

CBSE NCERT Solutions for Class 10 Mathematics Chapter 3

Back of Chapter Questions

1. Aftab tells his daughter, “Seven years ago, I was seven times as old as you were then. Also, three years from now, I shall be three times as old as you will be.” (Isn’t this interesting?) Represent this situation algebraically and graphically.

Solution:

Let the present age of Aftab be x and present age of daughter be y .

Hence, seven years ago,

$$\text{Age of Aftab} = x - 7$$

$$\text{Age of daughter} = y - 7$$

$$\text{Hence, as per the given condition, } (x - 7) = 7(y - 7)$$

$$\Rightarrow x - 7 = 7y - 49$$

$$\Rightarrow x - 7y = -42 \dots\dots\dots(i)$$

Three years later,

$$\text{Age of Aftab} = x + 3$$

$$\text{Age of daughter} = y + 3$$

$$\text{Hence, as per the given condition, } (x + 3) = 3(y + 3)$$

$$\Rightarrow x + 3 = 3y + 9$$

$$\Rightarrow x - 3y = 6 \dots\dots\dots(ii)$$

Hence, equation (i) and (ii) represent given conditions algebraically as:

$$x - 7y = -42$$

$$x - 3y = 6$$

Graphical Representation:

$$x - 7y = -42$$

$$\Rightarrow x = -42 + 7y$$

Two solutions of this equation are:

x	-7	0
y	5	6

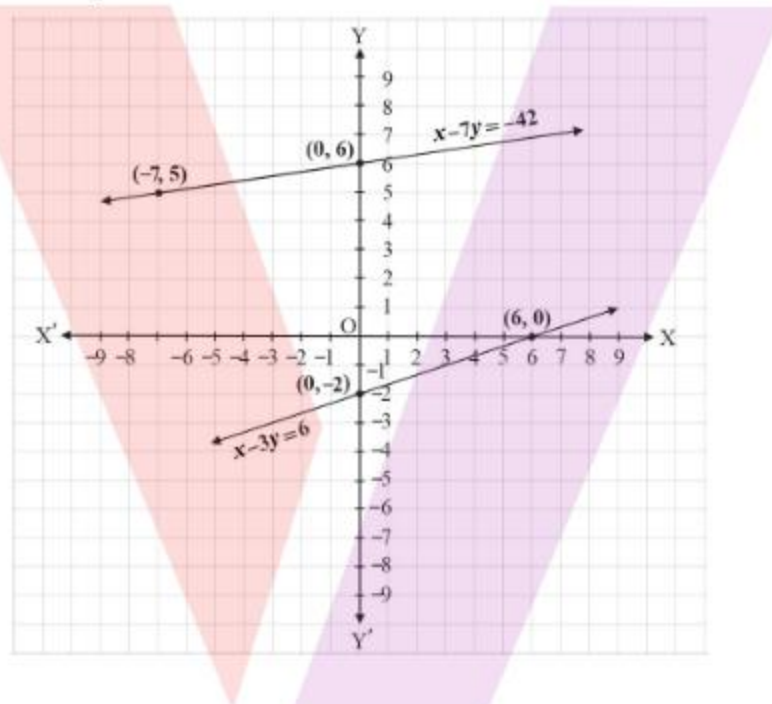
$$x - 3y = 6$$

$$\Rightarrow x = 6 + 3y$$

Two solutions of this equation are:

x	6	0
y	0	-2

The graphical representation is as follows:



2. The coach of a cricket team buys 3 bats and 6 balls for ₹ 3900. Later, she buys another bat and 3 more balls of the same kind for ₹ 1300. Represent this situation algebraically and geometrically.

Solution:

Let the price of a bat be ₹ x and a ball be ₹ y .

Hence, we can represent algebraically the given conditions as:

$$3x + 6y = 3900$$

$$x + 3y = 1300$$

$$3x + 6y = 3900 \Rightarrow x = \frac{3900 - 6y}{3}$$

Two solutions of this equation are:

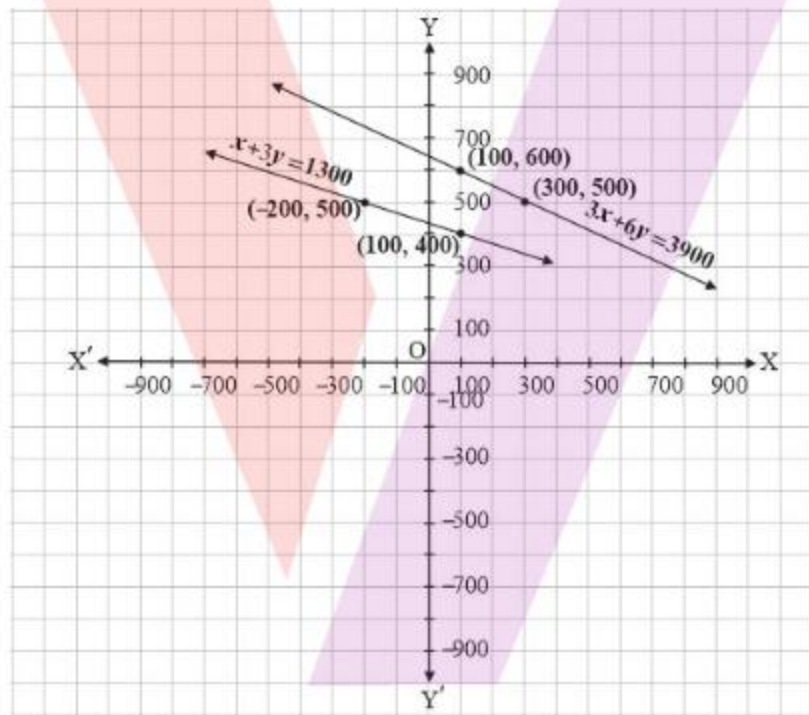
x	300	100
y	500	600

$$x + 3y = 1300 \Rightarrow x = 1300 - 3y$$

Two solutions of this equation are:

x	100	-200
y	400	500

The graphical representation is as follows:



3. The cost of 2 kg of apples and 1 kg of grapes on a day was found to be ₹ 160. After a month, the cost of 4 kg of apples and 2 kg of grapes is ₹ 300. Represent the situation algebraically and geometrically.

Solution:

Let the cost of 1 kg of apples be ₹ x and 1 kg grapes be ₹ y .

The given conditions can be algebraically represented as:

$$2x + y = 160 \quad (1)$$

$$4x + 2y = 300 \quad (2)$$

$$2x + y = 160 \Rightarrow y = 160 - 2x$$

Two solutions of this equation are:

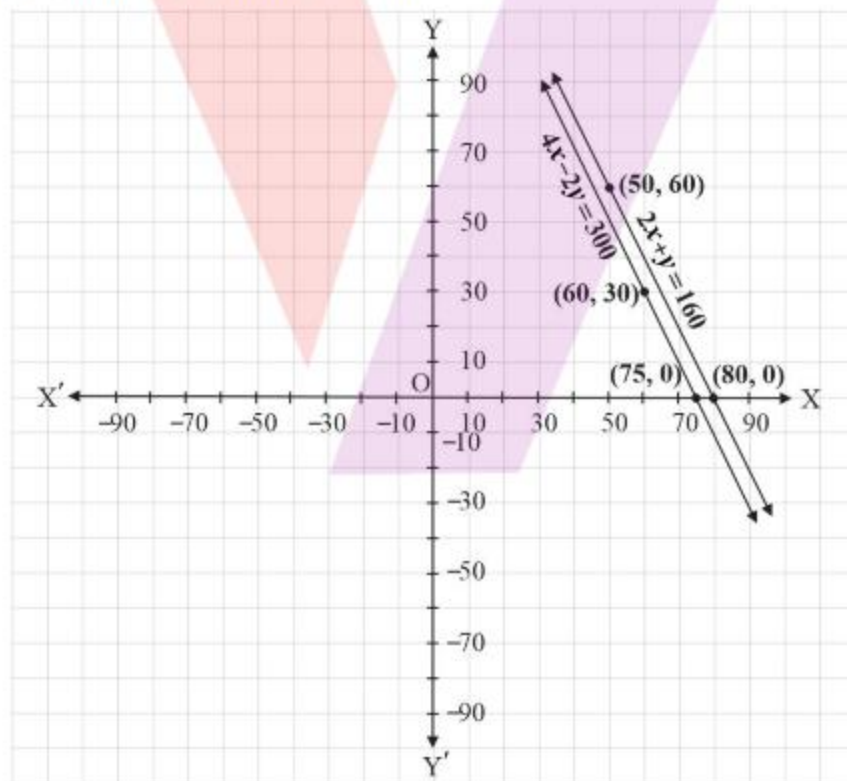
x	50	80
y	60	0

$$4x + 2y = 300 \Rightarrow y = \frac{300 - 4x}{2}$$

Two solutions of this equation are:

x	60	75
y	30	0

The graphical representation is as follows:



EXERCISE 3.2

1. Form the pair of linear equations in the following problems and find their solutions graphically.

(i) 10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

(ii) 5 Pencils and 7 pens together cost ₹ 50, whereas 7 pencils and 5 pens together cost ₹ 46. Find the cost of one pencil and that of one pen.

Solution:

(i) Let the number of girls in the class be x and number of boys in the class be y .

As per the given conditions

$$x + y = 10 \quad (1)$$

$$x - y = 4 \quad (2)$$

$$x + y = 10 \Rightarrow x = 10 - y$$

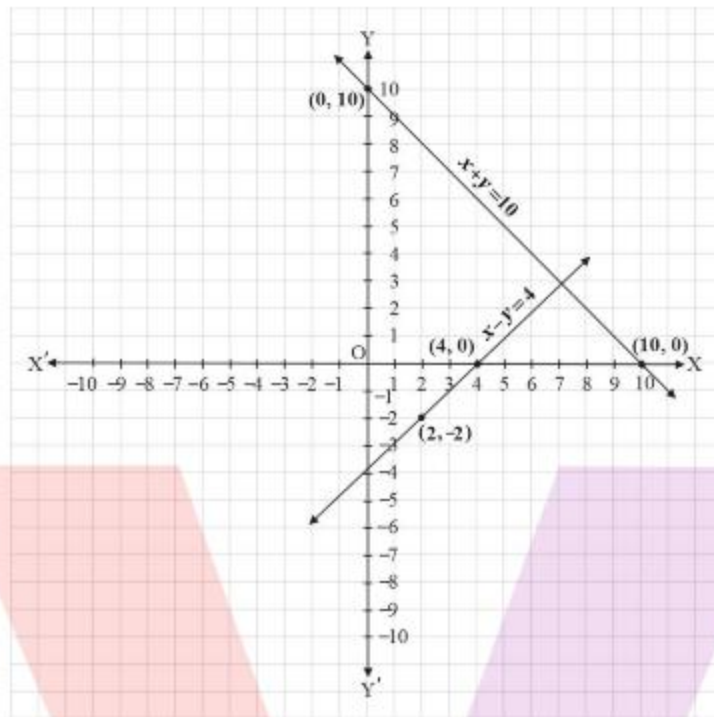
Two solutions of this equation are:

x	0	10
y	10	0

$$x - y = 4 \Rightarrow x = 4 + y$$

Two solutions of this equation are:

x	2	4
y	-2	0



The graphical representation is as follows:

From the graph, it can be observed that the two lines intersect each other at the point $(7, 3)$.

So, $x = 7$ and $y = 3$.

Hence, the number of girls and boys in the class are 7 and 3 respectively.

(ii) Let the cost of one pencil be ₹ x and one pen be ₹ y respectively.

As per the given conditions,

$$5x + 7y = 50 \quad (1)$$

$$7x + 5y = 46 \quad (2)$$

$$5x + 7y = 50 \Rightarrow x = \frac{50 - 7y}{5}$$

Two solutions of this equation are:

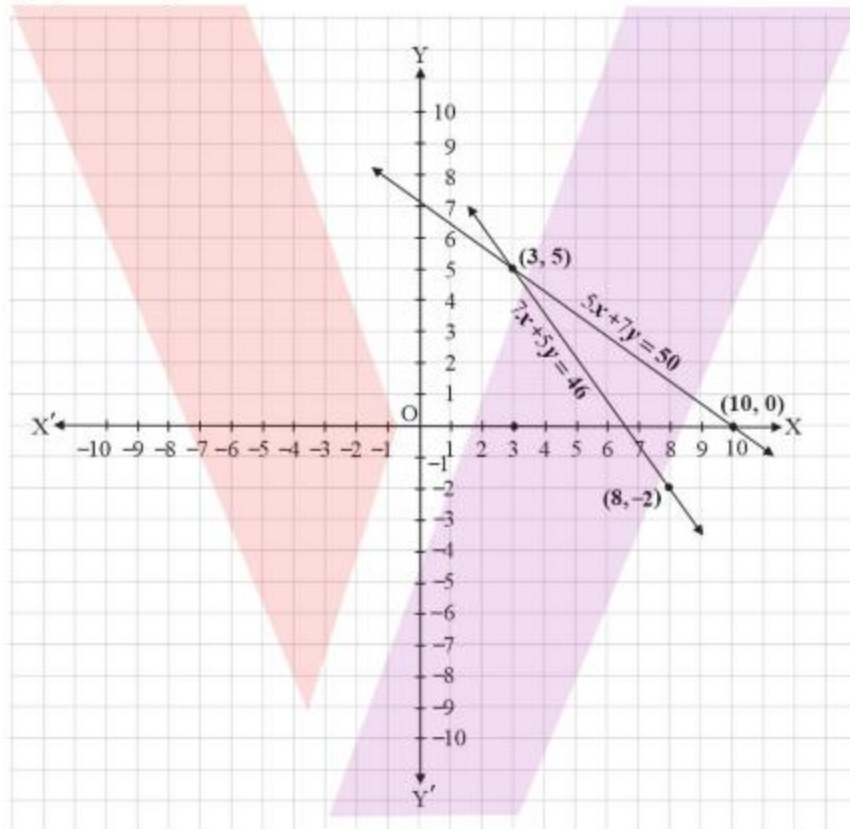
x	3	10
y	5	0

$$7x + 5y = 46 \Rightarrow x = \frac{46 - 5y}{7}$$

Two solutions of this equation are:

x	8	3
y	-2	5

The graphical representation is as follows:



From the graph, it can be observed that the two lines intersect each other at the point (3, 5).

So, $x = 3$ and $y = 5$.

Therefore, the cost of one pencil and one pen are ₹3 and ₹5 respectively.

2. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincident:

(i)	$5x - 4y + 8 = 0$	$7x + 6y - 9 = 0$
(ii)	$9x + 3y + 12 = 0$	$18x + 6y + 24 = 0$
(iii)	$6x - 3y + 10 = 0$	$2x - y + 9 = 0$

Solution:

(i) $5x - 4y + 8 = 0$

$7x + 6y - 9 = 0$

Comparing pair of equations with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we get

$a_1 = 5, b_1 = -4, c_1 = 8$

$a_2 = 7, b_2 = 6, c_2 = -9$

$\frac{a_1}{a_2} = \frac{5}{7}$

$\frac{b_1}{b_2} = \frac{-4}{6} = \frac{-2}{3}$

Since, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, the given pair of equations intersect at exactly one point.

(ii) $9x + 3y + 12 = 0$

$18x + 6y + 24 = 0$

Comparing pair of equations with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we get:

$a_1 = 9, b_1 = 3, c_1 = 12$

$a_2 = 18, b_2 = 6, c_2 = 24$

$\frac{a_1}{a_2} = \frac{9}{18} = \frac{1}{2}$

$\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$

$\frac{c_1}{c_2} = \frac{12}{24} = \frac{1}{2}$ Since, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ the given pair of equation are coincident.

(iii) $6x - 3y + 10 = 0$

$2x - y + 9 = 0$

Comparing pair of equations with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we get:

$a_1 = 6, b_1 = -3, c_1 = 10$

$a_2 = 2, b_2 = -1, c_2 = 9$

$\frac{a_1}{a_2} = \frac{6}{2} = \frac{3}{1}$

$\frac{b_1}{b_2} = \frac{-3}{-1} = \frac{3}{1}$

$\frac{c_1}{c_2} = \frac{10}{9}$

Since, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ the given pair of equation are parallel to each other.

3. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the following pair of linear equations are consistent, or inconsistent.

(i)	$3x + 2y = 5$	$2x - 3y = 7$
(ii)	$2x - 3y = 8$	$4x - 6y = 9$
(iii)	$\frac{3}{2}x + \frac{5}{3}y = 7$	$9x - 10y = 14$
(iv)	$5x - 3y = 11$	$-10x + 6y = -22$
(v)	$\frac{4}{3}x + 2y = 8$	$2x + 3y = 12$

Solution:

(i) $3x + 2y = 5$

$2x - 3y = 7$

Comparing pair of equations with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we get:

$a_1 = 3, b_1 = 2, c_1 = -5$

$a_2 = 2, b_2 = -3, c_2 = -7$

$$\frac{a_1}{a_2} = \frac{3}{2}, \frac{b_1}{b_2} = \frac{2}{-3} = -\frac{2}{3}, \frac{c_1}{c_2} = \frac{-5}{-7} = \frac{5}{7}$$

Since, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ the given pair of equation has only one solution.

Hence, the pair of linear equations is consistent.

(ii) $2x - 3y = 8$

$$4x - 6y = 9$$

Comparing pair of equations with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we get:

$$a_1 = 2, b_1 = -3, c_1 = -8$$

$$a_2 = 4, b_2 = -6, c_2 = -9$$

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{-3}{-6} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{-8}{-9} = \frac{8}{9}$$

Since, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ the given pair of equation has no solution.

Hence, the pair of linear equations is inconsistent.

(iii) $\frac{3}{2}x + \frac{5}{3}y = 7$

$$9x - 10y = 14$$

Comparing pair of equations with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we get:

$$a_1 = \frac{3}{2}, b_1 = \frac{5}{3}, c_1 = -7$$

$$a_2 = 9, b_2 = -10, c_2 = -14$$

$$\frac{a_1}{a_2} = \frac{\frac{3}{2}}{9} = \frac{1}{6}, \frac{b_1}{b_2} = \frac{\frac{5}{3}}{-10} = -\frac{1}{6}, \frac{c_1}{c_2} = \frac{-7}{-14} = \frac{1}{2}$$

Since, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ the given pair of equation has only one solution

Hence, the pair of linear equations is consistent.

(iv) $5x - 3y = 11$

$-10x + 6y = -22$

Comparing pair of equations with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we get:

$a_1 = 5, b_1 = -3, c_1 = -11$

$a_2 = -10, b_2 = 6, c_2 = 22$

$$\frac{a_1}{a_2} = \frac{5}{-10} = -\frac{1}{2}, \frac{b_1}{b_2} = \frac{-3}{6} = -\frac{1}{2}, \frac{c_1}{c_2} = \frac{-11}{22} = -\frac{1}{2}$$

Since, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ the given pair of equation has infinite number of solution.

Hence, the pair of linear equations is consistent.

(iv) $\frac{4}{3}x + 2y = 8$

$2x + 3y = 12$

Comparing pair of equations with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we get:

$a_1 = \frac{4}{3}, b_1 = 2, c_1 = -8$

$a_2 = 2, b_2 = 3, c_2 = -12$

$$\frac{a_1}{a_2} = \frac{\frac{4}{3}}{2} = \frac{2}{3}, \frac{b_1}{b_2} = \frac{2}{3}, \frac{c_1}{c_2} = \frac{-8}{-12} = \frac{2}{3}$$

Since, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ the given pair of equation has infinite number of Solution.

Hence, the pair of linear equations is consistent.

4. Which of the following pairs of linear equations are consistent/inconsistent? If consistent, obtain the solution graphically:

(i)	$x + y = 5$	$2x + 2y = 10$
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(ii)	$x - y = 8$	$3x - 3y = 16$
(iii)	$2x + y - 6 = 0$	$4x - 2y - 4 = 0$
(iv)	$2x - 2y - 2 = 0$	$4x - 4y - 5 = 0$

Solution:

(i) $x + y = 5$

$2x + 2y = 10$

Comparing pair of equations with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we get:

$a_1 = 1, b_1 = 1, c_1 = -5$

$a_2 = 2, b_2 = 2, c_2 = -10$

$\frac{a_1}{a_2} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{-5}{-10} = \frac{1}{2}$

Since, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ the given pair of linear equation has infinite number of Solution.

Hence, the pair of linear equations is consistent.

Now, $x + y = 5 \Rightarrow x = 5 - y$

Two solutions of this equation are:

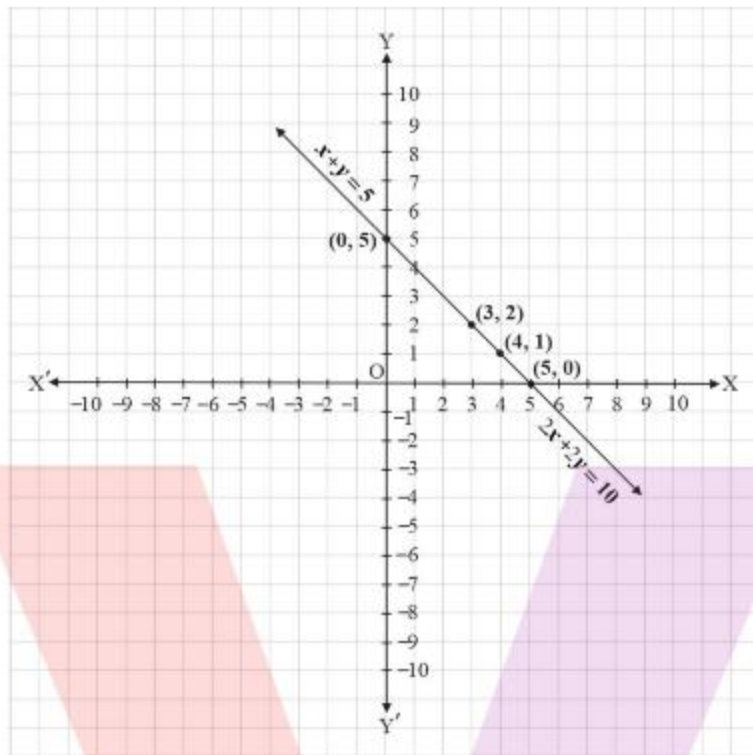
x	5	0
y	0	5

$2x + 2y = 10 \Rightarrow x = \frac{10 - 2y}{2}$

Two solutions of this equation are:

x	4	3
y	1	2

Thus, the graphical representation is as follows:



In the graph, it is observed that the two lines coincide. Hence, the given pair of equations has infinite solutions.

Let $x = t$, then $y = 5 - t$. So, the ordered pair $(t, 5 - t)$, where t is a constant, is the solution of the given pair of linear equations.

(ii) $x - y = 8$

$$3x - 3y = 16$$

Comparing pair of equations with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we get:

$$a_1 = 1, b_1 = -1, c_1 = -8$$

$$a_2 = 3, b_2 = -3, c_2 = -16$$

$$\frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{-1}{-3} = \frac{1}{3}, \frac{c_1}{c_2} = \frac{-8}{-16} = \frac{1}{2}$$

Since, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ the given pair of linear equation has no solution.

Hence, the pair of linear equations is inconsistent.

(iii) $2x + y - 6 = 0$

$4x - 2y - 4 = 0$

Comparing pair of equations with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we get:

$a_1 = 2, b_1 = 1, c_1 = -6$

$a_2 = 4, b_2 = -2, c_2 = -4$

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{1}{-2} = -\frac{1}{2}, \frac{c_1}{c_2} = \frac{-6}{-4} = \frac{3}{2}$$

Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, the given pair of linear equation has only one solution.

Hence, the pair of linear equations is consistent.

Now, $2x + y - 6 = 0 \Rightarrow y = 6 - 2x$

Two solutions of this equation are:

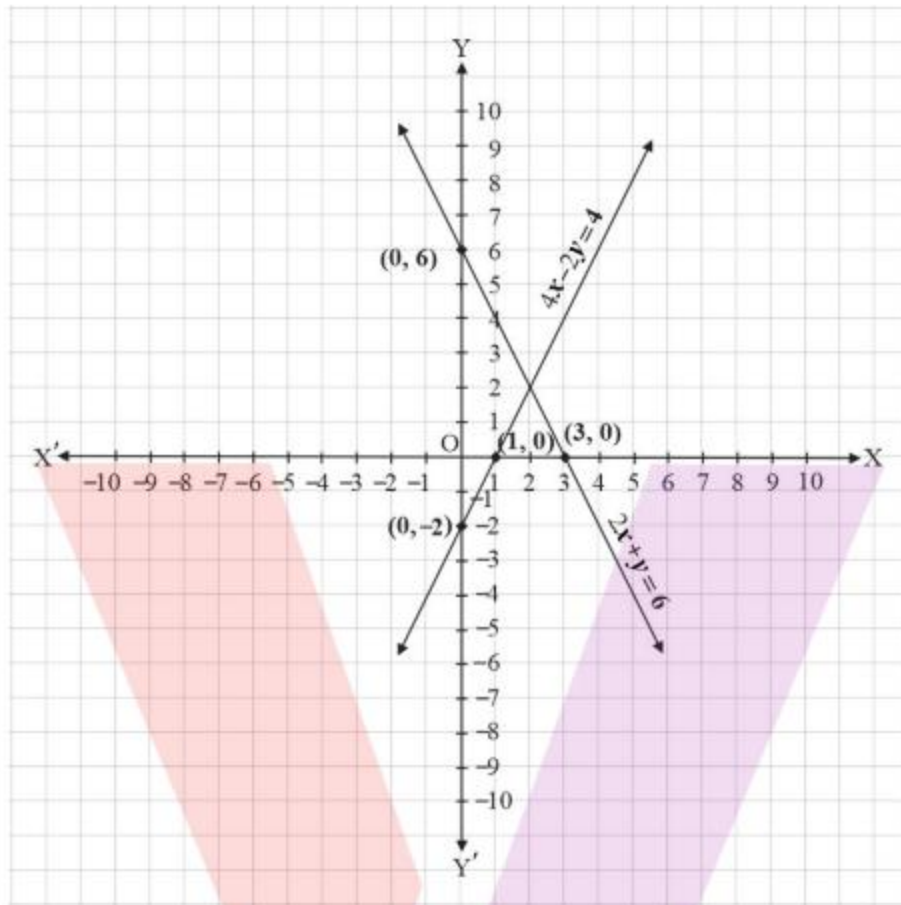
x	0	3
y	6	0

$$4x - 2y - 4 = 0 \Rightarrow y = \frac{4x - 4}{2}$$

Two solutions of this equation are:

x	1	0
y	0	-2

Hence, the graphical representation is as follows:



In the graph, it is observed that the two lines intersect each other at the point $(2, 2)$. Hence, the solution of the given pair of equations is $(2, 2)$.

(iv) $2x - 2y - 2 = 0$

$4x - 4y - 5 = 0$

Comparing pair of equations with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we get:

$a_1 = 2, b_1 = -2, c_1 = -2$

$a_2 = 4, b_2 = -4, c_2 = -5$

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{-2}{-4} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{-2}{-5} = \frac{2}{5}$$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, the given pair of linear equation has no solution

Hence, the pair of linear equations is inconsistent.

5. Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden.

Solution:

Let the length of the rectangular garden be x and width be y .

As per the given conditions,

$$x - y = 4$$

$$x + y = 36$$

$$x - y = 4 \Rightarrow x = y + 4$$

Two solutions of this equation are:

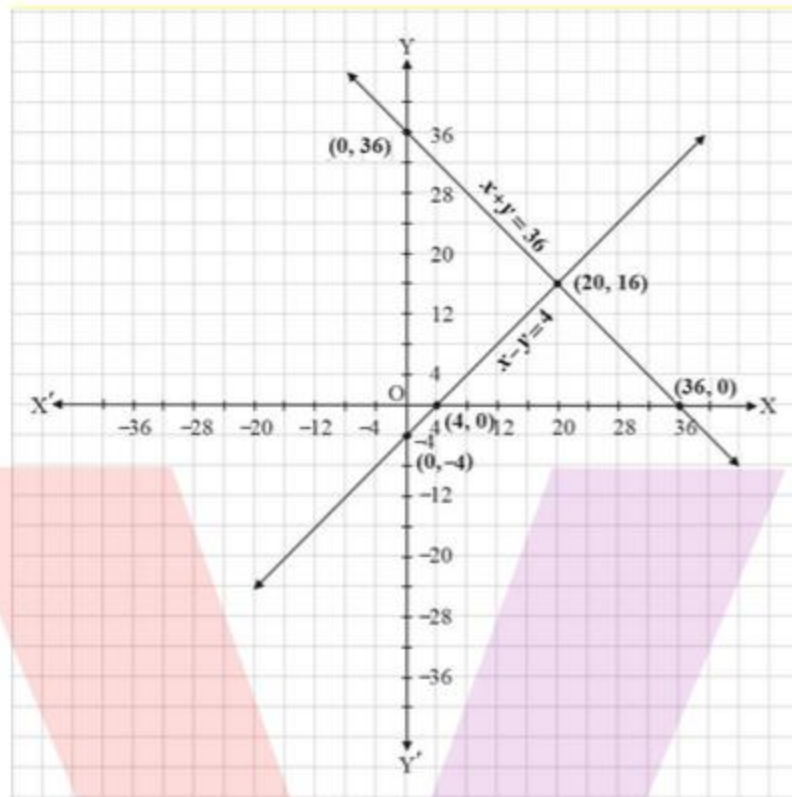
x	0	4
y	-4	0

$$x + y = 36$$

Two solutions of this equation are:

x	0	36
y	36	0

Hence, the graphical representation is as follows:



From the graph, it can be observed that the two lines intersect each other at the point $(20, 16)$. So, $x = 20$ and $y = 16$.

Thus, the length and width of the rectangular garden is 20 m and 16 m respectively.

6. Given the linear equation $2x + 3y - 8 = 0$, write another linear equation in two variables such that the geometrical representation of the pair so formed is:
- intersecting lines
 - parallel lines
 - coincident lines

Solution:

- (i) For the two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, to be intersecting, we must have $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence, the other linear equation can be $3x + 7y - 15 = 0$

$$\frac{a_1}{a_2} = \frac{2}{3}, \frac{b_1}{b_2} = \frac{3}{7}, \frac{c_1}{c_2} = \frac{-8}{-15} = \frac{8}{15}$$

- (ii) For the two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, to be parallel, we must have $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, the other linear equation can be $4x + 6y + 9 = 0$, as

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{-8}{9}$$

- (iii) For the two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, to be parallel, we must have $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

So, the other linear equation can be $6x + 9y - 24 = 0$,

$$\text{as } \frac{a_1}{a_2} = \frac{2}{6} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{3}{9} = \frac{1}{3}, \frac{c_1}{c_2} = \frac{-8}{-24} = \frac{1}{3}$$

7. Draw the graphs of the equations $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Determine the coordinates of the vertices of the triangle formed by these lines and the x-axis and shade the triangular region.

Solution:

$$x - y + 1 = 0 \Rightarrow x = y - 1$$

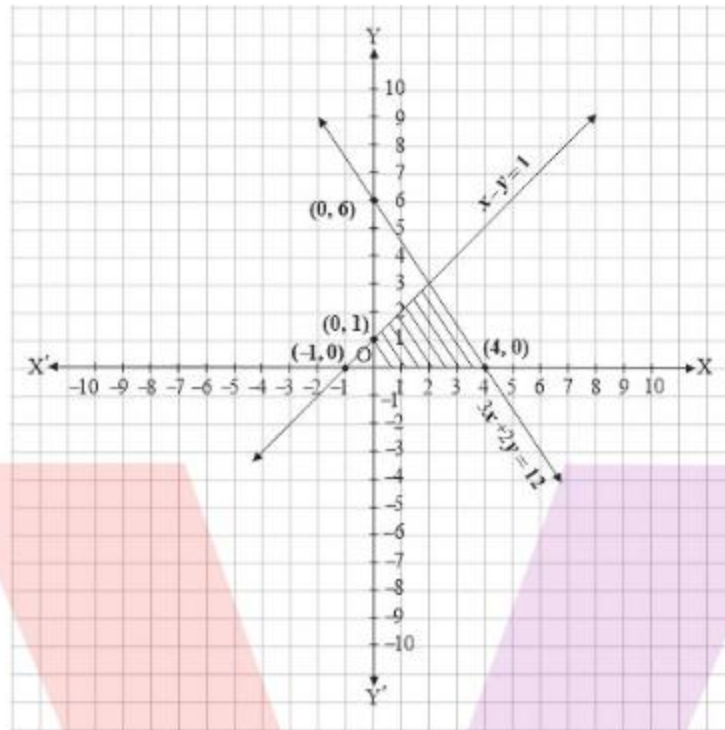
Two solutions of this equation are:

x	0	-1
y	1	0

$$3x + 2y - 12 = 0 \Rightarrow x = \frac{12 - 2y}{3}$$

Two solutions of this equation are:

x	4	0
y	0	6



Now, we have drawn these two equations on graph. Shaded part represents the triangle formed by given two lines and the x-axis:

In the graph, we can observe that the coordinates of the vertices of the triangle are $(2, 3)$, $(-1, 0)$, and $(4, 0)$.



EXERCISE 3.3

1. Solve the following pair of linear equations by the substitution method.

(i)	$x + y = 14$	$x - y = 4$
(ii)	$s - t = 3$	$\frac{s}{3} + \frac{t}{2} = 6$
(iii)	$3x - y = 3$	$9x - 3y = 9$
(iv)	$0.2x + 0.3y = 1.3$	$0.4x + 0.5y = 2.3$
(v)	$\sqrt{2}x + \sqrt{3}y = 0$	$\sqrt{3}x - \sqrt{8}y = 0$
(vi)	$\frac{3x}{2} - \frac{5y}{3} = -2$	$\frac{x}{3} + \frac{y}{2} = \frac{13}{6}$

Solution;

(i) $x + y = 14 \dots$ (i)

$x - y = 4 \dots$ (ii)

From (i), we get:

$x = 14 - y \dots$ (iii)

Substituting the value of x in equation (ii), we get:

$(14 - y) - y = 4$

$\Rightarrow 14 - 2y = 4$

$\Rightarrow 10 = 2y$

$\Rightarrow y = 5$

Substituting the value of y in equation (iii), we get:

$x = 9 \therefore x = 9, y = 5$

(ii) $s - t = 3 \dots$ (i)

$\frac{s}{3} + \frac{t}{2} = 6 \dots$ (ii)

From (i), we get:

(iii) $s = t + 3 \dots$ (iii)

Substituting the value of s in equation (ii), we get: $\frac{t+3}{3} + \frac{t}{2} = 6$

$\Rightarrow 2t + 6 + 3t = 36$

$\Rightarrow 5t = 30$

$\Rightarrow t = 6$

Substituting the value of t in equation (iii), we get:

$s = 9$

$\therefore s = 9, t = 6$

(iv) $3x - y = 3 \dots$ (i)

$9x - 3y = 9 \dots$ (ii)

From (I), we get

$$y = 3x - 3 \dots \text{(iii)}$$

Substituting the value of y in equation (ii), we get:

$$\text{(v)} \quad 9x - 3(3x - 3) = 9$$

$$\Rightarrow 9x - 9x = 9$$

$$\Rightarrow 9 = 9$$

This is always true.

Hence, the given pair of equations has infinitely many solutions and variables are related as

$$y = 3x - 3$$

$$\text{(vi)} \quad 0.2x + 0.3y = 1.3 \dots \text{(i)}$$

$$0.4x + 0.5y = 2.3 \dots \text{(ii)}$$

From (i), we get:

$$x = \frac{1.3 - 0.3y}{0.2} \dots \text{(iii)}$$

Substituting the value of x in equation (ii), we get:

$$0.4 \left(\frac{1.3 - 0.3y}{0.2} \right) + 0.5y = 2.3$$

$$\Rightarrow 2.6 - 0.6y + 0.5y = 2.3$$

$$\Rightarrow 2.6 - 2.3 = 0.1y$$

$$\Rightarrow 0.3 = 0.1y$$

$$\Rightarrow y = 3$$

Substituting the value of y in equation (iii), we get:

$$x = \frac{1.3 - 0.3 \times 3}{0.2} = \frac{0.4}{0.2} = 2$$

$$\therefore x = 2, y = 3$$

$$\text{(v)} \quad \sqrt{2}x + \sqrt{3}y = 0 \dots \text{(i)}$$

$$\sqrt{3}x - \sqrt{8}y = 0 \dots \text{(ii)}$$

From equation (i), we get:

$$x = \frac{-\sqrt{3}y}{\sqrt{2}} \dots \text{(iii)}$$

Substituting the value of x in equation (ii), we get:

$$\sqrt{3} \left(-\frac{\sqrt{3}y}{\sqrt{2}} \right) - \sqrt{8}y = 0$$

$$\Rightarrow -\frac{3y}{\sqrt{2}} - 2\sqrt{2}y = 0$$

$$\Rightarrow y \left(-\frac{3}{\sqrt{2}} - 2\sqrt{2} \right) = 0$$

$$\Rightarrow y = 0$$

Substituting the value of y in equation (iii), we get:

$$x = 0$$

$$\therefore x = 0, y = 0$$

$$\text{(vi)} \quad \frac{3}{2}x - \frac{5}{3}y = -2 \dots \text{(i)}$$

$$\frac{x}{3} + \frac{y}{2} = \frac{13}{6} \dots \text{(ii)}$$

From equation (i), we get:

$$9x - 10y = -12$$

$$\Rightarrow x = \frac{-12 + 10y}{9} \dots \text{(iii)}$$

Substituting the value of x in equation (ii), we get:

$$\frac{\frac{-12+10y}{9}}{3} + \frac{y}{2} = \frac{13}{6}$$

$$\Rightarrow \frac{-24 + 20y + 27y}{54} = \frac{13}{6}$$

$$\Rightarrow -24 + 47y = 13 \times 9$$

$$\Rightarrow 47y = 117 + 24$$

$$\Rightarrow 47y = 141$$

$$\Rightarrow y = 3$$

Substituting the value of y in equation (iii), we get:

$$x = \frac{-12 + 10 \times 3}{9} = 2$$

$$\therefore x = 2, y = 3$$

2. Solve $2x + 3y = 11$ and $2x - 4y = -24$ and hence find the value of 'm' for which $y = mx + 3$.

Solution:

$$2x + 3y = 11 \dots (i)$$

$$2x - 4y = -24 \dots (ii)$$

From equation (i), we get:

$$x = \frac{11 - 3y}{2} \dots (iii)$$

Substituting the value of x in equation (ii), we get:

$$2\left(\frac{11 - 3y}{2}\right) - 4y = -24$$

$$\Rightarrow 11 - 3y - 4y = -24$$

$$\Rightarrow -7y = -35$$

$$\Rightarrow y = 5$$

Substituting the value of y in equation (iii), we get:

$$x = \frac{11 - 3 \times 5}{2} = -2$$

$$\therefore x = -2, y = 5$$

Given, $y = mx + 3$

$$\Rightarrow 5 = -2m + 3$$

$$\Rightarrow -2m = 2$$

$$\Rightarrow m = -1$$

Hence, $m = -1$.

3. Form the pair of linear equations for the following problems and find their solution by substitution method.
- (i) The difference between two numbers is 26 and one number is three times the other. Find them.

- (ii) The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them.
- (ii) The coach of a cricket team buys 7 bats and 6 balls for ₹ 3800. Later, she buys 3 bats and 5 balls for ₹ 1750. Find the cost of each bat and each ball.
- (iv) The taxi charges in a city consist of a fixed charge together with the charge for the distance covered. For a distance of 10 km, the charge paid is ₹ 105 and for a journey of 15 km, the charge paid is ₹ 155. What are the fixed charges and the charge per km? How much does a person have to pay for travelling a distance of 25 km?
- (vii) A fraction becomes $\frac{9}{11}$, if 2 is added to both the numerator and the denominator. If, 3 is added to both the numerator and the denominator it becomes $\frac{5}{6}$. Find the fraction.
- (viii) Five years hence, the age of Jacob will be three times that of his son. Five years ago, Jacob's age was seven times that of his son. What are their present ages?

Solution:

- (i) Let two numbers are x and y such that $y > x$.

As per the question:

$$y = 3x \dots (1)$$

$$y - x = 26 \dots (2)$$

On substituting the value of y from equation (1) into equation (2), we get

$$26 + x = 3x$$

$$\Rightarrow 3x - x = 26$$

$$\Rightarrow x = 13 \dots (3)$$

Substituting the value of x in equation (1), we get

$$y = 39$$

Hence, two numbers are 13 and 39.

- (ii) Let larger angle be x and smaller angle be y

As we know, the sum of supplementary angles is always 180° .

Hence, as per the question,

$$x + y = 180 \dots (1)$$

$$x - y = 18 \dots (2)$$

From (1), we get

$$x = 180^\circ - y \dots (3)$$

Substituting x in equation (2), we obtain

$$180^\circ - y - y = 18^\circ$$

$$\Rightarrow 162^\circ = 2y$$

$$\Rightarrow 81^\circ = y \dots (4)$$

Putting this in equation (3), we obtain

$$x = 180^\circ - 81^\circ = 99^\circ$$

Hence, the angles are 99° and 81° .

(iii) Let the cost of a bat be x and a ball be y

As per the given information,

$$7x + 6y = 3800 \dots (1)$$

$$3x + 5y = 1750 \dots (2)$$

From (1), we get

$$y = \frac{3800 - 7x}{6} \dots (3)$$

Substituting x in equation (2), we get

$$3x + 5\left(\frac{3800 - 7x}{6}\right) = 1750$$

$$\Rightarrow 3x + \frac{9500}{3} - \frac{35x}{6} = 1750$$

$$\Rightarrow 3x - \frac{35x}{6} = 1750 - \frac{9500}{3}$$

$$\Rightarrow \frac{18x - 35x}{6} = \frac{5250 - 9500}{3}$$

$$\Rightarrow -\frac{17x}{6} = \frac{-4250}{3}$$

$$\Rightarrow -17x = -8500$$

$$\Rightarrow x = 500 \dots (4)$$

Substituting this in equation (3), we get

$$y = \frac{3800 - 7 \times 500}{6} = 50$$

Hence, the cost of a bat is ₹ 500 and that of a ball is ₹ 50.

- (iv) Let the fixed charge be ₹ x and charge per Km be ₹ y .

$$x + 10y = 105 \dots (1)$$

$$x + 15y = 155 \dots (2)$$

From (1), we get

$$x = 105 - 10y \dots (3)$$

Substituting this in equation (2), we get

$$105 - 10y + 15y = 155$$

$$\Rightarrow 5y = 50$$

$$\Rightarrow y = 10 \dots (4)$$

Putting y in equation (3), we get

$$x = 105 - 10 \times 10$$

$$x = 5$$

Hence, fixed charge = ₹ 5

And per Km charge = ₹ 10

Hence, charge for 25 km = $x + 25y$

$$= 5 + 250 = ₹ 255$$

- (v) Let the fraction be $\frac{x}{y}$

As per the given information, $\frac{x+2}{y+2} = \frac{9}{11}$

$$\Rightarrow 11x + 22 = 9y + 18$$

$$\Rightarrow 11x - 9y = -4 \dots (1)$$

$$\frac{x+3}{y+3} = \frac{5}{6}$$

$$\Rightarrow 6x + 18 = 5y + 15$$

$$\Rightarrow 6x - 5y = -3 \dots (2)$$

From equation (1), we get $x = \frac{-4+9y}{11} \dots (3)$

Substituting x in equation (2), we get

$$6\left(\frac{-4+9y}{11}\right) - 5y = -3$$

$$\Rightarrow -24 + 54y - 55y = -33$$

$$\Rightarrow -y = -9$$

$$\Rightarrow y = 9 \dots (4)$$

Substituting y in equation (3), we obtain

$$x = \frac{-4 + 81}{11} = 7$$

Hence, the fraction is $\frac{7}{9}$.

(vi) Let the age of Jacob be x and the age of his son be y

As per the given information,

$$(x + 5) = 3(y + 5)$$

$$\Rightarrow x - 3y = 10 \dots (1)$$

$$(x - 5) = 7(y - 5)$$

$$\Rightarrow x - 7y = -30 \dots (2)$$

From (1), we get

$$x = 3y + 10 \dots (3)$$

Substituting x in equation (2), we get

$$3y + 10 - 7y = -30$$

$$\Rightarrow -4y = -40$$

$$\Rightarrow y = 10 \dots (4)$$

Substituting y in equation (3), we get

$$x = 3 \times 10 + 10 = 40$$

Hence, the present age of Jacob is 40 years and the present age of his son is 10 years.



EXERCISE 3.4

1. Solve the following pair of linear equations by the elimination method and the substitution method:

(i) $x + y = 5$ and $2x - 3y = 4$

(ii) $3x + 4y = 10$ and $2x - 2y = 2$

(iii) $3x - 5y - 4 = 0$ and $9x = 2y + 7$

(iv) $\frac{x}{2} + \frac{2y}{3} = -1$ and $x - \frac{y}{3} = 3$

Solution:

(i) **Elimination method:**

$$x + y = 5 \quad \dots (1)$$

$$2x - 3y = 4 \quad \dots (2)$$

Multiplying equation (1) by 3, we get:

$$3x + 3y = 15 \quad \dots (3)$$

Adding equation (2) and (3), we get:

$$5x = 19$$

$$\Rightarrow x = \frac{19}{5}$$

Substituting x in equation (1), we get:

$$y = 5 - \frac{19}{5} = \frac{6}{5}$$

$$\therefore x = \frac{19}{5}, y = \frac{6}{5}$$

Substitution method:

From equation (1), we get:

$$x = 5 - y \quad \dots (4)$$

Putting x in equation (2), we get:

$$2(5 - y) - 3y = 4$$

$$-5y = -6$$

$$y = \frac{6}{5}$$

Putting y in equation (4), we get:

$$x = 5 - \frac{6}{5} = \frac{19}{5}$$

$$\text{Hence, } x = \frac{19}{5}, y = \frac{6}{5}$$

(ii) **Elimination method:**

$$3x + 4y = 10 \quad \dots (1)$$

$$2x - 2y = 2 \quad \dots (2)$$

Multiplying equation (2) by 2, we get:

$$4x - 4y = 4 \quad \dots (3)$$

Adding equation (1) and (3), we get:

$$7x = 14$$

$$\Rightarrow x = 2$$

Putting x in equation (1), we get:

$$6 + 4y = 10$$

$$\Rightarrow 4y = 4$$

$$\Rightarrow y = 1$$

Hence, $x = 2, y = 1$

Substitution method:

From equation (2), we get:

$$x = y + 1 \quad \dots (4)$$

Putting x in equation (1), we get:

$$3(y + 1) + 4y = 10$$

$$\Rightarrow 7y = 7$$

$$\Rightarrow y = 1$$

Putting y in equation (4), we get:

$$x = 1 + 1 = 2$$

$$\therefore x = 2, y = 1$$

(iii) **Elimination method:**

$$3x - 5y - 4 = 0 \quad \dots (1)$$

$$9x = 2y + 7$$

$$\Rightarrow 9x - 2y - 7 = 0 \quad \dots (2)$$

Multiplying equation (1) by 3, we get:

$$9x - 15y - 12 = 0 \quad \dots (3)$$

Subtracting equation (2) from equation (3), we get:

$$13y = -5$$

$$\Rightarrow y = -\frac{5}{13}$$

Putting y in equation (1), we get:

$$3x + \frac{25}{12} - 4 = 0$$

$$\Rightarrow x = \frac{9}{13}$$

$$\therefore x = \frac{9}{13}, y = -\frac{5}{13}$$

Substitution method:

From equation (1), we get:

$$x = \frac{5y + 4}{3} \quad \dots (4)$$

Putting x in equation (2), we get:

$$9\left(\frac{5y + 4}{3}\right) - 2y - 7 = 0$$

$$13y = -5$$

$$y = -\frac{5}{13}$$

Putting y in equation (4), we get:

$$x = \frac{5\left(-\frac{5}{13}\right) + 4}{3}$$

$$x = \frac{9}{13}$$

$$\therefore x = \frac{9}{13}, y = -\frac{5}{13}$$

(iv) **Elimination method:**

$$\frac{x}{2} + \frac{2y}{3} = -1 \dots (1)$$

$$x - \frac{y}{3} = 3 \dots (2)$$

Multiplying equation (2) by 2, we get:

$$2x - \frac{2y}{3} = 6 \dots (3)$$

Adding equation (1) and (3), we get:

$$\frac{5x}{2} = 5$$

$$\Rightarrow x = 2$$

Putting x in equation (1), we get:

$$1 + \frac{2y}{3} = -1$$

$$\Rightarrow y = -3$$

$$\therefore x = 2, y = -3$$

Substitution method:

From equation (2), we get:

$$y = 3x - 9 \dots (3)$$

Putting y in equation (1), we get:

$$\frac{x}{2} + \frac{2(3x - 9)}{3} = -1$$

$$\Rightarrow 3x + 4(3x - 9) = -6$$

$$\Rightarrow 15x = 30$$

$$\Rightarrow x = 2$$

Putting x in equation (3), we get:

$$y = 6 - 9 = -3$$

$$\therefore x = 2, y = -3$$

Note: Solution must be same in both the cases.

2. Form the pair of linear equations in the following problems, and find their solutions (if they exist) by the elimination method:
- (i) If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes $\frac{1}{2}$ if we only add 1 to the denominator. What is the fraction?
 - (ii) Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu?
 - (iv) The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.
 - (iv) Meena went to a bank to withdraw ₹ 2000. She asked the cashier to give her ₹ 50 and ₹ 100 notes only. Meena got 25 notes in all. Find how many notes of ₹ 50 and ₹ 100 she received.
 - (iv) A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid ₹ 27 for a book kept for seven days, while Susy paid ₹ 21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.

Solution:

- (i) Let the fraction be $\frac{x}{y}$

$$\frac{x+1}{y-1} = 1$$

$$\Rightarrow x - y = -2 \dots (1)$$

$$\frac{x}{y+1} = \frac{1}{2}$$

$$\Rightarrow 2x - y = 1 \dots (2)$$

As per the question, subtracting equation (1) from equation (2), we get: $x = 3$

Putting x in equation (2), we get:

$$6 - y = 1$$

$$\Rightarrow -y = -5$$

$$\Rightarrow y = 5$$

Hence, the fraction is $\frac{3}{5}$.

- (ii) Let present age of Nuri be x and Sonu be y .

As per the question,

$$(x - 5) = 3(y - 5)$$

$$\Rightarrow x - 3y = -10 \dots (1)$$

$$(x + 10) = 2(y + 10)$$

$$\Rightarrow x - 2y = 10 \dots (2)$$

Subtracting equation (1) from equation (2), we get:

$$y = 20$$

Putting the value of y in equation (2), we get:

$$x - 40 = 10$$

$$\Rightarrow x = 50$$

Hence, the age of Nuri is 50 years and the age of Sonu is 20 years.

- (iii) Let the unit digit of the number be x and tens digit of the number be y respectively.

Hence, number = $10y + x$

After reversing the digits, number = $10x + y$

As per the question,

$$x + y = 9 \dots (1)$$

$$9(10y + x) = 2(10x + y)$$

$$\Rightarrow 88y - 11x = 0$$

$$\Rightarrow -x + 8y = 0 \dots (2)$$

Adding equations (1) and (2), we get:

$$9y = 9$$

$$\Rightarrow y = 1$$

Putting y in equation (1), we get:

$$x = 8$$

Hence, the number is $10y + x = 10 \times 1 + 8 = 18$

- (v) Let the number of ₹ 50 notes be x and number of ₹ 100 notes be y .

- (vi) As per the question,

$$x + y = 25 \dots (1)$$

$$50x + 100y = 2000$$

$$\Rightarrow x + 2y = 40 \dots (2)$$

Subtracting equation (1) from equation (2), we get:

$$y = 15$$

Putting y in equation (1), we get:

$$x = 10$$

Hence, Meena received 10 notes of ₹ 50 and 15 notes of ₹ 100.

- (vii) Let the fixed charge for first three days be ₹ x and each day charge thereafter be ₹ y .

As per the question,

$$x + 4y = 27 \dots (1)$$

$$x + 2y = 21 \dots (2)$$

Subtracting equation (2) from equation (1), we get:

$$2y = 6$$

$$\Rightarrow y = 3$$

Putting y in equation (2), we get:

$$x + 6 = 21$$

$$\Rightarrow x = 15$$

Hence, the fixed charge is ₹ 15 and each day charge thereafter is ₹ 3.



EXERCISE 3.5

1. Which of the following pair of linear equation has unique solution, no solution, or infinitely many solutions. In case there is a unique solution, find it by using cross multiplication method.

(i)	$x - 3y - 3 = 0$	$3x - 9y - 2 = 0$
(ii)	$2x + y = 5$	$3x + 3y = 8$

(iii)	$3x - 5y = 20$	$6x - 10y = 40$
(iv)	$x - 3y - 7 = 0$	$3x - 3y - 15 = 0$

Solution:

(i) $x - 3y - 3 = 0$

$3x - 9y - 2 = 0$

Here, $a_1 = 1, b_1 = -3, c_1 = -3$

$a_2 = 3, b_2 = -9, c_2 = -2$

$$\frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{-3}{-9} = \frac{1}{3}, \frac{c_1}{c_2} = \frac{-3}{-2} = \frac{3}{2}$$

Since, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, the given pair of equations has no solution.

(ii) $2x + y = 5$

$\Rightarrow 2x + y - 5 = 0 \dots (1)$

$3x + 2y = 8$

$\Rightarrow 3x + 2y - 8 = 0 \dots (2)$

Here, $a_1 = 2, b_1 = 1, c_1 = -5$

$a_2 = 3, b_2 = 2, c_2 = -8$

$$\frac{a_1}{a_2} = \frac{2}{3}, \frac{b_1}{b_2} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{-5}{-8} = \frac{5}{8}$$

Since, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence, the given pair of equations has unique solution.

Now, using cross-multiplication method we know that,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\Rightarrow \frac{x}{-8 - (-10)} = \frac{y}{-15 - (-16)} = \frac{1}{4 - 3}$$

$$\Rightarrow \frac{x}{2} = \frac{y}{1} = \frac{1}{1}$$

$$\Rightarrow \frac{x}{2} = 1, \frac{y}{1} = 1$$

$$\therefore x = 2, y = 1$$

(iii) $3x - 5y = 20$

$$\Rightarrow 3x - 5y - 20 = 0 \quad \dots (1)$$

$$6x - 10y = 40$$

$$\Rightarrow 6x - 10y - 40 = 0 \quad \dots (2)$$

Here, $a_1 = 3, b_1 = -5, c_1 = -20$

$$a_2 = 6, b_2 = -10, c_2 = -40$$

$$\frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{-5}{-10} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{-20}{-40} = \frac{1}{2}$$

Since, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Hence, the given pair of equations has infinite solutions.

The solution will be found by assuming the value of x to be k . Hence the ordered pair $(k, \frac{3k-20}{5})$, where k is a constant, are the solutions of the given pair of equations.

(vi) $x - 3y - 7 = 0$

$$3x - 3y - 15 = 0$$

Here, $a_1 = 1, b_1 = -3, c_1 = -7$

$$a_2 = 3, b_2 = -3, c_2 = -15$$

$$\frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{-3}{-3} = 1, \frac{c_1}{c_2} = \frac{-7}{-15} = \frac{7}{15}$$

Since, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence, the given pair of equations has unique solution.

Now, using cross-multiplication method we know that,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\Rightarrow \frac{x}{45 - (21)} = \frac{y}{-21 - (-15)} = \frac{1}{-3 - (-9)}$$

$$\Rightarrow \frac{x}{24} = \frac{y}{-6} = \frac{1}{6}$$

$$\Rightarrow \frac{x}{24} = \frac{1}{6} \text{ and } \frac{y}{-6} = \frac{1}{6}$$

$$\therefore x = 4, y = -1$$

2. (i) For which values of a and b does the following pair of linear equations have an infinite number of solutions?

$$2x + 3y = 7$$

$$(a - b)x + (a + b)y = 3a + b - 2$$

- (ii) For which value of k will the following pair of linear equations have no solution?

$$3x + y = 1$$

$$(2k - 1)x + (k - 1)y = 2k + 1$$

Solution:

- (i) $2x + 3y = 7$

$$\Rightarrow 2x + 3y - 7 = 0 \dots (1)$$

$$(a - b)x + (a + b)y = 3a + b - 2$$

$$\Rightarrow (a - b)x + (a + b)y - (3a + b - 2) = 0 \dots (2)$$

$$\text{Here, } a_1 = 2, b_1 = 3, c_1 = -7$$

$$a_2 = (a - b), b_2 = (a + b), c_2 = -(3a + b - 2)$$

$$\frac{a_1}{a_2} = \frac{2}{a - b}, \frac{b_1}{b_2} = \frac{3}{a + b}, \frac{c_1}{c_2} = \frac{-7}{-(3a + b - 2)} = \frac{7}{(3a + b - 2)}$$

As we know that, for the equations to have infinitely many solutions:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\text{Hence, } \frac{2}{a - b} = \frac{7}{3a + b - 2}$$

$$\Rightarrow 6a + 2b - 4 = 7a - 7b$$

$$\Rightarrow a - 9b + 4 = 0 \dots (3)$$

$$\text{And } \frac{2}{a - b} = \frac{3}{a + b}$$

$$\Rightarrow 2a + 2b = 3a - 3b$$

$$\Rightarrow a - 5b = 0 \dots (4)$$

Now, using cross-multiplication method in equation (3) and (4),

$$\frac{a}{0 - (-20)} = \frac{b}{4 - 0} = \frac{1}{-5 - (-9)}$$

$$\Rightarrow \frac{a}{20} = \frac{b}{4} = \frac{1}{4}$$

$$\Rightarrow a = 5, b = 1$$

Hence, the values of a and b are 5 and 1 respectively.

(ii) $3x + y = 1$

$$\Rightarrow 3x + y - 1 = 0 \dots (1)$$

$$(2k - 1)x + (k - 1)y = (2k + 1)$$

$$\Rightarrow (2k - 1)x + (k - 1)y - (2k + 1) = 0 \dots (2)$$

Here, $a_1 = 3, b_1 = 1, c_1 = -1$

$$a_2 = (2k - 1), b_2 = (k - 1), c_2 = -(2k + 1)$$

$$\frac{a_1}{a_2} = \frac{3}{2k - 1}, \frac{b_1}{b_2} = \frac{1}{k - 1}, \frac{c_1}{c_2} = \frac{-1}{-2k - 1} = \frac{1}{2k + 1}$$

As we know that, for the equations to have no solutions:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\Rightarrow \frac{3}{2k - 1} = \frac{1}{k - 1} \neq \frac{1}{2k + 1}$$

$$\Rightarrow \frac{3}{2k - 1} = \frac{1}{k - 1}$$

$$\Rightarrow 3k - 3 = 2k - 1$$

$$\Rightarrow k = 2$$

Again for $k = 2, \frac{3}{2k-1} = \frac{1}{k-1} = 1$ but $\frac{1}{2k+1} = \frac{1}{5}$

Since, $1 \neq \frac{1}{5}$

Hence, the values of k is 2 for no solution.

3. Solve the following pair of linear equations by the substitution and cross-multiplication methods:

$$8x + 5y = 9$$

$$3x + 2y = 4$$

Solution:

Substitution method:

$$8x + 5y = 9 \dots (1)$$

$$3x + 2y = 4 \dots (2)$$

From equation (2), we get:

$$y = \frac{4 - 3x}{2} \dots (3)$$

Putting y in equation (1), we get:

$$8x + 5\left(\frac{4 - 3x}{2}\right) = 9$$

$$\Rightarrow 16x + 20 - 15x = 18$$

$$\text{So, } x = -2$$

Putting $x = -2$ in (3), we get

$$y = \left(\frac{4 - 3 \times -2}{2}\right) = 5$$

$$\therefore x = -2, y = 5$$

Cross-multiplication method:

$$8x + 5y - 9 = 0$$

$$3x + 2y - 4 = 0$$

$$\text{Here, } a_1 = 8, b_1 = 5, c_1 = -9$$

$$a_2 = 3, b_2 = 2, c_2 = -4$$

$$\frac{x}{-20 - (-18)} = \frac{y}{-27 - (-32)} = \frac{1}{16 - 15}$$

$$\Rightarrow \frac{x}{-2} = \frac{y}{5} = \frac{1}{1}$$

$$\Rightarrow \frac{x}{-2} = 1 \text{ and } \frac{y}{5} = 1$$

$$\Rightarrow x = -2 \text{ and } y = 5$$

4. Form the pair of linear equations in the following problems and find their solutions (if they exist) by any algebraic method:
- (i) A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days she has to pay ₹ 1000 as hostel charges whereas a student B, who takes food for 26 days, pays ₹ 1180 as hostel charges. Find the fixed charges and the cost of food per day.
 - (ii) A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.
 - (iii) Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test?
 - (iv) Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars?
 - (iv) The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and breadth is increased by 3 units. If we increase the length by 3 units and the breadth by 2 units, the area increases by 67 square units. Find the dimensions of the rectangle.

Solution:

- (i) Let the fixed charge of the food be x and the charge for food per day be y .

As per the question,

$$x + 20y = 1000 \dots (1)$$

$$x + 26y = 1180 \dots (2)$$

Subtracting equation (1) from equation (2), we get:

$$6y = 180$$

$$\Rightarrow y = 30$$

Putting y in equation (2), we get:

$$x + 26 \times 30 = 1180$$

$$\Rightarrow x = 1180 - 780$$

$$\Rightarrow x = 400$$

Hence, the fixed charge of the food and the charge per day are ₹ 400 and ₹ 30 respectively.

(ii) Let the fraction be $\frac{x}{y}$

As per the question,

$$\frac{x-1}{y} = \frac{1}{3}$$

$$\Rightarrow 3x - y = 3 \dots (1)$$

$$\frac{x}{y+8} = \frac{1}{4}$$

$$\Rightarrow 4x - y = 8 \dots (2)$$

Subtracting equation (1) from equation (2), we get:

$$x = 5$$

Putting x in equation (2), we get:

$$20 - y = 8$$

$$\Rightarrow y = 12$$

Hence, the fraction is $\frac{5}{12}$

(iii) Let the number of right answers be x and number of wrong answers be y.

As per the question,

$$3x - y = 40 \dots (1)$$

$$4x - 2y = 50$$

$$\Rightarrow 2x - y = 25 \dots (2)$$

Subtracting equation (2) from equation (1), we get:

$$x = 15$$

Putting x in equation (2), we get:

$$30 - y = 25$$

$$y = 5$$

Hence, the number of right answers and the number of wrong answers is 15 and 5 respectively.

Hence, the total number of questions is 20.

- (iv) Let the speed of first car be u km/h and speed of second car be v km/h respectively.

Speed of both cars while they are travelling in same direction = $(u - v)$ km/h

Speed of both cars while they are travelling in opposite directions i.e., when they are travelling towards each other = $(u + v)$ km/h

Distance travelled = speed \times Time

As per the question,

$$5(u - v) = 100$$

$$\Rightarrow u - v = 20 \dots (1)$$

$$1(u + v) = 100$$

$$\Rightarrow u + v = 100 \dots (2)$$

Adding equations (1) and (2), we get:

$$2u = 120$$

$$\Rightarrow u = 60$$

Putting u in equation (2), we get:

$$v = 40$$

Hence, speed of the first car is 60 km/h and speed of the second car is 40 km/h.

- (v) Let length of rectangle be x units and breadth of rectangle be y units.

Hence, area = xy

As per the question,

$$(x - 5)(y + 3) = xy - 9$$

$$\Rightarrow 3x - 5y - 6 = 0 \dots (1)$$

$$(x + 3)(y + 2) = xy + 67$$

$$\Rightarrow 2x + 3y - 61 = 0 \dots (2)$$

Using cross-multiplication method,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

we get:

$$\frac{x}{305 - (-18)} = \frac{y}{-12 - (-183)} = \frac{1}{9 - (-10)}$$

$$\Rightarrow \frac{x}{323} = \frac{y}{171} = \frac{1}{19}$$

$$\Rightarrow x = 17, y = 9$$

Hence, the length and breadth of the rectangle are 17 units and 9 units respectively.

EXERCISE 3.6

1. Solve the following pairs of equations by reducing them to a pair of linear equations:

(i)	$\frac{1}{2x} + \frac{1}{3y} = 2$	$\frac{1}{3x} + \frac{1}{2y} = \frac{13}{6}$
(ii)	$\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2$	$\frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1$
(iii)	$\frac{4}{x} + 3y = 14$	$\frac{3}{x} - 4y = 23$
(iv)	$\frac{5}{x-1} + \frac{1}{y-2} = 2$	$\frac{6}{x-1} - \frac{3}{y-2} = 1$
(v)	$\frac{7x-2y}{xy} = 5$	$\frac{8x+7y}{xy} = 15$
(vi)	$6x + 3y = 6xy$	$2x + 4y = 5xy$
(vii)	$\frac{10}{x+y} + \frac{2}{x-y} = 4$	$\frac{15}{x+y} - \frac{5}{x-y} = -2$
(viii)	$\frac{1}{3x+y} + \frac{1}{3x-y} = \frac{3}{4}$	$\frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = \frac{-1}{8}$

Solutions:

$$(i) \quad \frac{1}{2x} + \frac{1}{3y} = 2$$

$$\frac{1}{3x} + \frac{1}{2y} = \frac{13}{6}$$

$$\text{Let } \frac{1}{x} = p \text{ and } \frac{1}{y} = q$$

Hence both equations convert to:

$$\frac{p}{2} + \frac{q}{3} = 2$$

$$\Rightarrow 3p + 2q - 12 = 0 \dots (1)$$

$$\frac{p}{3} + \frac{q}{2} = \frac{13}{6}$$

$$\Rightarrow 2p + 3q - 13 = 0 \dots (2)$$

Using cross-multiplication method, we get:

$$\frac{p}{-26 - (-36)} = \frac{q}{-24 - (-39)} = \frac{1}{9-4}$$

$$\Rightarrow \frac{p}{10} = \frac{q}{15} = \frac{1}{5}$$

$$\Rightarrow p = 2, q = 3$$

$$\therefore \frac{1}{x} = p = 2, \frac{1}{y} = q = 3$$

$$\text{Hence, } x = \frac{1}{2}, y = \frac{1}{3}$$

$$(ii) \quad \frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2$$

$$\frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1$$

let $\frac{1}{\sqrt{x}} = p$ and $\frac{1}{\sqrt{y}} = q$

Hence both equations convert to:

$$2p + 3q = 2 \dots (1)$$

$$4p - 9q = -1 \dots (2)$$

Multiplying equation (1) by 2, we get:

$$4p + 6q = 4 \dots (3)$$

Subtracting equation (2) from equation (3), we get:

$$15q = 5$$

$$\Rightarrow q = \frac{1}{3}$$

Putting the value of q in equation (1), we get:

$$2p + 3 \times \frac{1}{3} = 2$$

$$\Rightarrow 2p = 1$$

$$\Rightarrow p = \frac{1}{2}$$

Since, $p = \frac{1}{\sqrt{x}} = \frac{1}{2}$

$$\Rightarrow \sqrt{x} = 2$$

$$\Rightarrow x = 4$$

$$q = \frac{1}{\sqrt{y}} = \frac{1}{3}$$

$$\Rightarrow \sqrt{y} = 3$$

$$\Rightarrow y = 9$$

$$\therefore x = 4, y = 9$$

(iii) $\frac{4}{x} + 3y = 14$

$$\frac{3}{x} - 4y = 23$$

Let $\frac{1}{x} = p$

Hence both equations convert to:

$$4p + 3y = 14$$

$$\Rightarrow 4p + 3y - 14 = 0 \dots (1)$$

$$3p - 4y = 23$$

$$\Rightarrow 3p - 4y - 23 = 0 \dots (2)$$

Using cross-multiplication method, we get:

$$\frac{p}{-69-56} = \frac{y}{-42-(-92)} = \frac{1}{-16-9}$$

$$\Rightarrow \frac{p}{-125} = \frac{y}{50} = \frac{-1}{25}$$

$$\Rightarrow \frac{p}{-125} = \frac{-1}{25}, \frac{y}{50} = \frac{-1}{25}$$

$$\Rightarrow p = 5, y = -2$$

$$\text{Since, } p = \frac{1}{x} = 5$$

$$\Rightarrow x = \frac{1}{5} \text{ and } y = -2$$

$$(iv) \quad \frac{5}{x-1} + \frac{1}{y-2} = 2$$

$$\frac{6}{x-1} - \frac{3}{y-2} = 1$$

$$\text{Let } \frac{1}{x-1} = p \text{ and } \frac{1}{y-2} = q$$

Hence both equations convert to:

$$5p + q = 2 \dots (1)$$

$$6p - 3q = 1 \dots (2)$$

Multiplying equation (1) by 3, we get:

$$15p + 3q = 6 \dots (3)$$

Adding (2) and (3), we get:

$$21p = 7$$

$$\Rightarrow p = \frac{1}{3}$$

Putting p in equation (1), we get:

$$5 \times \frac{1}{3} + q = 2$$

$$\Rightarrow q = 2 - \frac{5}{3} = \frac{1}{3}$$

$$\text{Since, } p = \frac{1}{x-1} = \frac{1}{3}$$

$$\Rightarrow x - 1 = 3$$

$$\Rightarrow x = 4$$

$$q = \frac{1}{y-2} = \frac{1}{3}$$

$$\Rightarrow y - 2 = 3$$

$$\Rightarrow y = 5$$

$$\therefore x = 4, y = 5$$

$$(v) \quad \frac{7x-2y}{xy} = 5$$

$$\Rightarrow \frac{7}{y} - \frac{2}{x} = 5 \dots (1)$$

$$\frac{8x+7y}{xy} = 15$$

$$\Rightarrow \frac{8}{y} + \frac{7}{x} = 15 \dots (2)$$

$$\text{Let } \frac{1}{x} = p \text{ and } \frac{1}{y} = q$$

Hence both equations convert to:

$$-2p + 7q = 5$$

$$\Rightarrow -2p + 7q - 5 = 0 \dots (3)$$

$$7p + 8q = 15$$

$$\Rightarrow 7p + 8q - 15 = 0 \dots (4)$$

using cross-multiplication method, we get:

$$\frac{p}{-105 - (-40)} = \frac{q}{-35 - 30} = \frac{1}{-16 - 49}$$

$$\Rightarrow \frac{p}{-65} = \frac{q}{-65} = \frac{1}{-65}$$

$$\Rightarrow \frac{p}{-65} = \frac{1}{-65}, \frac{q}{-65} = \frac{1}{-65}$$

$$\Rightarrow p = 1, q = 1$$

$$\text{Since, } p = \frac{1}{x} = 1, q = \frac{1}{y} = 1$$

$$\text{Hence, } x = 1, y = 1$$

(vi) $6x + 3y = 6xy$
 $\Rightarrow \frac{6}{y} + \frac{3}{x} = 6 \dots (1)$

$2x + 4y = 5xy$
 $\Rightarrow \frac{2}{y} + \frac{4}{x} = 5 \dots (2)$

Let $\frac{1}{x} = p$ and $\frac{1}{y} = q$

Hence both equations convert to:

$3p + 6q - 6 = 0$

$4p + 2q - 5 = 0$

Using cross-multiplication method, we get:

$$\frac{p}{-30 - (-12)} = \frac{q}{-24 - (-15)} = \frac{1}{6 - 24}$$

$$\Rightarrow \frac{p}{-18} = \frac{q}{-9} = \frac{1}{-18}$$

$$\Rightarrow \frac{p}{-18} = \frac{1}{-18}, \frac{q}{-9} = \frac{1}{-18}$$

$$\Rightarrow p = 1, q = \frac{1}{2}$$

Since, $p = \frac{1}{x} = 1, q = \frac{1}{y} = \frac{1}{2}$

Hence, $x = 1, y = 2$

(vii) $\frac{10}{x+y} + \frac{2}{x-y} = 4$

$\frac{15}{x+y} - \frac{5}{x-y} = -2$

Let $\frac{1}{x+y} = p$ and $\frac{1}{x-y} = q$

Hence both equations convert to:

$10p + 2q = 4$

$\Rightarrow 10p + 2q - 4 = 0 \dots (1)$

$15p - 5q = -2$

$\Rightarrow 15p - 5q + 2 = 0 \dots (2)$

Using cross-multiplication method, we get:

$$\frac{p}{4-20} = \frac{q}{-60-20} = \frac{1}{-50-30}$$

$$\Rightarrow \frac{p}{-16} = \frac{1}{-80} \text{ and } \frac{q}{-80} = \frac{1}{-80}$$

$$\Rightarrow p = \frac{1}{5} \text{ and } q = 1$$

Since, $p = \frac{1}{x+y} = \frac{1}{5}$ and $q = \frac{1}{x-y} = 1$

$$x + y = 5 \dots (3)$$

$$x - y = 1 \dots (4)$$

Adding both equations (3) and (4), we get:

$$2x = 6$$

$$\Rightarrow x = 3$$

Putting x in equation (3), we get:

$$y = 2$$

$$\therefore x = 3, y = 2$$

(viii) $\frac{1}{3x+y} + \frac{1}{3x-y} = \frac{3}{4}$

$$\frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = \frac{-1}{8}$$

let $\frac{1}{3x+y} = p$ and $\frac{1}{3x-y} = q$

Hence both equations convert to:

$$p + q = \frac{3}{4} \dots (1)$$

$$\frac{p}{2} - \frac{q}{2} = \frac{-1}{8}$$

$$\Rightarrow p - q = \frac{-1}{4} \dots (2)$$

Adding (1) and (2), we get:

$$2p = \frac{3}{4} - \frac{1}{4}$$

$$\Rightarrow 2p = \frac{1}{2}$$

$$\Rightarrow p = \frac{1}{4}$$

Putting p in (1), we get:

$$\frac{1}{4} + q = \frac{3}{4}$$

$$\Rightarrow q = \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$$

$$\text{Since, } p = \frac{1}{3x+y} = \frac{1}{4}$$

$$\Rightarrow 3x + y = 4 \quad \dots (3)$$

$$q = \frac{1}{3x-y} = \frac{1}{2}$$

$$\Rightarrow 3x - y = 2 \quad \dots (4)$$

Adding equations (3) and (4), we get:

$$6x = 6$$

$$\Rightarrow x = 1$$

Putting the value of x in (3), we get:

$$3(1) + y = 4$$

$$\Rightarrow y = 1$$

$$\therefore x = 1, y = 1$$

2. Formulate the following problems as a pair of equations, and hence find their solutions:
- Ritu can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of the current.
 - 2 women and 5 men can together finish an embroidery work in 4 days, while 3 women and 6 men can finish it in 3 days. Find the time taken by 1 woman alone to finish the work, and also that taken by 1 man alone.
 - Roohi travels 300 km to her home partly by train and partly by bus. She takes 4 hours if she travels 60 km by train and the remaining by bus. If she travels 100 km by train and the remaining by bus, she takes 10 minutes longer. Find the speed of the train and the bus separately.

Solution:

- Let the speed of Ritu be x km/h in still water and the speed of stream be y km/h.

Hence, speed of Ritu while rowing upstream = $(x - y)$ km/h

And speed of Ritu while rowing downstream = $(x + y)$ km/h

According to the question,

$$2(x + y) = 20$$

$$\Rightarrow x + y = 10 \dots (1)$$

$$2(x - y) = 4$$

$$\Rightarrow x - y = 2 \dots (2)$$

Adding equations (1) and (2), we get:

$$2x = 12$$

$$\Rightarrow x = 6$$

Putting the value of x in equation (2), we get:

$$y = 4$$

Thus, Rita's speed is 6km/h in still water and the speed of the current is 4 km/h.

- (ii) Let the number of days taken by a woman to finish the work be x and a man to finish the work be y .

$$\text{Work done by a woman in 1 day} = \frac{1}{x}$$

$$\text{Work done by a man in 1 day} = \frac{1}{y}$$

According to the question,

$$4\left(\frac{2}{x} + \frac{5}{y}\right) = 1$$

$$\Rightarrow \frac{2}{x} + \frac{5}{y} = \frac{1}{4} \dots (1)$$

$$3\left(\frac{3}{x} + \frac{6}{y}\right) = 1$$

$$\Rightarrow \frac{3}{x} + \frac{6}{y} = \frac{1}{3} \dots (2)$$

$$\text{Let } \frac{1}{x} = p \text{ and } \frac{1}{y} = q$$

Hence given equations convert to:

$$2p + 5q = \frac{1}{4}$$

$$\Rightarrow 8p + 20q - 1 = 0 \dots (3)$$

$$3p + 6q = \frac{1}{3}$$

$$\Rightarrow 9p + 18q - 1 = 0 \dots (4)$$

Using cross-multiplication method, we get:

$$\frac{p}{-20 - (-18)} = \frac{q}{-9 - (-8)} = \frac{1}{144 - 180}$$

$$\Rightarrow \frac{p}{-2} = \frac{q}{-1} = \frac{1}{-36}$$

$$\Rightarrow \frac{p}{-2} = \frac{1}{-36}, \frac{q}{-1} = \frac{1}{-36}$$

$$\Rightarrow p = \frac{1}{18}, q = \frac{1}{36}$$

$$\text{Since, } p = \frac{1}{x} = \frac{1}{18}, q = \frac{1}{y} = \frac{1}{36}$$

$$\text{Hence, } x = 18, y = 36$$

Hence, the number of days taken by a woman and a man to finish the work is 18 and 36.

- (iii) Let the speed of train be u km/h and speed of bus be v km/h.

$$\text{Now as we know, Time taken} = \frac{\text{Distance travelled}}{\text{Speed}}$$

Hence, As per the question,

$$\frac{60}{u} + \frac{240}{v} = 4 \dots (1)$$

$$\frac{100}{u} + \frac{200}{v} = \frac{25}{6} \dots (2) \text{ (Since, 10 minutes} = \frac{1}{6} \text{ hours)}$$

$$\text{Let } \frac{1}{u} = p \text{ and } \frac{1}{v} = q$$

Hence given equations convert to:

$$60p + 240q = 4 \dots (3)$$

$$100p + 200q = \frac{25}{6}$$

$$\Rightarrow 600p + 1200q = 25 \dots (4)$$

Multiplying equation (3) by 10, we get:

$$600p + 2400q = 40 \dots (5)$$

Subtracting equation (4) from equation (5), we get:

$$1200q = 15$$

$$\Rightarrow q = \frac{15}{1200} = \frac{1}{80}$$

Putting q in equation (3), we get:

$$60p + 3 = 4$$

$$\Rightarrow 60p = 1$$

$$\Rightarrow p = \frac{1}{60}$$

$$\text{Since, } p = \frac{1}{u} = \frac{1}{60}, q = \frac{1}{v} = \frac{1}{80}$$

$$\Rightarrow u = 60 \text{ km/h, } v = 80 \text{ km/h}$$

Hence, the speed of train and the speed of bus are 60 km/h and 80 km/h respectively.



EXERCISE 3.7 (Optional)*

1. The ages of two friends Ani and Biju differ by 3 years. Ani's father Dharam is twice as old as Ani and Biju is twice as old as his sister Cathy. The ages of Cathy and Dharam differ by 30 years. Find the ages of Ani and Biju.

Solution:

The difference of ages of Ani and Biju is 3 years. Hence, either Biju is 3 years older than Ani or Ani is 3 years older than Biju.

Let the age of Ani and Biju be x years and y years respectively.

$$\text{Age of Dharam} = 2x \text{ years}$$

$$\text{Age of Cathy} = \frac{y}{2} \text{ years}$$

Case I:

Ani is older than Biju by 3 years

$$x - y = 3 \dots (1)$$

$$2x - \frac{y}{2} = 30$$

$$\Rightarrow 4x - y = 60 \dots (2)$$

Subtracting (1) from (2), we get:

$$3x = 60 - 3 = 57 \Rightarrow x = 19$$

Hence, age of Ani = 19 years

And age of Biju = $19 - 3 = 16$ years

Case II: Biju is older than Ani by 3 years

$$y - x = 3 \dots (3)$$

$$2x - \frac{y}{2} = 30$$

$$\Rightarrow 4x - y = 60 \dots (4)$$

Adding (3) and (4), we get:

$$3x = 63$$

$$\Rightarrow x = 21$$

Hence, age of Ani = 21 years

And age of Biju = $21 + 3 = 24$ years

2. One says, "Give me a hundred, friend! I shall then become twice as rich as you". The other replies, "If you give me ten, I shall be six times as rich as you". Tell me what is the amount of their (respective) capital? [From the *Bijaganita of Bhaskara II*]

[Hint: $x + 100 = 2(y - 100), y + 10 = 6(x - 10)$].

Solution:

Let the money with the first person be ₹ x and money with the second person be ₹ y .

As per the question,

$$x + 100 = 2(y - 100)$$

$$\Rightarrow x + 100 = 2y - 200$$

$$\Rightarrow x - 2y = -300 \dots (1)$$

$$6(x - 10) = (y + 10)$$

$$\Rightarrow 6x - 60 = y + 10$$

$$\Rightarrow 6x - y = 70 \dots (2)$$

Multiplying equation (2) by 2, we get:

$$12x - 2y = 140 \dots (3)$$

Subtracting equation (1) from equation (3), we get:

$$11x = 140 + 300$$

$$\Rightarrow 11x = 440$$

$$\Rightarrow x = 40$$

Putting x in equation (1), we get:

$$40 - 2y = -300$$

$$\Rightarrow 40 + 300 = 2y$$

$$\Rightarrow 2y = 340$$

$$\Rightarrow y = 170$$

Hence, the two friends has ₹ 40 and ₹ 170 respectively.

3. A train covered a certain distance at a uniform speed. If the train would have been 10 km/h faster, it would have taken 2 hours less than the scheduled time. And, if the train were slower by 10 km/h; it would have taken 3 hours more than the scheduled time. Find the distance covered by the train.

Solution:

Let total distance travel by train be d km, the speed of the train be x km/h and the time taken by train to travel d km be t hours.

$$\text{Since, Speed} = \frac{\text{Distance travlled}}{\text{Time taken to tranvel that distance}}$$

$$\Rightarrow x = \frac{d}{t}$$

$$\Rightarrow d = xt \dots (1)$$

$$\text{As per the question, } (x + 10) = \frac{d}{(t-2)} \Rightarrow (x + 10)(t - 2) = d$$

$$\Rightarrow xt + 10t - 2x - 20 = d$$

By using equation (1), we get:

$$-2x + 10t = 20 \dots (2)$$

$$(x - 10) = \frac{d}{(t + 3)}$$

$$\Rightarrow (x - 10)(t + 3) = d$$

By using equation (1), we get:

$$3x - 10t = 30 \dots (3)$$

Adding equations (2) and (3), we get:

$$x = 50$$

Substituting x in equation (2), we get:

$$(-2) \times (50) + 10t = 20$$

$$\Rightarrow -100 + 10t = 20$$

$$\Rightarrow 10t = 120$$

$$\Rightarrow t = 12$$

From equation (1), we get:

$$d = xt = 50 \times 12 = 600$$

Hence, the distance covered by the train is 600 km.

4. The students of a class are made to stand in rows. If 3 students are extra in a row, there would be 1 row less. If 3 students are less in a row, there would be 2 rows more. Find the number of students in the class.

Solution:

Let the number of rows be x and number of students in a row be y .

Total number of students in the class = Number of rows \times Number of students in a row = xy

As per the question,

$$\text{Total number of students} = (x - 1)(y + 3)$$

$$\Rightarrow xy = (x - 1)(y + 3)$$

$$\Rightarrow xy = xy - y + 3x - 3$$

$$\Rightarrow 3x - y - 3 = 0$$

$$\Rightarrow 3x - y = 3 \quad \dots (1)$$

$$\text{Again, Total number of students} = (x + 2)(y - 3)$$

$$\Rightarrow xy = xy + 2y - 3x - 6$$

$$\Rightarrow 3x - 2y = -6 \quad \dots (2)$$

Subtracting equation (2) from (1), we get:

$$y = 9$$

Substituting the value of y in equation (1), we get:

$$3x - 9 = 3$$

$$\Rightarrow 3x = 9 + 3 = 12$$

$$\Rightarrow x = 4$$

Hence, number of rows, $x = 4$

And number of students in a row, $y = 9$

Hence, total number of students in a class = $xy = 4 \times 9 = 36$

5. In a $\triangle ABC$, $\angle C = 3 \angle B = 2(\angle A + \angle B)$. Find the three angles.

Solution:

Given,

$$\angle C = 3 \angle B = 2(\angle A + \angle B)$$

$$\Rightarrow 3 \angle B = 2(\angle A + \angle B)$$

$$\Rightarrow \angle B = 2 \angle A \Rightarrow 2 \angle A - \angle B = 0 \dots (1)$$

We know that the sum of all angles of a triangle is 180°

$$\text{Hence, } \angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow \angle A + \angle B + 3 \angle B = 180^\circ$$

$$\Rightarrow \angle A + 4 \angle B = 180^\circ \dots (2)$$

Multiplying equation (1) by 4, we get:

$$8 \angle A - 4 \angle B = 0 \dots (3)$$

Adding equations (2) and (3), we get:

$$9 \angle A = 180^\circ$$

$$\Rightarrow \angle A = 20^\circ$$

From equation (2), we get:

$$20^\circ + 4 \angle B = 180^\circ$$

$$\Rightarrow 4 \angle B = 160^\circ$$

$$\Rightarrow \angle B = 40^\circ$$

$$\text{Hence, } \angle C = 3 \angle B = 3 \times 40^\circ = 120^\circ$$

Thus, the value of $\angle A$, $\angle B$ and $\angle C$ are 20° , 40° and 120° respectively.

6. Draw the graphs of the equations $5x - y = 5$ and $3x - y = 3$. Determine the coordinates of the vertices of the triangle formed by these lines and the y axis.

Solution:

$$5x - y = 5$$

$$\Rightarrow y = 5x - 5$$

Two solutions of this equation are:

x	0	1
y	-5	0

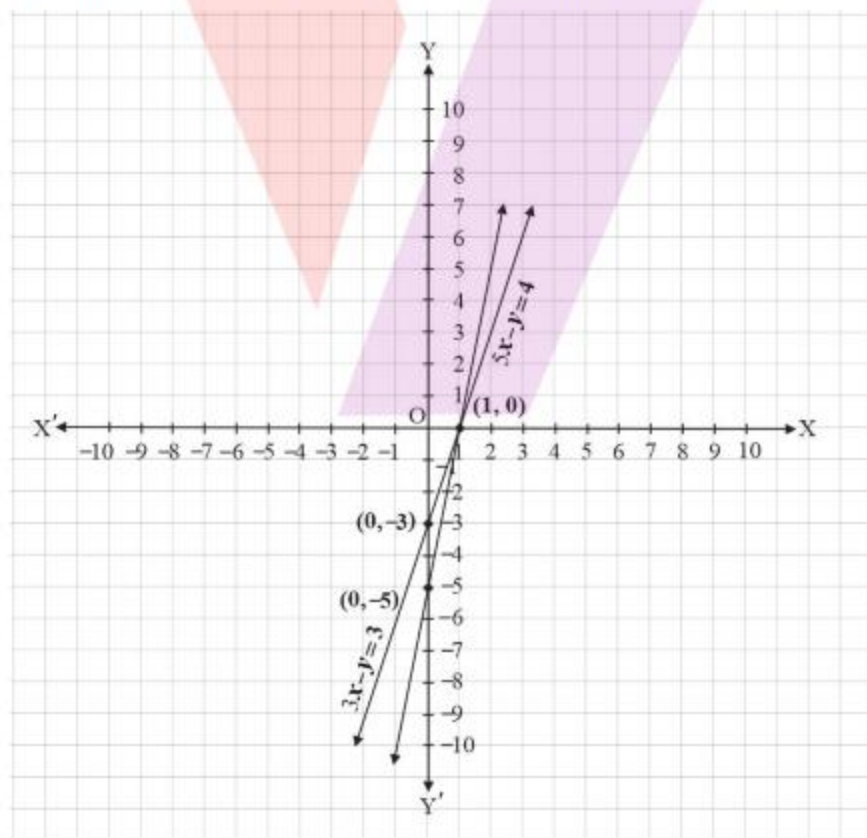
$$3x - y = 3$$

$$\Rightarrow y = 3x - 3$$

Two solutions of this equation are:

x	0	1
y	-3	0

The graphical representation of the two lines will be as follows:



We have shown required triangle $\triangle ABC$ in the graph. The coordinates of its vertices are A (1, 0), B (0, -3), C (0, -5).

7. Solve the following pair of linear equations:

(i)	$px + qy = p - q$	$qx - py = p + q$
(ii)	$ax + by = c$	$bx + ay = 1 + c$
(iii)	$\frac{x}{a} - \frac{y}{b} = 0$	$ax + by = a^2 + b^2$
(iv)	$(a - b)x + (a + b)y = a^2 - 2ab - b^2$	$(a + b)(x + y) = a^2 + b^2$
(v)	$152x - 378y = -74$	$-378x + 152y = -604$

Solution:

- (i) Both equations can be written as:

$$px + qy - (p - q) = 0 \dots (1)$$

$$qx - py - (p + q) = 0 \dots (2)$$

By using cross multiplication method, we get:

$$\frac{x}{-q(p + q) - p(p - q)} = \frac{y}{-q(p - q) + p(p + q)} = \frac{1}{-p^2 - q^2}$$

$$\Rightarrow \frac{x}{-p^2 - q^2} = \frac{y}{p^2 + q^2} = \frac{1}{-p^2 - q^2}$$

$$\Rightarrow x = 1, y = -1$$

- (ii) Both equations can be written as:

$$ax + by - c = 0 \dots (1)$$

$$bx + ay - (1 + c) = 0 \dots (2)$$

By using cross multiplication method, we get:

$$\frac{x}{-b(1 + c) + ac} = \frac{y}{-bc + a(1 + c)} = \frac{1}{a^2 - b^2}$$

$$\Rightarrow \frac{x}{c(a-b)-b} = \frac{y}{c(a-b)+a} = \frac{1}{a^2-b^2}$$

$$\Rightarrow x = \frac{c(a-b)-b}{a^2-b^2}, y = \frac{c(a-b)+a}{a^2-b^2}$$

(iii) $\frac{x}{a} - \frac{y}{b} = 0$

$$bx - ay = 0 \dots (1)$$

$$ax + by = a^2 + b^2 \dots (2)$$

Multiplying equation (1) and (2) by b and a respectively, we get:

$$b^2x - aby = 0 \dots (3)$$

$$a^2x + aby = a^3 + ab^2 \dots (4)$$

Adding equations (3) and (4), we get:

$$b^2x + a^2x = a^3 + ab^2$$

$$\Rightarrow x(b^2 + a^2) = a(a^2 + b^2)$$

$$\Rightarrow x = a$$

Substituting x in equation (1), we get:

$$b(a) - ay = 0$$

$$ab - ay = 0$$

$$ay = ab$$

$$y = b$$

Hence, $x = a, y = b$

(iv) $(a - b)x + (a + b)y = a^2 - 2ab - b^2 \dots (1)$

$$(a + b)(x + y) = a^2 + b^2$$

$$\Rightarrow (a + b)x + (a + b)y = a^2 + b^2 \dots (2)$$

Subtracting equation (2) from (1), we get:

$$(a - b)x - (a + b)x = (a^2 - 2ab - b^2) - (a^2 + b^2)$$

$$(a - b - a - b)x = -2ab - 2b^2 - 2bx = -2b(a + b)$$

$$x = a + b$$

Substituting x in equation (1), we get:

$$(a - b)(a + b) + (a + b)y = a^2 - 2ab - b^2$$

$$a^2 - b^2 + (a + b)y = a^2 - 2ab - b^2$$

$$(v) \quad (a + b)y = -2ab$$

$$y = \frac{-2ab}{a + b}$$

$$\text{Hence, } x = a + b, y = \frac{-2ab}{a + b}$$

$$(vi) \quad 152x - 378y = -74 \dots (1)$$

$$-378x + 152y = -604 \dots (2)$$

Adding the equations (1) and (2), we get:

$$-226x - 226y = -678$$

$$\Rightarrow x + y = 3 \dots (3)$$

Subtracting the equation (2) from equation (1), we get:

$$530x - 530y = 530$$

$$\Rightarrow x - y = 1 \dots (4)$$

Adding equations (3) and (4), we get:

$$2x = 4$$

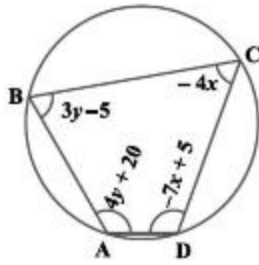
$$\Rightarrow x = 2$$

Substituting x in equation (3), we get:

$$y = 1$$

$$\text{Hence, } x = 2, y = 1$$

8. ABCD is a cyclic quadrilateral (see Fig.) Find the angles of the cyclic quadrilateral.



Solution:

As we know, the sum of opposite angles in a cyclic quadrilateral is 180° .

$$\angle A + \angle C = 180$$

$$\Rightarrow 4y + 20^\circ - 4x = 180$$

$$\Rightarrow -4x + 4y = 160^\circ$$

$$\Rightarrow x - y = -40 \dots (1)$$

Also, $\angle B + \angle D = 180^\circ$

$$\Rightarrow 3y - 5^\circ - 7x + 5^\circ = 180^\circ$$

$$\Rightarrow -7x + 3y = 180 \dots (2)$$

Multiplying equation (1) by 3, we get:

$$3x - 3y = -120 \dots (3)$$

Adding equations (2) and (3), we get:

$$-4x = 60^\circ$$

$$\Rightarrow x = -15^\circ$$

Substituting x in equation (1), we get:

$$-15^\circ - y = -40^\circ$$

$$\Rightarrow y = -15^\circ + 40^\circ = 25$$

Hence,

$$\angle A = 4y + 20^\circ = 4(25^\circ) + 20^\circ = 120^\circ$$

$$\angle B = 3y - 5^\circ = 3(25^\circ) - 5^\circ = 70^\circ$$

$$\angle C = -4x = -4(-15^\circ) = 60^\circ$$

$$\angle D = -7x + 5^\circ = -7(-15^\circ) + 5^\circ = 110^\circ$$



