1)d$

$a_{17} = a + (17 - 1)d$

$\Rightarrow a_{17} = a + 16d$

Similarly, $a_{10} = a + 9d$

But according to the question, it is given that

$a_{17} - a_{10} = 7$

$\Rightarrow (a + 16d) - (a + 9d) = 7$

$\Rightarrow 7d = 7$

$\Rightarrow d = 1$

Hence, the common difference is 1.

11. Which term of the AP: $3, 15, 27, 39, \ldots$ will be 132 more than its 54th term?

Solution:

Given: AP is $3,15,27,39,\ldots.$

$a = 3$

$d = a_2 - a_1 = 15 - 3 = 12$

$a_{54} = a + (54 - 1)d$

$= 3 + (53)(12)$

$= 3 + 636 = 639$

Required term = $132 + 639 = 771$ (Given that term to be found is 132 more than a_{54})

Now, we need to determine the term of the given AP i.e., 771.

Let n^{th} term be 771.

$a_n = a + (n - 1)d$

Substituting the values,

$771 = 3 + (n - 1)12$

$\Rightarrow 768 = (n - 1)12$

$\Rightarrow (n - 1) = 64$

$\Rightarrow n = 65$

Hence, the 65^{th} term will be 132 more than 54^{th} term.

12. Two APs have the same common difference. The difference between their 100th terms is 100, what is the difference between their 1000th terms?

Solution:

Let the first term of the given two APs be a_1 and a_2 respectively and the common difference of these APs be d.

For first AP,

$a_{100} = a_1 + (100 - 1)d$

$= a_1 + 99d$

$a_{1000} = a_1 + (1000 - 1)d$

$= a_{1000} = a_1 + 999d$

For Second AP,

$a'_{100} = a_2 + (100 - 1)d$

$= a_2 + 99d$

$a'_{1000} = a_2 + (1000 - 1)d$

$= a_2 + 999d$

According to the question, it is given that the difference between

100^{th} term of these APs = 100

Therefore, $(a_1 + 99d) - (a_2 + 99d) = 100$

$a_1 - a_2 = 100$ ... (1)

Difference between 1000^{th} terms of these APs

$(a_1 + 999d) - (a_2 + 999d) = a_1 - a_2$

From equation (1),

This difference, $a_1 - a_2 = 100$

Hence difference between 1000^{th} terms of these AP will be 100.

13. How many three-digit numbers are divisible by 7?

Solution:

First three-digit number that is divisible by 7 = 105

Next number = 105 + 7 = 112

So, 105,112,119,…..

All are 3-digit numbers that are divisible by 7 and therefore, all these are terms of an AP having first term as 105 and common difference as 7.

The maximum possible three-digit number is 999. When we divide it by 7, the remainder will be 5.

Clearly, 999 − 5 = 994 is the maximum possible 3-digit number that is divisible by 7.

The AP is as follows.

105, 112, 119, ……, 994

Let 994 be the n^{th} term of this AP

$a = 105$

$d = 7$

$a_n = 994$

$n = ?$

$a_n = a + (n - 1)d$

$\Rightarrow 994 = 105 + (n - 1)7$

$\Rightarrow 889 = (n - 1)7$

$\Rightarrow (n-1) = 127$

$\Rightarrow n = 128$

Hence, there are 128 three-digit numbers that are divisible by 7.

14. How many multiples of 4 lie between 10 and 250?

Solution:

First multiple of 4 that is greater than 10 is 12. Next will be 16.

So, $12, 16, 20, 24, \ldots$

All these are divisible by 4 and therefore, all these are terms of an AP with first term as 12 and common difference as 4.

when we divide 250 by 4, the remainder will be 2.

Hence, $250 - 2 = 248$ is divisible by 4

The AP is as follows.

$12, 16, 20, 24, \ldots \ldots 248$

Let 248 be the nth term of this AP

$a = 12$

$d = 4$

$a_n = 248$

$a_n = a + (n-1)d$

$\Rightarrow 248 = 12 + (n-1)4$

$\Rightarrow \frac{236}{4} = n - 1$

$\Rightarrow n = 60$

So, there are 60 multiples of 4 lying between 10 and 250.

15. For what value of n, are the nth terms of two APs: $63, 65, 67, \ldots$ and $3, 10, 17, \ldots$ equal?

Solution:

Let us consider the first AP:

$63, 65, 67, \ldots \ldots$

$a = 63$

$d = a_2 - a_1 = 65 - 63 = 2$

n^{th} term of this AP $= a + (n - 1)d$

$\Rightarrow a_n = 63 + (n - 1)2 = 63 + 2n - 2$

$\Rightarrow a_n = 61 + 2n \quad ...(1)$

Now, let us consider the second AP:

$3, 10, 17, \ldots..$

$a = 3$

$d = a_2 - a_1 = 10 - 3 = 7$

n^{th} term of this AP $= 3 + (n - 1)7$

$a_n = 7n - 4 \quad ...(2)$

In question, it is given that, n^{th} term of the two APs are equal to each other.

Equating both (1) and (2), we get

$61 + 2n = 7n - 4$

$\Rightarrow 61 + 4 = 5n$

$\Rightarrow 5n = 65$

$\Rightarrow n = 13$

Hence, 13^{th} terms of two APs are equal.

16. Determine the AP whose third term is 16 and the 7th term exceeds the 5th term by 12.

Solution:

$a_3 = 16$

Recall formula for n^{th} term of an AP: $a_n = a + (n - 1)d$

Substituting the value of $n = 3$, we get

$a + (3 - 1)d = 16$

$\Rightarrow a + 2d = 16 \quad ...(1)$

$a_7 - a_5 = 12$

$\Rightarrow [a + (7 - 1)d] - [a + (5 - 1)d] = 12$

$\Rightarrow (a + 6d) - (a + 4d) = 12$

$\Rightarrow 2d = 12$

$\Rightarrow d = 6$

From equation (1), we get

$a + 2(6) = 16$

$\Rightarrow a + 12 = 16$

$\Rightarrow a = 4$

So, AP will be

$4, 10, 16, 22, \dots\dots\dots,$

17. Find the 20th term from the last term of the AP : $3, 8, 13, \dots, 253$.

Solution:

According to the question, given AP is

$3, 8, 13, \dots\dots\dots\dots, 253$

Common difference for this AP is 5.

So, this AP can be written in reverse order as

$253, 248, 243, \dots\dots, 13, 8, 5$

For this AP,

$a = 253$

$d = 248 - 253 = -5$

$n = 20$

$a_{20} = a + (20 - 1)d$

$\Rightarrow a_{20} = 253 + (19)(-5)$

$\Rightarrow a_{20} = 253 - 95$

$a_{20} = 158$

Hence, 20^{th} term from the last term of the given AP is 158.

18. The sum of the 4th and 8th terms of an AP is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the AP.

Solution:

We know that,

$a_n = a + (n - 1)d$

$\Rightarrow a_4 = a + (4 - 1)d$

$\Rightarrow a_4 = a + 3d$

Similarly,

$a_8 = a + 7d$

$a_6 = a + 5d$

$a_{10} = a + 9d$

According to the question, it is given that, $a_4 + a_8 = 24$

$\Rightarrow (a + 3d) + (a + 7d) = 24$

$\Rightarrow 2a + 10d = 24$

$\Rightarrow a + 5d = 12 \quad ...(1)$

It is also given in the question that, $a_6 + a_{10} = 44$

$\Rightarrow a + 5d + a + 9d = 44$

$\Rightarrow 2a + 14d = 44$

$\Rightarrow a + 7d = 22 \quad ...(2)$

On subtracting equation (1) from (2), we get

$2d = 22 - 12$

$\Rightarrow 2d = 10$

$\Rightarrow d = 5$

From equation (1), we get

$a + 5d = 12$

$\Rightarrow a + 5(5) = 12$

$\Rightarrow a + 25 = 12$

$\Rightarrow a = -13$

$a_2 = a + d = -13 + 5 = -8$

$a_3 = a_2 + d = -8 + 5 = -3$

Hence, the first three terms of the given AP are $-13, -8$, and -3.

19. Subba Rao started work in 1995 at an annual salary of ₹ 5000 and received an increment of ₹ 200 each year. In which year did his income reach ₹ 7000?

Solution:

We can observe that the income Subba Rao obtained in various years are in AP as every year, his salary is increased by ₹ 200.

So, the salaries of each year from 1995 are

$5000, 5200, 5400, \ldots\ldots$

Here, $a = 5000$

$d = 200$

Let after n^{th} year, his salary be ₹ 7000.

Therefore, $a_n = a + (n-1)d$

$\Rightarrow 7000 = 5000 + (n-1)200$

$\Rightarrow 200(n-1) = 2000$

$\Rightarrow (n-1) = 10$

$\Rightarrow n = 11$

Hence, in 11^{th} year, Subba Rao's salary will become ₹ 7000.

20. Ramkali saved ₹ 5 in the first week of a year and then increased her weekly savings by ₹ 1.75. If in the nth week, her weekly savings become ₹ 20.75, find n.

Solution:

According to the question, it is given that,

Savings made by Ramkali in first week = $a = 5$

Increment in savings = $d = 1.75$

Let her savings become ₹ 20.75 in nth week, i.e., $a_n = 20.75$

$n = ?$

$a_n = a + (n-1)d$

Substituting the values

$20.75 = 5 + (n-1)1.75$

$\Rightarrow 15.75 = (n-1)1.75$

$$\Rightarrow (n-1) = \frac{15.75}{1.75} = \frac{1575}{175}$$

$= \frac{63}{7} = 9$

$\Rightarrow n - 1 = 9$

$\Rightarrow n = 10$

So, her savings become ₹ 20.75 in10th week.

♦ ♦ ♦

EXERCISE 5.3

1. Find the sum of the following APs:

(i) 2, 7, 12, …, to 10 terms.

(ii) $-37, -33, -29, \ldots$, to 12 terms.

(iii) 0.6, 1.7, 2.8, …, to 100 terms.

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \ldots$ to 11 terms.

Solution:

(i) 2, 7 ,12,……,to 10 terms

For the given AP,

$a = 2$

$d = a_2 - a_1 = 7 - 2 = 5$

$n = 10$

We know that,

$S_n = \frac{n}{2}[2a + (n-1)d]$

Substituting the values,

$S_{10} = \frac{10}{2}[2(2) + (10-1)5]$

$= 5 \times [4 + (9) \times (5)]$

$= 5 \times 49 = 245$

(ii) $-37, -33, -29, \ldots\ldots$,to 12 terms

For the given AP,

$a = -37$

$d = a_2 - a_1 = (-33) - (-37)$

$= -33 + 37 = 4$

$n = 12$

We know that,

$S_n = \frac{n}{2}[2a + (n-1)d]$

Substituting the values,

$S_{12} = \frac{12}{2}[2(-37) + (12-1)4]$

$= 6[-74 + 11 \times 4]$

$= 6[-74 + 44]$

$= 6(-30) = -180$

(iii) $0.6, 1.7, 2.8, \ldots\ldots,$ to 100 terms

For the given AP,

$a = 0.6$

$d = a_2 - a_1 = 1.7 - 0.6 = 1.1$

$n = 100$

We know that,

$S_n = \frac{n}{2}[2a + (n-1)d]$

Substituting the values,

$S_{100} = \frac{100}{2}[2(0.6) + (100-1)1.1]$

$= 50[1.2 + (99) \times (1.1)]$

$= 50[1.2 + 108.9]$

$= 50[110.1]$

$= 5505$

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \ldots\ldots,$ to 11 terms

For the given AP,

$a = \frac{1}{15}$

$n = 11$

$d = a_2 - a_1 = \frac{1}{12} - \frac{1}{15}$

$= \frac{5-4}{60} = \frac{1}{60}$

We know that,

$S_n = \frac{n}{2}[2a + (n-1)d]$

Substituting the values,

$S_{11} = \frac{11}{2}\left[2\left(\frac{1}{15}\right) + (11-1)\frac{1}{60}\right]$

$= \frac{11}{2}\left[\frac{2}{15} + \frac{10}{60}\right]$

$= \left(\frac{11}{2}\right)\left(\frac{9}{30}\right) = \frac{33}{20}$

2. Find the sums given below:

(i) $7 + 10\frac{1}{2} + 14 + \cdots + 84$

(ii) $34 + 32 + 30 + \ldots + 10$

(iii) $-5 + (-8) + (-11) + \ldots + (-230)$

Solution:

(i) $7 + 10\frac{1}{2} + 14 + \ldots\ldots + 84$

For the given AP,

$a = 7$

Last term = $l = 84$

$d = a_2 - a_1 = 10\frac{1}{2} - 7 = \frac{21}{2} - 7 = \frac{7}{2}$

Let 84 be the n^{th} term of the given AP

We know that,

$a_n = a + (n-1)d$

Substituting the values,

$$84 = 7 + (n-1)\frac{7}{2}$$

$$\Rightarrow 77 = (n-1)\frac{7}{2}$$

$$\Rightarrow 22 = \text{n} - 1$$

So, $\text{n} = 23$

We know that,

$$S_n = \frac{n}{2}(a + l)$$

Substituting the values,

$$S_{23} = \frac{23}{2}[7 + 84]$$

$$= \frac{23 \times 91}{2} = \frac{2093}{2}$$

$$= 1046\frac{1}{2}$$

(ii) $34 + 32 + 30 + \ldots\ldots\ldots + 10$

For the given AP,

$\text{a} = 34$

$\text{d} = \text{a}_2 - \text{a}_1 = 32 - 34 = -2$

Last term $= l = 10$

Let 10 be the nth term of the given AP

We know that,

$$\text{a}_\text{n} = \text{a} + (\text{n} - 1)\text{d}$$

$$\Rightarrow 10 = 34 + (\text{n} - 1)(-2)$$

$$\Rightarrow -24 = (\text{n} - 1)(-2)$$

$$\Rightarrow 12 = \text{n} - 1$$

So, $\text{n} = 13$

We know that,

$$S_n = \frac{n}{2}(a + l)$$

Substituting the values,

$S_{13} = \frac{13}{2}(34 + 10)$

$= \frac{13 \times 44}{2} = 13 \times 22$

$= 286$

(iii) $(-5) + (-8) + (-11) + \ldots\ldots\ldots\ldots + (-230)$

For the given AP,

$a = -5$

last term $= l = -230$

$d = a_2 - a_1 = (-8) - (-5)$

$= -8 + 5 = -3$

Let -230 be the nth term of the given AP

We know that,

$a_n = a + (n-1)d$

$\Rightarrow -230 = -5 + (n-1)(-3)$

$\Rightarrow -225 = (n-1)(-3)$

$\Rightarrow (n-1) = 75$

So, $n = 76$

And, $S_n = \frac{n}{2}(a + l)$

Substituting the values,

$S_{76} = \frac{76}{2}[(-5) + (-230)]$

$= 38(-235)$

$= -8930$

3. In an AP:

(i) given $a = 5, d = 3, a_n = 50$, find n and S_n.

(ii) given $a = 7, a_{13} = 35$, find d and S_{13}.

(iii) given $a_{12} = 37, d = 3$, find a and S_{12}.

(iv) given $a_3 = 15, S_{10} = 125$, find d and a_{10}.

(v) given $d = 5, S_9 = 75$, find a and a_9.

(vi) given $a = 2, d = 8, S_n = 90$, find n and a_n.

(vii) given $a = 8, a_n = 62, S_n = 210$, find n and d.

(viii) given $a_n = 4, d = 2, S_n = -14$, find n and a.

(ix) given $a = 3, n = 8, S = 192$, find d.

(x) given $l = 28, S = 144$, and there are total 9 terms. Find a.

Solution:

(i) Given that, $a = 5, d = 3, a_n = 50$

We know that, $a_n = a + (n-1)d$,

Substituting the values,

$\therefore 50 = 5 + (n-1)3$

$\Rightarrow 45 = (n-1)3$

$\Rightarrow 15 = n - 1$

So, $n = 16$

We also know that,

$S_n = \frac{n}{2}[a + a_n]$

Substituting the values,

$S_{16} = \frac{16}{2}[5 + 50]$

$= 8 \times 55$

$= 440$

(ii) Given that, $a = 7, a_{13} = 35$

We Know that, $a_n = a + (n-1)d$,

$\therefore a_{13} = a + (13-1)d$

$\Rightarrow 35 = 7 + 12d$

$\Rightarrow 28 = 12d$

$\Rightarrow d = \frac{7}{3}$

We also know that,

$S_n = \frac{n}{2}[a + a_n]$

$S_{13} = \frac{n}{2}[a + a_{13}]$

Substituting the values,

$= \frac{13}{2}[7 + 35]$

$= \frac{13 \times 42}{2} = 13 \times 21$

$= 273$

(iii) Given that, $a_{12} = 37, d = 3$

We know that, $a_n = a + (n - 1)d$,

Substituting the values,

$a_{12} = a + (12 - 1)3$

$\Rightarrow 37 = a + 33$

So, $a = 4$

We also know that,

$S_n = \frac{n}{2}[a + a_n]$

Substituting the values,

$S_{12} = \frac{12}{2}[4 + 37]$

$\Rightarrow S_{12} = 6(41)$

$\Rightarrow S_{12} = 246$

(iv) Given that, $a_3 = 15, S_{10} = 125$

We know that, $a_n = a + (n - 1)d$,

Substituting the values,

$a_3 = a + (3 - 1)d$

$\Rightarrow 15 = a + 2d$ ... (i)

We also know that,

$S_n = \frac{n}{2}[2a + (n - 1)d]$

Substituting the values,

$S_{10} = \frac{10}{2}[2a + (10 - 1)d]$

$\Rightarrow 125 = 5(2a + 9d)$

$\Rightarrow 25 = 2a + 9d$ ... (ii)

By multiplying equation (1) by 2, we get $30 = 2a + 4d$ ... (iii)

By subtracting equation (iii) from (ii), we get

$-5 = 5d$

So, $d = -1$

From equation (i),

$15 = a + 2(-1)$

$\Rightarrow 15 = a - 2$

$\Rightarrow a = 17$

We know that, $a_n = a + (n - 1)d,$

Substituting the values,

$a_{10} = a + (10 - 1)d$

$\Rightarrow a_{10} = 17 + (9)(-1)$

So, $a_{10} = 17 - 9 = 8$

(v) Given that, $d = 5, S_9 = 75$

We know that, $S_n = \frac{n}{2}[2a + (n - 1)d],$

Substituting the values,

$S_9 = \frac{9}{2}[2a + (9 - 1)5]$

$\Rightarrow 75 = \frac{9}{2}(2a + 40)$

$\Rightarrow 25 = 3(a + 20)$

$\Rightarrow 25 = 3a + 60$

$\Rightarrow 3a = 25 - 60$

$\Rightarrow a = \frac{-35}{3}$

We also know that,

$a_n = a + (n-1)d$

Substituting the values,

$a_9 = a + (9-1)(5)$

$= \frac{-35}{3} + 8(5)$

$= \frac{-35}{3} + 40$

$= \frac{-35+120}{3} = \frac{85}{3}$

(vi) Given that, $a = 2, d = 8, S_n = 90$

We know that,

$S_n = \frac{n}{2}[2a + (n-1)d]$,

Substituting the values,

$90 = \frac{n}{2}[4 + (n-1)8]$

$\Rightarrow 90 = n[2 + (n-1)4]$

$\Rightarrow 90 = n[2 + 4n - 4]$

$\Rightarrow 90 = n(4n-2) = 4n^2 - 2n$

$\Rightarrow 4n^2 - 2n - 90 = 0$

$\Rightarrow 4n^2 - 20n + 18n - 90 = 0$ (Factorisation by splitting the middle term)

$\Rightarrow 4n(n-5) + 18(n-5) = 0$

$\Rightarrow (n-5)(4n+18) = 0$

Either $n - 5 = 0$ or $4n + 18 = 0$

$\Rightarrow n = 5$ or $n = -\frac{18}{4} = \frac{-9}{2}$

But, n can neither be negative nor fractional.

Hence, $n = 5$

We also know that,

$a_n = a + (n-1)d$

$\Rightarrow a_5 = 2 + (5-1)8$

$= 2 + (4)(8)$

$= 2 + 32 = 34$

So, $n = 5$ and $a_5 = 34$.

(vii) Given that, $a = 8, a_n = 62, S_n = 210$

We know that,

$S_n = \frac{n}{2}[a + a_n]$

Substituting the values,

$210 = \frac{n}{2}[8 + 62]$

$\Rightarrow 210 = \frac{n}{2}(70)$

So, $n = 6$

We also know that,

$a_n = a + (n - 1)d$

Substituting the values,

$62 = 8 + (6 - 1)d$

$\Rightarrow 62 - 8 = 5d$

Hence, $d = \frac{54}{5}$

(viii) Given that, $a_n = 4, d = 2, S_n = -14$

We know that,

$a_n = a + (n - 1)d$

Substituting the values,

$4 = a + (n - 1)2$

$\Rightarrow 4 = a + 2n - 2$

$\Rightarrow a + 2n = 6$

$\Rightarrow a = 6 - 2n$ …………………. (i)

We also know that,

$S_n = \frac{n}{2}[a + a_n]$

Substituting the values,

$-14 = \frac{n}{2}[a + 4]$

$\Rightarrow -28 = n(a + 4)$

$\Rightarrow -28 = n(6 - 2n + 4)$ {From equation (i)}

$\Rightarrow -28 = n(-2n + 10)$

$\Rightarrow -28 = -2n^2 + 10n$

$\Rightarrow 2n^2 - 10n - 28 = 0$

$\Rightarrow n^2 - 5n - 14 = 0$

$\Rightarrow n^2 - 7n + 2n - 14 = 0$ [Factorisation by splitting the middle term]

$\Rightarrow n(n - 7) + 2(n - 7) = 0$

$\Rightarrow (n - 7)(n + 2) = 0$

Either $n - 7 = 0$ or $n + 2 = 0$

$\Rightarrow n = 7$ or $n = -2$

But, n can only take positive integral values.

So, $n = 7$

From equation (i), we obtain

$a = 6 - 2n$

$\Rightarrow a = 6 - 2(7)$

$= 6 - 14$

$= -8$

Hence, $n = 7\ and\ a = -8$.

(ix) Given that, $a = 3, n = 8, S = 192$

We know that,

$S_n = \frac{n}{2}[2a + (n - 1)\text{d}]$

Substituting the values,

$192 = \frac{8}{2}[2 \times 3 + (8 - 1)d]$

$\Rightarrow 192 = 4[6 + 7d]$

$\Rightarrow 48 = 6 + 7d$

$\Rightarrow 42 = 7d$

So, $d = 6$

(x) Given that, $l = 28, S = 144$ and there are total 9 terms.

We know that,

$$S_n = \frac{n}{2}(a + l)$$

Substituting the values,

$$144 = \frac{9}{2}(a + 28)$$

$\Rightarrow (16) \times (2) = a + 28$

$\Rightarrow 32 = a + 28$

Hence, $a = 4$

4. How many terms of the AP : $9, 17, 25, \ldots$ must be taken to give a sum of 636?

Solution:

Let the number of terms in the given AP $= n$

It is given that for this AP, $a = 9$

$d = a_2 - a_1 = 17 - 9 = 8$

We know that,

$$S_n = \frac{n}{2}[2a + (n - 1)d)$$

Substituting the values,

$$636 = \frac{n}{2}[2 \times 9 + (n - 1)8]$$

$$\Rightarrow 636 = \frac{n}{2}[18 + (n - 1)8]$$

$\Rightarrow 636 = n[9 + 4n - 4]$

$\Rightarrow 636 = n(4n + 5)$

$\Rightarrow 4n^2 + 5n - 636 = 0$

$\Rightarrow 4n^2 + 53n - 48n - 636 = 0$ [Factorisation by splitting the middle term]

$\Rightarrow n(4n + 53) - 12(4n + 53) = 0$

$\Rightarrow (4n + 53)(n - 12) = 0$

Either $4n + 53 = 0$ or $n - 12 = 0$

$\Rightarrow n = \frac{-53}{4}$ or $n = 12$

n cannot be $-\frac{53}{4}$. As the number of terms can neither be negative nor fractional, Hence, $n = 12$.

5. The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

Solution:

According to the question, it is given that,

$a = 5$

$a_n = l = 45$

$S_n = 400$

We know that,

$$S_n = \frac{n}{2}(a + l)$$

Substituting the values,

$$400 = \frac{n}{2}(5 + 45)$$

$$400 = \frac{n}{2}(50)$$

So, $n = 16$

We also know that,

$a_n = l = a + (n - 1)d$

$45 = 5 + (16 - 1)d$

$40 = 15d$

Hence, $d = \frac{40}{15} = \frac{8}{3}$

6. The first and the last terms of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

Solution:

According to the question, it is given that,

$a = 17$

$a_n = l = 350$

$d = 9$

Let the number of terms in the given AP = n

We know that,

$a_n = l = a + (n-1)d$

Substituting the values,

$350 = 17 + (n-1)9$

$\Rightarrow 333 = (n-1)9$

$\Rightarrow (n-1) = 37$

So, $n = 38$

We also know that,

$S_n = \frac{n}{2}(a + l)$

Substituting the values,

$\Rightarrow S_{38} = \frac{38}{2}(17 + 350) = 19(367) = 6973$

Hence, the given AP contains 38 terms and their sum is 6973.

7. Find the sum of first 22 terms of an AP in which $d = 7$ and 22nd term is 149.

Solution:

According to the question, for this AP,

$d = 7$

$a_{22} = 149$

$S_{22} = ?$

We know that,

$a_n = a + (n-1)d$

$\Rightarrow a_{22} = a + (22-1)d$

$\Rightarrow 149 = a + 21 \times 7$

$\Rightarrow 149 = a + 147$

$\Rightarrow a = 2$

We also know that,

$S_n = \frac{n}{2}(a + a_n)$

Substituting the values,

$S_{22} = \frac{22}{2}(2 + 149)$

$\Rightarrow S_{22} = 11(151) = 1661$

Therefore, the sum of first 22 terms of the given AP is 1661.

8. Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

Solution:

As per the question, for the given AP:

$a_2 = 14$

$a_3 = 18$

$d = a_3 - a_2 = 18 - 14 = 4$

$a_2 = a + d$

$\Rightarrow 14 = a + 4$

$\Rightarrow a = 10$

We know that,

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_{51} = \frac{51}{2}[2 \times 10 + (51 - 1)4]$

$= \frac{51}{2}[20 + (50)(4)]$

$= \frac{51(220)}{2} = 51(110)$

$= 5610$

Therefore, the sum of first 51 terms of the given AP is 5610.

9. If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

Solution:

As per the question, for the given AP:

$S_7 = 49, S_{17} = 289$

We know that,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow S_7 = \frac{7}{2}[2a + (7-1)d]$$

$$\Rightarrow 49 = \frac{7}{2}(2a + 6d)$$

$$\Rightarrow 7 = (a + 3d)$$

$$\Rightarrow a + 3d = 7 \quad \text{... (i)}$$

Similarly, $S_{17} = \frac{17}{2}[2a + (17-1)d]$

$$\Rightarrow 289 = \frac{17}{2}[2a + 16d]$$

$$\Rightarrow 17 = (a + 8d)$$

$$\Rightarrow a + 8d = 17 \quad \text{... (ii)}$$

By subtracting equation (i) from equation (ii),

$$\Rightarrow 5d = 10$$

$$\Rightarrow d = 2$$

From equation (i),

$$a + 3(2) = 7$$

$$\Rightarrow a + 6 = 7$$

$$\Rightarrow a = 1$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Substituting the values,

$$= \frac{n}{2}[2(1) + (n-1)(2)]$$

$= \frac{n}{2}(2 + 2n - 2)$

$= \frac{n}{2}(2n)$

$= n^2$

10. Show that $a_1, a_2, \dots, a_n, \dots$ form an AP where a_n is defined as below:

(i) $a_n = 3 + 4n$

(ii) $a_n = 9 - 5n$

Also find the sum of the first 15 terms in each case.

Solution:

(i) $a_n = 3 + 4n \ \ldots\ldots (1)$

$a_{k+1} = 3 + 4(k + 1)$

$= 3 + 4k + 4$

Here, $a_{k+1} - a_k = (3 + 4k + 4) - (3 + 4k) = 4$ which is independent of k.

So, this is an AP with common difference, $d = 4$.

Also, by substituting $n = 1$ in equation (1), we get first term of AP i.e., $a = 7$.

We know that,

$S_n = \frac{n}{2}[2a + (n - 1)d]$

Substituting the values,

$S_{15} = \frac{15}{2}[2(7) + (15 - 1)4]$

$= \frac{15}{2}[(14) + 56]$

$= \frac{15}{2}(70)$

$= 15 \times 35$

$= 525$

Therefore, the sum of the first 15 terms of this AP is 525.

(ii) $a_n = 9 - 5n$ ………… (1)

Here $a_{k+1} - a_k = (9 - 5k - 5) - (9 - 5k)$

$= -5$ which is independent of k.

So, this is an AP with common difference, $d = -5$ and

We obtain first term, $a = 4$, by substituting $n = 1$ in equation (1)

As, $S_n = \frac{n}{2}[2a + (n-1)d]$

Substituting the values,

$S_{15} = \frac{15}{2}[2(4) + (15-1)(-5)]$

$= \frac{15}{2}[8 + 14(-5)]$

$= \frac{15}{2}(8 - 70)$

$= \frac{15}{2}(-62) = 15(-31)$

$= -465$

Therefore, the sum of the first 15 terms of this AP is -465.

11. If the sum of the first n terms of an AP is $4n - n^2$, what is the first term (that is S_1)? What is the sum of first two terms? What is the second term? Similarly, find the 3rd, the 10th and the nth terms.

Solution:

According to the question, for a given AP: $S_n = 4n - n^2$

First term, $a = S_1 = 4(1) - (1)^2 = 4 - 1 = 3$

Sum of first two terms $= S_2$

$= 4(2) - (2)^2 = 8 - 4 = 4$

We know that,

$a_n = S_n - S_{n-1}$

Substituting the values,

Second term, $a_2 = S_2 - S_1 = 4 - 3 = 1$

$d = a_2 - a_1 = a_2 - a = 1 - 3 = -2$

$a_n = a + (n-1)d$

$= 3 + (n-1)(-2)$

$= 3 - 2n + 2$

$= 5 - 2n$

Thus, $a_3 = 5 - 2(3) = 5 - 6 = -1$

$a_{10} = 5 - 2(10) = 5 - 20 = -15$

Therefore, the first term is 3 and the sum of first two terms is 4. The second term is $1.3^{\text{rd}}, 10^{\text{th}}$ and n^{th} terms are $-1, -15$ and $5 - 2n$ respectively.

12. Find the sum of the first 40 positive integers divisible by 6.

Solution:

The positive integers that are divisible by 6 are $6, 12, 18, 24$ ...

Now, an AP is formed with these terms, whose first term is 6 and common difference is 6.

$a = 6, d = 6, S_{40} = ?$

We know that,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow S_{40} = \frac{40}{2}[2(6) + (40-1)6]$$

$= 20[12 + (39)(6)]$

$= 20(12 + 234)$

$= 20 \times 246$

$= 4920$

Hence, the sum of the first 40 positive integers divisible by 6 is 4920.

13. Find the sum of the first 15 multiples of 8.

Solution:

Multiples of 8 are $8, 16, 24, 32$ ...

Now, these terms are in an AP, whose first term is 8 and common difference is 8.

Therefore, $a = 8$, $d = 8, S_{15} = ?$

We know that,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Substituting the values,

$$S_{15} = \frac{15}{2}[2(8) + (15-1)8]$$

$$= \frac{15}{2}[16 + 14(8)]$$

$$= \frac{15}{2}(16 + 112)$$

$$= \frac{15(128)}{2} = 15 \times 64$$

$$= 960$$

Hence, the sum of the first 15 multiples of 8 is 960.

14. Find the sum of the odd numbers between 0 and 50.

Solution:

We can observe clearly that the odd numbers between 0 and 50 are $1, 3, 5, 7, 9 \ldots 49$

So, it can be noticed that these odd numbers are in an AP.

$a = 1, d = 2, l = 49$

We know that,

$$a_n = l = a + (n-1)d$$

$$\Rightarrow 49 = 1 + (n-1)2$$

$$\Rightarrow 48 = 2(n-1)$$

$$\Rightarrow n - 1 = 24$$

So, $n = 25$

We also know that,

$$S_n = \frac{n}{2}(a + l)$$

$$S_{25} = \frac{25}{2}(1 + 49)$$

$$= \frac{25(50)}{2} = (25)(25)$$

$= 625$

Hence, the sum of the odd numbers between 0 and 50 is 625.

15. A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: ₹ 200 for the first day, ₹ 250 for the second day, ₹ 300 for the third day, etc., the penalty for each succeeding day being ₹ 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days?

Solution:

As per the question, penalties are in AP having first term as 200 and common difference as 50.

$a = 200, d = 50$

The penalty that has to be paid if contractor has delayed the work by 30 days = S_{30}

We know that,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Substituting the values,

$$S_{30} = \frac{30}{2}[2(200) + (30-1)50]$$

$= 15[400 + 1450]$

$= 15(1850)$

$= 27750$

Hence, the contractor has to pay ₹ 27750 as penalty, if he has delayed the work by 30 days.

16. A sum of ₹ 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is ₹ 20 less than its preceding prize, find the value of each of the prizes.

Solution:

Let the cost of 1^{st} prize be C.

Cost of 2^{nd} prize $= C - 20$

and cost 3^{ed} prize $= (C - 20) - 20$

Cost of the prizes are forming an AP with common difference, -20 and first term as C.

$a = C, d = -20$

Given that, $S_7 = 700$

We know that,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Substituting the values,

$$\frac{7}{2}[2C + (7-1)d] = 700$$

$$\Rightarrow \frac{[2C + (6)(-20)]}{2} = 100$$

$$\Rightarrow C + 3(-20) = 100$$

$$\Rightarrow C - 60 = 100$$

$$\Rightarrow C = 160$$

Hence, the values of the prizes (in ₹) are $160, 140, 120, 100, 80, 60$ and 40.

17. In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of Class I will plant 1 tree, a section of Class II will plant 2 trees and so on till Class XII. There are three sections of each class. How many trees will be planted by the students?

Solution:

To solve this question, let's first calculate the number of trees planted by students of one section of all the classes from I to XII.

Then the number of trees planted by students of all the three sections of all the classes from I to XII gives us the total number of trees planted by the students.

As per the question, number of trees planted by the students of one section of all classes forms as AP as follows:

$1, 2, 3, 4, 5, \ldots, 12$

First term, $a = 1$

Common difference, $d = 2 - 1 = 1$

We know that,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Substituting the values,

$$S_{12} = \frac{12}{2}[2(1) + (12-1)(1)]$$

$= 6(2 + 11)$

$= 6(13)$

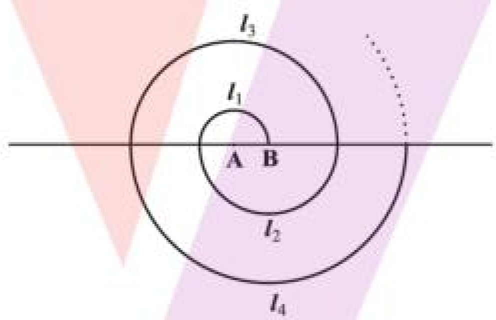
$= 78$

Thus, the number of trees planted by one section of the classes $= 78$

Then the number of trees planted by three sections of the classes $= 3 \times 78 = 234$

Hence, the total number of trees planted by the school students is 234.

18. A spiral is made up of successive semicircles, with centres alternately at A and B, starting with centre at A, of radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm, . . . as shown in Fig. What is the total length of such a spiral made up of thirteen consecutive semicircles? (Take $\pi = \frac{22}{7}$)



[*Hint:* Length of successive semicircles is $l_1, l_2, l_3, l_4, \ldots$ with centres at A, B, A, B, . . . ,respectively.]

Solution:

Semi-perimeter of circle $= \pi r$

$l_1 = \pi(0.5) = \frac{\pi}{2}$ cm

$l_2 = \pi(1) = \pi$ cm

$l_3 = \pi(1.5) = \frac{3\pi}{2}$ cm

So, $l_1, l_2, l_3, \dots$ i. e. the lengths of the semi-circles form an AP,

$$\frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi, \dots$$

$$a = \frac{\pi}{2}$$

$$d = \pi - \frac{\pi}{2} = \frac{\pi}{2}$$

$$S_{13} = ?$$

We know that the sum of n terms of an AP is given by

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow S_{13} = \frac{13}{2}\left[2\left(\frac{\pi}{2}\right) + (13-1)\left(\frac{\pi}{2}\right)\right]$$

$$= \frac{13}{2}\left[\pi + \frac{12\pi}{2}\right]$$

$$= \left(\frac{13}{2}\right)(7\pi)$$

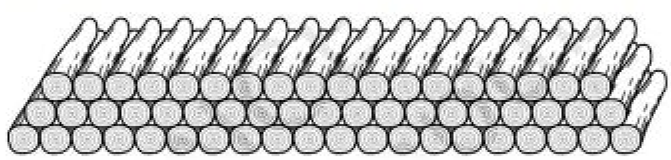
$$= \frac{91\pi}{2}$$

$$= \frac{91 \times 22}{2 \times 7} = 13 \times 11$$

$$= 143$$

Hence, the total length of such spiral of thirteen consecutive semi-circles is 143 cm.

19. 200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row,18 in the row next to it and so on (see Figure). In how many rows are the 200 logs placed and how many logs are in the top row?



Solution:

According to the question, here the number of logs in each row are in an AP

20, 19, 18 ...

For the given AP,

$a = 20$

$d = a_2 - a_1 = 19 - 20 = -1$

Let a total of 200 logs be placed in n rows.

$S_n = 200$

We know that,

$S_n = \frac{n}{2}[2a + (n-1)d]$

Substituting the values,

$200 = \frac{n}{2}[2(20) + (n-1)(-1)]$

$\Rightarrow 400 = n(40 - n + 1)$

$\Rightarrow 400 = n(41 - n)$

$\Rightarrow 400 = 41n - n^2$

$\Rightarrow n^2 - 41n + 400 = 0$

$\Rightarrow n^2 - 16n - 25n + 400 = 0$ [Factorisation by splitting the middle term]

$\Rightarrow n(n-16) - 25(n-16) = 0$

$\Rightarrow (n-16)(n-25) = 0$

Either $(n-16) = 0$ or $(n-25) = 0$

$\Rightarrow n = 16$ or $n = 25$

$a_n = a + (n-1)d$

$a_{16} = 20 + (16-1)(-1)$

$a_{16} = 20 - 15$

$a_{16} = 5$

Similarly,

$a_{25} = 20 + (25-1)(-1)$

$a_{25} = 20 - 24$

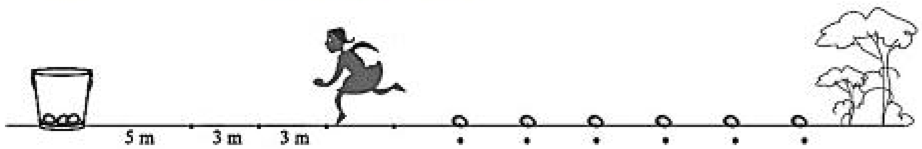
$= -4$

Clearly, the number of logs in 16^{th} row i.e., the top row is 5.

But, the number of logs in 25^{th} row is negative, which is not possible.

Hence, 200 logs can be placed in 16 rows and the number of logs in the 16^{th} row (top) is 5.

20. In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato, and the other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line (see Figure).



A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?

[*Hint:* To pick up the first potato and the second potato, the total distance (in metres) run by a competitor is $2 \times 5 + 2 \times (5 + 3)$]

Solution:

The total distance covered by a competitor to pick the first potato $= 2 \times 5 = 10$

The total distance covered by a competitor to pick the second potato $= 2 \times (5 + 3) = 16$

So, the distances covered by a competitor to pick the potatoes form an AP as follows: $10, 16, 22, 28, \ldots$

Here, $a = 10, d = 16 - 10 = 6$ and $n = 10$.

We know that the sum of n terms of an AP is given by $S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_{10} = \frac{10}{2}[2(10) + (10-1)(6)]$

$= 5[20 + 54]$

$= 5(74)$

$= 370$ m

Hence, the total distance covered by the competitor is 370 m.

◆ ◆ ◆

EXERCISE 5.4 (Optional)*

1. Which term of the AP : $121, 117, 113, \ldots$, is its first negative term?

[*Hint*: Find n for $a_n < 0$]

Solution:

Given AP is $121, 117, 113$ ...

$a = 121$

$d = 117 - 121 = -4$

$a_n = a + (n-1)d$

$= 121 + (n-1)(-4)$

$= 121 - 4n + 4$

$= 125 - 4n$

We have to find the first negative term of this AP

Therefore, $a_n < 0$

$\Rightarrow 125 - 4n < 0$

$\Rightarrow 125 < 4n$

$\Rightarrow n > \frac{125}{4}$

$\Rightarrow n > 31.25$

Therefore, 32^{nd} term will be the first negative term of this AP

2. The sum of the third and the seventh terms of an AP is 6 and their product is 8. Find the sum of first sixteen terms of the AP.

Solution:

We know that,

$a_n = a + (n-1)d$

$a_3 = a + (3-1)d$

$\Rightarrow a_3 = a + 2d$

Similarly, $a_7 = a + 6d$

Given that, $a_3 + a_7 = 6$

$\Rightarrow (a + 2d) + (a + 6d) = 6$

$\Rightarrow 2a + 8d = 6$

$\Rightarrow a + 4d = 3$

$\Rightarrow a = 3 - 4d \quad ...(i)$

Also, it is given that $(a_3) \times (a_7) = 8$

$\Rightarrow (a + 2d) \times (a + 6d) = 8$

From equation (i),

$(3 - 4d + 2d) \times (3 - 4d + 6d) = 8$

$\Rightarrow (3 - 2d) \times (3 + 2d) = 8$

$\Rightarrow 9 - 4d^2 = 8$

$\Rightarrow 4d^2 = 9 - 8 = 1$

$\Rightarrow d^2 = \frac{1}{4}$

$\Rightarrow d = \pm\frac{1}{2}$

$\Rightarrow d = \frac{1}{2}$ or $-\frac{1}{2}$

From equation (i),

(When d is $\frac{1}{2}$)

$a = 3 - 4d$

$\Rightarrow a = 3 - 4\left(\frac{1}{2}\right)$

$= 3 - 2 = 1$

(When d is $-\frac{1}{2}$)

$a = 3 - 4\left(-\frac{1}{2}\right)$

$\Rightarrow a = 3 + 2 = 5$

$S_n = \frac{n}{2}[2a + (n - 1)d]$

(When a is 1 and d is $\frac{1}{2}$)

$S_{16} = \frac{16}{2}\left[2(1) + (16 - 1)\left(\frac{1}{2}\right)\right]$

$= 8\left[2 + \frac{15}{2}\right]$

$= 4(19) = 76$

(When a is 5 and d is $-\frac{1}{2}$)

$$S_{16} = \frac{16}{2}\left[2(5) + (16-1)\left(-\frac{1}{2}\right)\right]$$

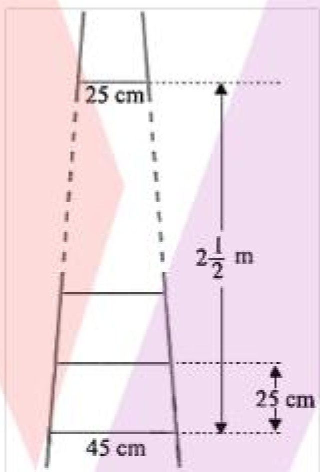
$$= 8\left[10 + (15)\left(-\frac{1}{2}\right)\right]$$

$$= 8\left(\frac{5}{2}\right)$$

$$= 20$$

3. A ladder has rungs 25 cm apart. (See Fig.). The rungs decrease uniformly in length from 45 cm at the bottom to 25 cm at the top. If the top and the bottom rungs are $2\frac{1}{2}$ m apart, what is the length of the wood required for the rungs?

[*Hint:* Number of rungs $= \frac{250}{25} + 1$]



Solution:

It is given that the rungs are 25 cm apart and top and bottom rungs are $2\frac{1}{2}$ m apart.

$\therefore$ Total number of rungs $= \frac{2\frac{1}{2}\times 100}{25} + 1 = \frac{250}{25} + 1 = 11$

Now, as the lengths of the rungs decrease uniformly, they will be in an AP

The length of the wood required for the rungs equals the sum of all the terms of this AP

First term, $a = 45$

Last term, $l = 25$

$S_n = \frac{n}{2}(a + l)$

$\therefore\ S_{11} = \frac{11}{2}(45 + 25) = \frac{11}{2}(70) = 385$ cm

Therefore, the length of the wood required for the rungs is 385 cm.

4. The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it. Find this value of x.

[*Hint*: $S_{x-1} = S_{49} - S_x$]

Solution:

The number of houses was $1, 2, 3, \dots, 49$

It can be observed that the number of houses is in an AP having a as 1 and d also as 1.

We know that,

Sum of n terms in an AP $= \frac{n}{2}[21 + (n-1)d]$

Sum of number of houses preceding x^{th} house $= S_{x-1}$

$= \frac{(x-1)}{2}[2a + (x-1-1)d]$

$= \frac{x-1}{2}[2(1) + (x-2)(1)]$

$= \frac{x-1}{2}[2 + x - 2]$

$= \frac{(x)(x-1)}{2}$

Sum of number of houses following x^{th} house $S_{49} - S_x$

$= \frac{49}{2}[2(1) + (49-1)(1)] - \frac{x}{2}[2(1) + (x-1)(1)]$

$= \frac{49}{2}(2 + 49 - 1) - \frac{x}{2}(2 + x - 1)$

$= \left(\frac{49}{2}\right)(50) - \frac{x}{2}(x + 1)$

$= 25(49) - \frac{x(x+1)}{2}$

It is given that these sums are equal to each other.

Hence, $\frac{x(x-1)}{2} = 25(49) - x\frac{(x+1)}{2}$

$\frac{x^2}{2} - \frac{x}{2} = 1225 - \frac{x^2}{2} - \frac{x}{2}$

$x^2 = 1225$

$x = \pm 35$

However, the house numbers are positive integers.

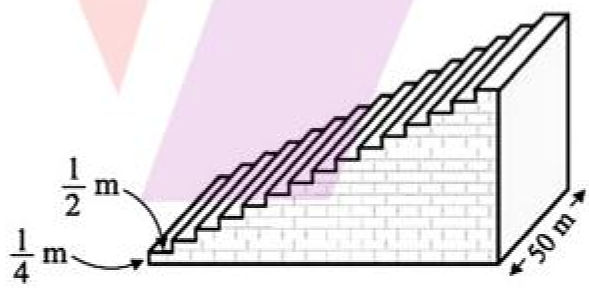
The value of x will be 35 only.

Therefore, house number 35 is such that the sum of the numbers of houses preceding the house numbered 35 is equal to the sum of the numbers of the houses following it.

5. A small terrace at a football ground comprises of 15 steps each of which is 50 m long and built of solid concrete.

Each step has a rise of $\frac{1}{4}$ m and a tread of $\frac{1}{2}$ m. (see Fig.). Calculate the total volume of concrete required to build the terrace.

[*Hint:* Volume of concrete required to build the first step $= \frac{1}{4} \times \frac{1}{2} \times 50 \text{ m}^3$]



Solution:

From the figure, we see that

1^{st} step has $\frac{1}{4}$ m height,

2^{nd} step has $\frac{1}{2}$ m height,

3^{ed} step has $\frac{3}{4}$ m height.

Therefore, the height of each step is increasing by $\frac{1}{4}$ m each time whereas their width $\frac{1}{2}$ m and length 50 m remains the same.

Volume of concrete in 1st step $= \frac{1}{4} \times \frac{1}{2} \times 50 = \frac{25}{4}$

Volume of concrete in 2^{ed} step $= \frac{1}{2} \times \frac{1}{2} \times 50 = \frac{25}{2}$

Volume of concrete in 3^{ed} step $= \frac{1}{2} \times \frac{3}{4} \times 50 = \frac{75}{4}$

It can be observed that the volumes of concrete in these steps are in an AP

$$\frac{25}{4}, \frac{25}{2}, \frac{75}{4}, \ldots$$

$$a = \frac{25}{4}$$

$$d = \frac{25}{2} - \frac{25}{4} = \frac{25}{4}$$

and $S_n = \frac{n}{2}[2a + (n-1)d]$

$$S_{15} = \frac{15}{2}\left[2\left(\frac{25}{4}\right) + \frac{(15-1)25}{4}\right]$$

$$= \frac{15}{2}\left[\frac{25}{2} + \frac{(14)25}{4}\right]$$

$$= \frac{15}{2}\left[\frac{25}{2} + \frac{175}{2}\right]$$

$$= \frac{15}{2}(100) = 750$$

Volume of concrete required to build the terrace is 750 m^3.

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