# CBSE NCERT Solutions for Class 10 Science Chapter 2 – Ex 2.1

1. The graphs of y = p(x) are given in following figure, for some polynomials p(x). Find the number of zeroes of p(x), in each case.

(i)

Y

Y

Y

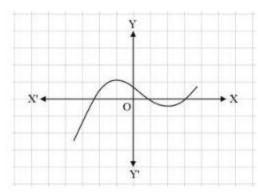
(ii)

Y

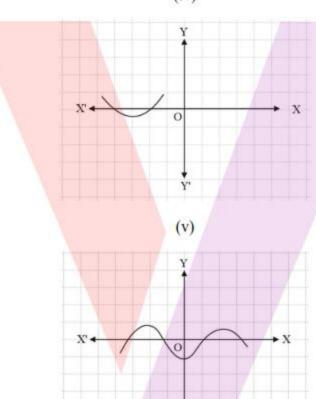
Y

(iii)

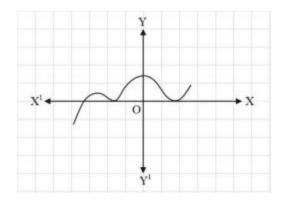
(iii)



(iv)



(vi)



Solution:

- (i) Since the graph of p(x) does not cut the X-axis at all. Therefore, the number of zeroes is 0.
- (ii) As the graph of p(x) intersects the X-axis at only 1 point. Therefore, the number of zeroes is 1.
- (iii) Since the graph of p(x) intersects the X-axis at 3 points. Hence, the number of zeroes is 3.
- (iv) As the graph of p(x) intersects the X-axis at 2 points. So, the number of zeroes is 2.
- (v) Since the graph of p(x) intersects the X-axis at 4 points. Therefore, the number of zeroes is 4.
- (vi) As the graph of p(x) intersects the X-axis at 3 points. So, the number of zeroes is 3.

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## CBSE NCERT Solutions for Class 10 Science Chapter 2 – Ex 2.2

- Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.
  - (i)  $x^2 2x 8$
  - (ii)  $4s^2 4s + 1$
  - (iii)  $6x^2 3 7x$
  - (iv)  $4u^2 + 8u$

(v) 
$$t^2 - 15$$

(vi) 
$$3x^2 - x - 4$$

#### Solution:

(i) 
$$x^2 - 2x - 8$$
  
 $= x^2 - 4x + 2x - 8$  [Factorisation by splitting the middle term]  
 $= x(x - 4) + 2(x - 4)$   
 $= (x - 4)(x + 2)$ 

We know that the zeroes of the quadratic polynomial  $ax^2 + bx + c$  are the same as the roots of the quadratic equation  $ax^2 + bx + c = 0$ .

Therefore, by equating the given polynomial to zero. We get,

$$x^{2} - 2x - 8 = 0$$

$$\Rightarrow (x - 4)(x + 2) = 0$$

$$\Rightarrow x - 4 = 0 \text{ or } x + 2 = 0$$

$$\Rightarrow x = 4 \text{ or } x = -2$$

Therefore, the zeroes of  $x^2 - 2x - 8$  are 4 and -2.

Sum of zeroes = 
$$4 - 2 = 2 = \frac{-(-2)}{1} = \frac{-(Coefficient of x)}{Coefficient of x^2}$$

Product of zeroes = 
$$4 \times (-2) = -8 = \frac{(-8)}{1} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

Hence, the relationship between the zeroes and the coefficients is verified.

(ii) 
$$4s^2 - 4s + 1 = (2s - 1)^2$$
 [Since,  $a^2 - 2ab + b^2 = (a - b)^2$ ]

We know that the zeroes of the quadratic polynomial  $ax^2 + bx + c$  are the same as the roots of the quadratic equation  $ax^2 + bx + c = 0$ .

Therefore, by equating the given polynomial to zero. We get,

$$4s^2 - 4s + 1 = 0$$

$$\Rightarrow (2s-1)^2 = 0$$

Cancelling square on both the sides,

$$\Rightarrow 2s - 1 = 0$$

$$\Rightarrow s = \frac{1}{2}$$

Therefore, the zeroes of  $4s^2 - 4s + 1$  are  $\frac{1}{2}$  and  $\frac{1}{2}$ .

Sum of zeroes = 
$$\frac{1}{2} + \frac{1}{2} = 1 = \frac{-(-4)}{4} = \frac{-(\text{Coefficient of } s)}{(\text{Coefficient of } s^2)}$$

Product of zeroes = 
$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = \frac{\text{Constant term}}{\text{Coefficient of } s^2}$$

Hence, the relationship between the zeroes and the coefficients is verified.

(iii) 
$$6x^2 - 3 - 7x = 6x^2 - 7x - 3$$
  
=  $6x^2 - 9x + 2x - 3$  [Factorisation by splitting the middle term]  
=  $3x(2x - 3) + (2x - 3)$   
=  $(3x + 1)(2x - 3)$ 

We know that the zeroes of the quadratic polynomial  $ax^2 + bx + c$  are the same as the roots of the quadratic equation  $ax^2 + bx + c = 0$ .

Therefore, by equating the given polynomial to zero. We get,

$$6x^2 - 3 - 7x = 0$$
  
 $\Rightarrow 3x + 1 = 0 \text{ or } 2x - 3 = 0$ 

$$\Rightarrow x = \frac{-1}{3} \text{ or } x = \frac{3}{2}$$

Therefore, the zeroes of  $6x^2 - 3 - 7x$  are  $\frac{-1}{3}$  and  $\frac{3}{2}$ .

Sum of zeroes = 
$$\frac{-1}{3} + \frac{3}{2} = \frac{7}{6} = \frac{-(-7)}{6} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

Product of zeroes 
$$=\frac{-1}{3} \times \frac{3}{2} = \frac{-1}{2} = \frac{-3}{6} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

Hence, the relationship between the zeroes and the coefficients is verified.

(iv) 
$$4u^2 + 8u = 4u^2 + 8u + 0 = 4u(u + 2)$$

We know that the zeroes of the quadratic polynomial  $ax^2 + bx + c$  are the same as the roots of the quadratic equation  $ax^2 + bx + c = 0$ .

Therefore, by equating the given polynomial to zero. We get,

$$4u^2 + 8u = 0$$

$$\Rightarrow 4u = 0 \text{ or } u + 2 = 0$$

$$\Rightarrow u = 0 \text{ or } u = -2$$

So, the zeroes of  $4u^2 + 8u$  are 0 and -2.

Sum of zeroes = 
$$0 + (-2) = -2 = \frac{-8}{4} = \frac{-(\text{Coefficient of } u)}{\text{Coefficient of } u^2}$$

Product of zeroes = 
$$0 \times (-2) = 0 = \frac{0}{4} = \frac{\text{Constant term}}{\text{Coefficient of } u^2}$$

Hence, the relationship between the zeroes and the coefficients is verified.

(v) 
$$t^2 - 15 = t^2 - 0.t - 15 = (t - \sqrt{15})(t + \sqrt{15})$$
 [Since,  $a^2 - b^2 = (a + b)(a - b)$ ]

We know that the zeroes of the quadratic polynomial  $ax^2 + bx + c$  are the same as the roots of the quadratic equation  $ax^2 + bx + c = 0$ .

Therefore, by equating the given polynomial to zero. We get,

$$t^2 - 15 = 0$$

$$\Rightarrow t - \sqrt{15} = 0 \text{ or } t + \sqrt{15} = 0$$

$$\Rightarrow t = \sqrt{15} \text{ or } t = -\sqrt{15}$$

Therefore, the zeroes of  $t^2 - 15$  are  $\sqrt{15}$  and  $-\sqrt{15}$ 

Sum of zeroes = 
$$\sqrt{15} + (-\sqrt{15}) = 0 = \frac{-0}{1} = \frac{-(\text{Coefficient of } t)}{(\text{Coefficient of } t^2)}$$

Product of zeroes = 
$$(\sqrt{15})(-\sqrt{15}) = -15 = \frac{-15}{1} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

Hence, the relationship between the zeroes and the coefficients is verified.

(vi) 
$$3x^2 - x - 4$$
  
 $= 3x^2 - 4x + 3x - 4$  [Factorisation by splitting the middle term]  
 $= x(3x - 4) + (3x - 4)$   
 $= (3x - 4)(x + 1)$ 

We know that the zeroes of the quadratic polynomial  $ax^2 + bx + c$  are the same as the roots of the quadratic equation  $ax^2 + bx + c = 0$ .

Therefore, by equating the given polynomial to zero. We get,

$$3x^2 - x - 4 = 0$$

$$\Rightarrow 3x - 4 = 0 \text{ or } x + 1 = 0$$

$$\Rightarrow x = \frac{4}{3} \text{ or } x = -1$$

Hence, the zeroes of  $3x^2 - x - 4$  are  $\frac{4}{3}$  and -1.

Sum of zeroes = 
$$\frac{4}{3} + (-1) = \frac{1}{3} = \frac{-(-1)}{3} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

Product of zeroes = 
$$\frac{4}{3}(-1) = \frac{-4}{3} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

Hence, the relationship between the zeroes and the coefficients is verified.

- Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.
  - (i)  $\frac{1}{4}$ , -1
  - (ii)  $\sqrt{2}, \frac{1}{3}$
  - (iii) 0, √5
  - (iv) 1,1
  - (v)  $-\frac{1}{4}, \frac{1}{4}$
  - (vi) 4,1

#### Solution:

- (i) We know that if  $\alpha$  and  $\beta$  are the zeroes of a quadratic polynomial p(x), then, the polynomial p(x) can be written as  $p(x) = a\{x^2 (\alpha + \beta)x + \alpha\beta\}$  or,
  - $p(x) = a\{x^2 (\text{Sum of the zeroes})x + \text{Product of the zeroes}\}\$ , where a is a non-zero real number.

Given: sum of the roots =  $\alpha + \beta = \frac{1}{4}$  and product of the roots =  $\alpha\beta = -1$ 

Hence, the quadratic polynomial p(x) can be written as:

$$p(x) = a\{x^2 - \frac{1}{4}x - 1\}$$

$$=a\left\{\frac{4x^2-x-4}{4}\right\}$$

By taking a = 4, we get one of the quadratic polynomials which satisfy the given conditions.

Therefore, the quadratic polynomial is  $(4x^2 - x - 4)$ .

- (ii) We know that if  $\alpha$  and  $\beta$  are the zeroes of a quadratic polynomial p(x), then, the polynomial p(x) can be written as  $p(x) = a\{x^2 (\alpha + \beta)x + \alpha\beta\}$  or,
  - $p(x) = a\{x^2 (Sum \text{ of the zeroes})x + Product \text{ of the zeroes}\}\$ , where a is a non-zero real number.

Given: sum of the roots =  $\alpha + \beta = \sqrt{2}$  and product of the roots =  $\alpha\beta = \frac{1}{3}$ 

Hence, the quadratic polynomial p(x) can be written as:

$$p(x) = a\{x^2 - \sqrt{2}x + \frac{1}{3}\}$$
$$= a\{\frac{3x^2 - 3\sqrt{2}x + 1}{3}\}$$

By taking a = 3, we get one of the quadratic polynomials which satisfy the given conditions.

Therefore, the quadratic polynomial is  $(3x^2 - 3\sqrt{2}x + 1)$ .

(iii) We know that if  $\alpha$  and  $\beta$  are the zeroes of a quadratic polynomial p(x), then, the polynomial p(x) can be written as  $p(x) = a\{x^2 - (\alpha + \beta)x + \alpha\beta\}$  or,

 $p(x) = a\{x^2 - (\text{Sum of the zeroes})x + \text{Product of the zeroes}\}\$ , where a is a non-zero real number.

Given: sum of the roots =  $\alpha + \beta = 0$  and product of the roots =  $\alpha\beta = \sqrt{5}$ 

Hence, the quadratic polynomial p(x) can be written as:

$$p(x) = a\{x^2 - 0.x + \sqrt{5}\}$$
$$= a\{x^2 + \sqrt{5}\}$$

By taking a = 1, we get one of the quadratic polynomials which satisfy the given conditions.

Therefore, the quadratic polynomial is  $(x^2 + \sqrt{5})$ .

(iv) We know that if  $\alpha$  and  $\beta$  are the zeroes of a quadratic polynomial p(x), then, the polynomial p(x) can be written as  $p(x) = a\{x^2 - (\alpha + \beta)x + \alpha\beta\}$  or,

 $p(x) = a\{x^2 - (\text{Sum of the zeroes})x + \text{Product of the zeroes}\}\$ , where a is a non-zero real number.

Given: sum of the roots =  $\alpha + \beta = 1$  and product of the roots =  $\alpha\beta = 1$ 

Hence, the quadratic polynomial p(x) can be written as:

$$p(x) = a\{x^2 - 1.x + 1\}$$
$$= a\{x^2 - x + 1\}$$

By taking a = 1, we get one of the quadratic polynomials which satisfy the given conditions.

Therefore, the quadratic polynomial is  $(x^2 - x + 1)$ .

(v) We know that if  $\alpha$  and  $\beta$  are the zeroes of a quadratic polynomial p(x), then, the polynomial p(x) can be written as  $p(x) = a\{x^2 - (\alpha + \beta)x + \alpha\beta\}$  or,

 $p(x) = a\{x^2 - (\text{Sum of the zeroes})x + \text{Product of the zeroes}\}\$ , where a is a non-zero real number.

Given: sum of the roots =  $\alpha + \beta = -\frac{1}{4}$  and product of the roots =  $\alpha\beta = \frac{1}{4}$ 

Hence, the quadratic polynomial p(x) can be written as:

$$p(x) = a\{x^2 + \frac{1}{4}x + \frac{1}{4}\}$$

$$=a\left\{\frac{4x^2+x+1}{4}\right\}$$

By taking a = 4, we get one of the quadratic polynomials which satisfy the given conditions.

Therefore, the quadratic polynomial is  $(4x^2 + x + 1)$ .

(vi) We know that if  $\alpha$  and  $\beta$  are the zeroes of a quadratic polynomial p(x), then, the polynomial p(x) can be written as  $p(x) = a\{x^2 - (\alpha + \beta)x + \alpha\beta\}$  or,

 $p(x) = a\{x^2 - (\text{Sum of the zeroes})x + \text{Product of the zeroes}\}\$ , where a is a non-zero real number.

Given: sum of the roots =  $\alpha + \beta = 4$  and product of the roots =  $\alpha\beta = 1$ 

Hence, the quadratic polynomial p(x) can be written as:

$$p(x) = a\{x^2 - 4x + 1\}$$

By taking a = 1, we get one of the quadratic polynomials which satisfy the given conditions.

Therefore, the quadratic polynomial is  $(x^2 - 4x + 1)$ .

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## CBSE NCERT Solutions for Class 10 Science Chapter 2 – Ex 2.3

1. Divide the polynomial p(x) by the polynomial g(x) and find the quotient and remainder in each of the following:

(i) 
$$p(x) = x^3 - 3x^2 + 5x - 3$$
,  $g(x) = x^2 - 2$ 

(ii) 
$$p(x) = x^4 - 3x^2 + 4x + 5$$
,  $g(x) = x^2 + 1 - x$ 

(iii) 
$$p(x) = x^4 - 5x + 6$$
,  $g(x) = 2 - x^2$ 

Solution:

(i) 
$$p(x) = x^3 - 3x^2 + 5x - 3$$
,  $g(x) = x^2 - 2$ 

Here, both the polynomials are already arranged in the descending powers of variable.

The polynomial p(x) can be divided by the polynomial g(x) as follows:

$$\begin{array}{r}
x-3 \\
x^2-2 \overline{\smash)} \quad x^3-3x^2+5x-3 \\
x^3 \qquad -2x \\
- \qquad \qquad + \\
-3x^2+7x-3 \\
-3x^2 \qquad +6 \\
+ \qquad \qquad -7x-9
\end{array}$$

Quotient = x - 3

Remainder = 7x - 9

(ii) 
$$p(x) = x^4 - 3x^2 + 4x + 5 = x^4 + 0 \cdot x^3 - 3x^2 + 4x + 5$$
,

Here, the polynomial p(x) is already arranged in the descending powers of variable.

$$g(x) = x^2 + 1 - x$$

Here, the polynomial g(x) is not arranged in the descending powers of variable.

Now, 
$$g(x) = x^2 - x + 1$$

The polynomial p(x) can be divided by the polynomial g(x) as follows:

$$\begin{array}{r} x^2 + x - 3 \\ x^2 - x + 1 \overline{\smash)} \ \, x^4 + 0 \, x^3 - 3 x^2 + 4 x + 5 \\ x^4 - x^3 + x^2 \\ - + - \\ \hline x^3 - 4 x^2 + 4 x + 5 \\ x^3 - x^2 + x \\ - + - \\ \hline - 3 x^2 + 3 x + 5 \\ - 3 x^2 + 3 x - 3 \\ + - + \\ 8 \end{array}$$

Quotient = 
$$x^2 + x - 3$$

Remainder = 8

(iii) 
$$p(x) = x^4 - 5x + 6 = x^4 + 0.x^2 - 5x + 6$$
  
 $g(x) = 2 - x^2$ 

Here, the polynomial g(x) is not arranged in the descending powers of variable.

Now, 
$$g(x) = -x^2 + 2$$

The polynomial p(x) can be divided by the polynomial g(x) as follows:

$$\begin{array}{r}
-x^{2}-2 \\
-x^{2}+2 \overline{)} & x^{4}+0.x^{2}-5x+6 \\
x^{4}-2x^{2} \\
-+ \\
2x^{2}-5x+6 \\
2x^{2}-4 \\
-+ \\
-5x+10
\end{array}$$

Quotient = 
$$-x^2 - 2$$

$$Remainder = -5x + 10$$

Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:

(i) 
$$t^2 - 3$$
,  $2t^4 + 3t^3 - 2t^2 - 9t - 12$ 

(ii) 
$$x^2 + 3x + 1$$
,  $3x^4 + 5x^3 - 7x^2 + 2x + 2$ 

(iii) 
$$x^3 - 3x + 1$$
,  $x^5 - 4x^3 + x^2 + 3x + 1$ 

#### Solution:

(i) The polynomial  $2t^4 + 3t^3 - 2t^2 - 9t - 12$  can be divided by the polynomial  $t^2 - 3 = t^2 + 0$ . t - 3 as follows:

Since the remainder is 0, hence  $t^2 - 3$  is a factor of  $2t^4 + 3t^3 - 2t^2 - 9t - 12$ .

(ii) The polynomial  $3x^4 + 5x^3 - 7x^2 + 2x + 2$  can be divided by the polynomial  $x^2 + 3x + 1$  as follows:

$$\begin{array}{r}
3x^2 - 4x + 2 \\
x^2 + 3x + 1 \overline{\smash)3x^4 + 5x^3 - 7x^2 + 2x + 2} \\
3x^4 + 9x^3 + 3x^2 \\
- - - \\
-4x^3 - 10x^2 + 2x + 2 \\
-4x^3 - 12x^2 - 4x \\
+ + + \\
2x^2 + 6x + 2 \\
0
\end{array}$$

Since the remainder is 0, hence  $x^2 + 3x + 1$  is a factor of  $3x^4 + 5x^3 - 7x^2 + 2x + 2$ 

(iii) The polynomial  $x^5 - 4x^3 + x^2 + 3x + 1$  can be divided by the polynomial  $x^3 - 3x + 1$  as follows:

Since the remainder is not equal to 0, hence  $x^3 - 3x + 1$  is not a factor of  $x^5 - 4x^3 + x^2 + 3x + 1$ .

3. Obtain all other zeroes of  $3x^4 + 6x^3 - 2x^2 - 10x - 5$ , if two of its zeroes are  $\sqrt{\frac{5}{3}}$  and  $-\sqrt{\frac{5}{3}}$ .

#### Solution:

Let 
$$p(x) = 3x^4 + 6x^3 - 2x^2 - 10x - 5$$

It is given that the two zeroes of p(x) are  $\sqrt{\frac{5}{3}}$  and  $-\sqrt{\frac{5}{3}}$ 

$$\therefore \left(x - \sqrt{\frac{5}{3}}\right) \left(x + \sqrt{\frac{5}{3}}\right) = \left(x^2 - \frac{5}{3}\right) \text{ is a factor of } p(x) \quad \{\text{Since, } (a - b)(a + b) = a^2 - b^2\}$$

Therefore, on dividing the given polynomial by  $x^2 - \frac{5}{3}$ , we obtain remainder as 0.

$$x^{2} + 0 \times -\frac{5}{3} ) \begin{array}{r} 3x^{2} + 6x + 3 \\ 3x^{4} + 6x^{3} - 2x^{2} - 10x - 5 \\ 3x^{4} + 0x^{3} - 5x^{2} \\ - & - & + \\ \hline 6x^{3} + 3x^{2} - 10x - 5 \\ 6x^{3} + 0x^{2} - 10x \\ - & - & + \\ \hline 3x^{2} + 0x - 5 \\ 3x^{2} + 0x - 5 \\ - & - & + \\ \hline 0 \end{array}$$

Hence, 
$$3x^4 + 6x^3 - 2x^2 - 10x - 5 = \left(x^2 - \frac{5}{3}\right)(3x^2 + 6x + 3)$$

$$=3\left(x^2-\frac{5}{3}\right)(x^2+2x+1)$$

Now, 
$$x^2 + 2x + 1 = (x+1)^2$$

Thus, the two zeroes of  $x^2 + 2x + 1$  are -1 and -1

Therefore, the zeroes of the given polynomial are  $\sqrt{\frac{5}{3}}$ ,  $-\sqrt{\frac{5}{3}}$ , -1 and -1.

4. On dividing  $x^3 - 3x^2 + x + 2$  by a polynomial g(x), the quotient and remainder were x - 2 and -2x + 4, respectively. Find g(x).

#### Solution:

Dividend, 
$$p(x) = x^3 - 3x^2 + x + 2$$

Quotient = 
$$(x-2)$$

$$Remainder = (-2x + 4)$$

g(x) be the divisor.

According to the division algorithm,

Dividend = Divisor × Quotient + Remainder

$$x^3 - 3x^2 + x + 2 = g(x) \times (x - 2) + (-2x + 4)$$

$$x^3 - 3x^2 + x + 2 + 2x - 4 = g(x)(x - 2)$$

$$x^3 - 3x^2 + 3x - 2 = g(x)(x - 2)$$

Now, g(x) is the quotient when  $x^3 - 3x^2 + 3x - 2$  is divided by x - 2. (Since, Remainder = 0)

$$g(x) = x^2 - x + 1$$

- 6. Give examples of polynomials p(x), g(x), q(x) and r(x), which satisfy the division algorithm and
  - (i)  $\deg p(x) = \deg q(x)$
  - (ii)  $\deg q(x) = \deg r(x)$
  - (iii)  $\deg r(x) = 0$

#### Solution:

According to the division algorithm, if p(x) and g(x) are two polynomials with  $g(x) \neq 0$ , then we can find polynomials q(x) and r(x) such that

$$p(x) = g(x) \times q(x) + r(x)$$
, where  $r(x) = 0$  or degree of  $r(x) < degree$  of  $g(x)$ .

 Degree of quotient will be equal to degree of dividend when divisor is constant.

Let us consider the division of  $2x^2 + 2x - 16$  by 2.

Here, 
$$p(x) = 2x^2 + 2x - 16$$
 and  $g(x) = 2$ 

$$q(x) = x^2 + x - 8$$
 and  $r(x) = 0$ 

Clearly, the degree of p(x) and q(x) is the same which is 2.

Verification:

$$p(x) = g(x) \times q(x) + r(x)$$

$$2x^2 + 2x - 16 = 2(x^2 + x - 8) + 0$$

$$= 2x^2 + 2x - 16$$

Thus, the division algorithm is satisfied.

(ii) Let us consider the division of 4x + 3 by x + 2.

Here, 
$$p(x) = 4x + 3$$
 and  $g(x) = x + 2$ 

$$q(x) = 4$$
 and  $r(x) = -5$ 

Here, degree of q(x) and r(x) is the same which is 0.

Verification:

$$p(x) = g(x) \times q(x) + r(x)$$

$$4x + 3 = (x + 2) \times 4 + (-5)$$

$$4x + 3 = 4x + 3$$

Thus, the division algorithm is satisfied.

(iii) Degree of remainder will be 0 when remainder obtained on division is a constant.

Let us consider the division of 4x + 3 by x + 2.

Here, 
$$p(x) = 4x + 3$$
 and  $g(x) = x + 2$ 

$$q(x) = 4 \text{ and } r(x) = -5$$

Here, we get remainder as a constant. Therefore, the degree of r(x) is 0.

Verification:

$$p(x) = g(x) \times q(x) + r(x)$$

$$4x + 3 = (x + 2) \times 4 + (-5)$$

$$4x + 3 = 4x + 3$$

Thus, the division algorithm is satisfied.

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