CBSE NCERT Solutions for Class 10 Mathematics Chapter 10

Back of Chapter Questions

1. How many tangents can a circle have?

Solution:

As we know that if a point lies on a circle then at this point only one tangent can be drawn to the circle. Since the circle has infinite points on it and at each point, we can draw one tangent, hence infinite tangents can be drawn to a circle.

2. Fill in the blanks:

(i) A tangent to a circle intersects it in ________ point (s).

(ii) A line intersecting a circle in two points is called a ________.

(iii) A circle can have ________ parallel tangents at the most.

(iv) The common point of a tangent to a circle and the circle is called ________.

Solution:

(i) one

If a line is a tangent to a circle then it meets the circle only at one point.

(ii) secant

If a line intersects a circle at two distinct points, then this line is called a secant of the circle.

(iii) two

tangents drawn at the ends of any diameter are Parallel. A diameter contains only two ends, hence maximum two parallel tangents can be drawn to a circle.

(iv) point of contact

A line meets a circle at exactly one point is called a tangent to the circle and the point where line touches the circle is called point of contact.

3. A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that $OQ = 12$ cm. Length PQ is:

(A) 12 cm

(B) 13 cm

(C) 8.5 cm

(D) $\sqrt{119}$ cm

Solution: **(D)**

As we know radius is always perpendicular to the tangent at the point of contact i.e. $OP \perp PQ$.

Now, applying Pythagoras theorem in ΔOPQ,

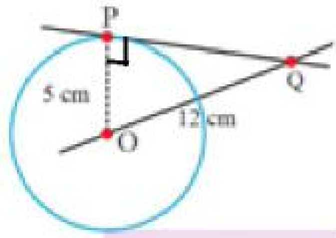
$OP^2 + PQ^2 = OQ^2$

$\Rightarrow 5^2 + PQ^2 = 12^2$

$\Rightarrow PQ^2 = 144 - 25$

$\Rightarrow PQ = \sqrt{119}$ cm

Hence length of PQ is $\sqrt{119}$ cm



4. Draw a circle and two lines parallel to a given line such that one is a tangent and the other, a secant to the circle.

Solution:

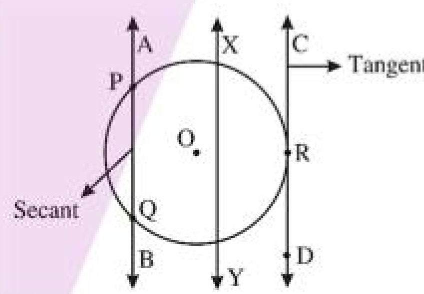
We know how to draw a line parallel to some other line. Yes, we can draw infinite such lines which are parallel to a given line.

Consider the line XY.

We also know if a line intersects a circle at two distinct points then this line is called secant to the circle.

So, if we shift the line XY parallel to itself on left-hand or right-hand side then these lines intersect the circle at two distinct points and represent secant to the circle for example line AB.

Also, a line which meet the circle at exactly one point is called tangent. So, if we shift the line XY on right hand side such that it meets the circle at exactly one point like the line CD, will represent tangent to the circle.



EXERCISE 10.2

In Q.1 to 3, choose the correct option and give justification.

1. From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is

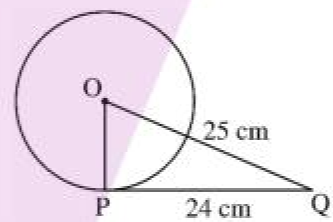
(A) 7 cm

(B) 12 cm

(C) 15 cm

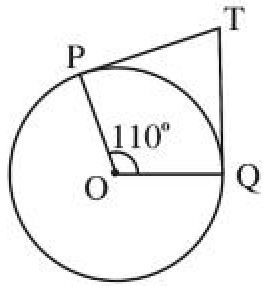
(D) 24.5 cm

Solution: **(A)**

Let O be the center of the circle and the tangent from Q meet the circle at P. Hence the length PQ will represent length of the tangent from Q which is given 24 cm i.e. PQ = 24 cm…………(i) Also OQ = 25 cm……………(ii) As we know radius is perpendicular to tangent at the point of contact i.e. $OP \perp PQ$. Hence applying Pythagoras theorem in ΔOPQ, we get $OP^2 + PQ^2 = OQ^2$ $\Rightarrow OP^2 + 24^2 = 25^2$ $\Rightarrow OP^2 = 625 - 576$ $\Rightarrow OP^2 = 49$ $\Rightarrow OP = 7$ Thus, the radius of the circle is 7 cm.	

2. In Fig., if TP and TQ are the two tangents to a circle with centre O so that

$\angle POQ = 110^o$, then $\angle PTQ$ is equal to



(A) 60^o

(B) 70^o

(C) 80^o

(D) 90^o

Solution: **(B)**

As we know radius is perpendicular to tangent at the point of contact, hence $OP \perp TP$ and $OQ \perp TQ$.

$\Rightarrow \angle OPT = 90^o$ and $\angle OQT = 90^o$………(i)

Also, in a quadrilateral sum of interior angles is 360^o.

Hence for the quadrilateral POQT, we can write

$\angle OPT + \angle POQ + \angle OQT + \angle PTQ = 360^o$………(ii)

$\Rightarrow 90^o + 110^o + 90^o + \angle PTQ = 360^o$

$\Rightarrow \angle PTQ = 70^o$

3. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80^o, then $\angle POA$ is equal to

(A) 50^o

(B) 60^o

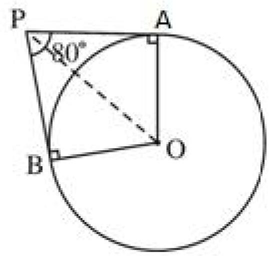
(C) 70^o

(D) 80^o

Solution: **(A)**

As we know radius is perpendicular to tangent at the point of contact, hence $OA \perp PA$ and $OB \perp PB$.

$\Rightarrow \angle OBP = 90^o$ and $\angle OAP = 90^o$………(i)



Also, in a quadrilateral sum of interior angles is 360°, hence for the quadrilateral AOBP, we can write

$\angle OAP + \angle APB + \angle PBO + \angle BOA = 360^o$………(ii)

$\Rightarrow 90^o + 80^o + 90^o + \angle BOA = 360^o$

$\Rightarrow \angle BOA = 100^o$

In ΔOPA and ΔOPB

$AP = BP$ (length of tangents from an external point to a circle is equal)

$OA = OB$ (radius of circle)

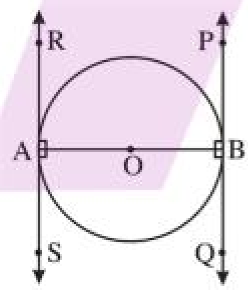
$OP = OP$ (common side)

Hence, $\Delta OPB \cong \Delta OPA$ (by SSS congruency)

Hence, $\angle POA = \angle POB = \frac{1}{2}\angle BOA$

$\Rightarrow \angle POA = \dfrac{100^o}{2} = 50^o$

4. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.



Solution:

Consider AB as a diameter of the circle. PQ and RS are two tangents drawn at the end points of the diameter AB.

As we know that the radius is perpendicular to tangent at the point of contact.

Hence $\angle OAR = 90^o$ and $\angle OBQ = 90^o$………(i)

From alternate interior angles theorem, we can say

$\angle OAR = \angle OBQ$ ....(ii)

Since, alternate interior angles are equal, hence lines PQ and RS must be parallel.

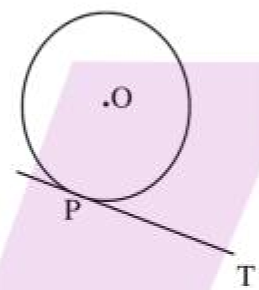
5. Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

Solution:

Let P be the point of contact and PT be the tangent at the point P on the circle with centre O.

Since OP is radius of the circle and PT is a tangent at P, $OP \perp PT$.

Thus, the perpendicular at the point of contact to the tangent passes through the centre.



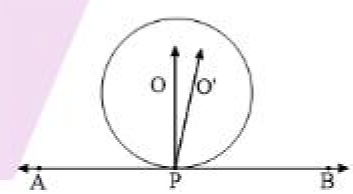
Let us consider a circle with centre O. Let AB be a tangent which touches the circle at point P.

We have to prove that a line perpendicular to AB at point P passes through the centre.

Let us assume that the perpendicular to AB at P does not pass through centre O. Let it pass through another point O′.

As perpendicular to AB at P passes through O′, therefore,

$\angle O'PB = 90^o$.........(1)



But O is the centre of the circle. As we know the line joining the centre and the point of contact to the tangent of the circle are perpendicular to each other.

$\therefore \angle OPB = 90^o$ ....(2)

From equation (1) & equation (2)

$\angle O'PB = \angle OPB$

From the figure,

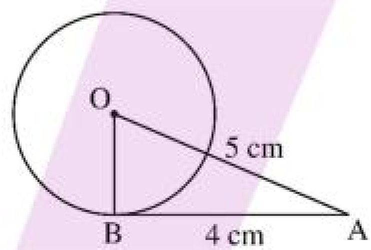
$\angle O'PB < \angle OPB$

$\therefore \angle O'PB < \angle OPB$ is not possible.

It is only possible, when the line O'P coincides with OP. Therefore, the perpendicular to AB through P passes through centre O.	

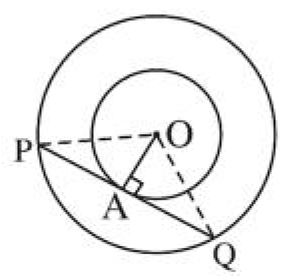
6. The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.

Solution:

Let tangent to the circle from A meet the circle at B and O be the centre of the circle. Given $OA = 5$ cm and $AB = 4$ cm……….(i) As we know radius is perpendicular to tangent at the point of contact i.e. $OB \perp AB$………(ii) Applying Pythagoras theorem in ΔABO, $AB^2 + OB^2 = OA^2$ $\Rightarrow 4^2 + OB^2 = 5^2$ $\Rightarrow OB^2 = 9$ $\Rightarrow OB = 3$ Hence, the radius of the circle is 3 cm.	

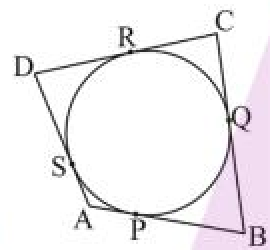
7. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Solution:

Let the centre of the two concentric circles be O and chord PQ of the larger circle touches the smaller circle at A. Since PQ is tangent to the smaller circle, hence $OA \perp PQ$. Applying Pythagoras theorem in ΔOAP, we get $OA^2 + AP^2 = OP^2$ $\Rightarrow 3^2 + AP^2 = 5^2$	

$\Rightarrow AP^2 = 16$ $\Rightarrow AP = 4$ cm Since $AP = AQ$ (Perpendicular from center of circle bisects the chord) $\Rightarrow PQ = 2AP = 2 \times 4$ cm $= 8$ cm Length of chord of larger circle is 8 cm.	

8. A quadrilateral ABCD is drawn to circumscribe a circle (see Fig.). Prove that $AB + CD = AD + BC$



Solution:

As we know,

Length of the tangents from an external point to a circle is always equal. Hence from the given diagram we can easily say

Length of the tangents from the point A: $AP = AS$………..(i)

Length of the tangents from the point B: $BP = BQ$……….(ii)

Length of the tangents from the point C: $CR = CQ$……….(iii)

Length of the tangents from the point D: $DR = DS$………..(iv)

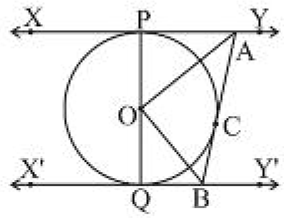
Adding the above four equations, we get

$AP + BP + CR + DR = AS + BQ + CQ + DS$

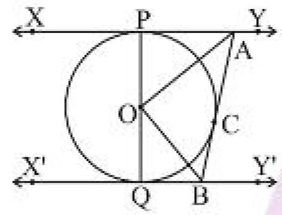
$\Rightarrow (DR + CR) + (BP + AP) = (DS + AS) + (CQ + BQ)$

$\Rightarrow CD + AB = AD + BC$

9. In Fig., XY and X′Y′ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X′Y′ at B. Prove that $\angle AOB = 90^o$.



Solution:



First, we will show that $\angle POA = \angle COA$ and $\angle QOB = \angle COB$.

Join O with the point of contact C.

Now in ΔOPA and ΔOCA, we can easily observe

$OP = OC$ (radius of the same circle)

$AP = AC$ (length of tangents from external point)

$AO = AO$ (common side)

Hence $\Delta OPA \cong \Delta OCA$ (by SSS congruency)

$\Rightarrow \angle POA = \angle COA$ (by CPCT) ………..(i)

Similarly, $\Delta OQB \cong \Delta OCB$

$\Rightarrow \angle QOB = \angle COB$ ………..(ii)

Since, POQ is a diameter of circle, we can say $\angle POQ = 180^o$

$\Rightarrow \angle POA + \angle AOC + \angle COB + \angle BOQ = 180^o$

Now from equations (i) and (ii),

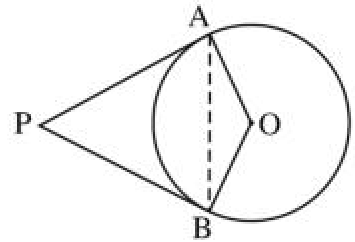
$2\angle AOC + 2\angle COB = 180^o$

$\Rightarrow (\angle AOC + \angle COB) = 90^o$

$\Rightarrow \angle AOB = 90^o$

10. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

Solution:



Here we have to show that $\angle APB + \angle BOA = 180^o$.

Let the centre of the circle be O. Tangents from PA and PB drawn to the circle from P meet the circle at A and B.

AB is the line segment joining point of contacts A and B together such that it subtends $\angle AOB$ at center O of the circle.

As the radius is perpendicular to the tangent at the point of contact. Hence, we can say

$\angle OAP = 90^o$ and $\angle OBP = 90^o$ ………...(i)

As we know sum of interior angles in a quadrilateral is 360^o, hence in quadrilateral OAPB

$\angle OAP + \angle APB + \angle PBO + \angle BOA = 360^o$

$\Rightarrow 90^o + \angle APB + 90^o + \angle BOA = 360^o$

$\Rightarrow \angle APB + \angle BOA = 180^o$

Hence, the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

11. Prove that the parallelogram circumscribing a circle is a rhombus.

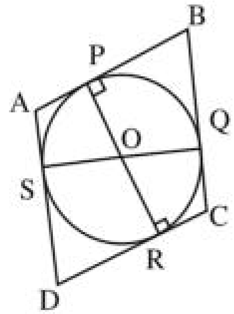
Solution:

Let ABCD be the parallelogram circumscribed about a circle with centre O and the sides AB, BC, CD and DA touches the circle at points P, Q, R and S respectively.

Since, ABCD is a parallelogram.

$AB = CD$ ………..(i)

$BC = AD$ ………..(ii)



As we know length of the tangents from an external point to a circle is always equal. Hence from the given diagram we can easily say

Length of the tangents from the point A: $AP = AS$………..(iii)

Length of the tangents from the point B: $BP = BQ$……….(iv)

Length of the tangents from the point C: $CR = CQ$……….(v)

Length of the tangents from the point D: $DR = DS$………..(vi)

Adding (iii), (iv),(v) and (vi), we get

$DR + CR + BP + AP = DS + CQ + BQ + AS$

$\Rightarrow (DR + CR) + (BP + AP) = (DS + AS) + (CQ + BQ)$

$\Rightarrow CD + AB = AD + BC$ ………..(vii)

From equation (i), (ii) and (vii):

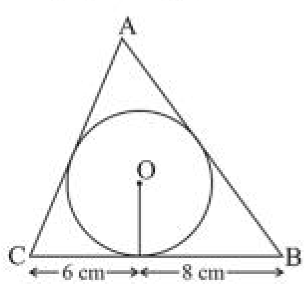
$2AB = 2BC$

$\Rightarrow AB = BC$

$\Rightarrow AB = BC = CD = DA$

Hence, ABCD is a rhombus.

12. A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively (see Fig.). Find the sides AB and AC.



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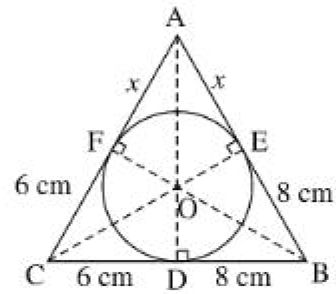


Let the sides AB and AC of the triangle ABC touches the circle at E and F respectively. Also side BC touches the circle at D.

Consider the length of the line segment AF be x.

As we know length of the tangents from an external point to a circle is always equal. Hence from the given diagram we can easily say

$AE = AF = x$ (Length of the tangents from the point A)

$BE = BD = 8$ cm (Length of the tangents from the point B)

$CF = CD = 6$ cm (Length of the tangents from the point C)

Let the semi-perimeter of the triangle be s.

Perimeter $= AB + BC + CA = x + 8 + 14 + 6 + x = 28 + 2x$

$\Rightarrow s = 14 + x$

Area of Δ ABC $= \sqrt{s(s-a)(s-b)(s-c)}$ (Heron's formula)

$= \sqrt{(14+x)((14+x)-14)((14+x)-(6+x))((14+x)-(8+x))}$

$= \sqrt{(14+x)(x)(8)(6)}$

Area of $\Delta ABC = 4\sqrt{3(14x+x^2)}$…………(i)

Area of $\Delta OBC = \frac{1}{2} \times OD \times BC = \frac{1}{2} \times 4 \times 14 = 28$………..(ii)

Area of $\times\ \Delta OCA = \frac{1}{2} \times OF \times AC = \frac{1}{2} \times 4 \times (16 + x) = 12 + 2x$………..(iii)

Area of $\Delta OAB = \frac{1}{2} \times OE \times AB = \frac{1}{2} \times 4 \times (8 + x) = 16 + 2x$………..(iv)

Area of ΔABC = Area of ΔOBC + Area of ΔOCA + Area of ΔOAB

$\Rightarrow 4\sqrt{3(14x+x^2)} = 28 + 12 + 2x + 16 + 2x$

$\Rightarrow 4\sqrt{3(14x+x^2)} = 56 + 4x$

$\Rightarrow \sqrt{3(14x+x^2)} = 14 + x$

$\Rightarrow 3(14x + x^2) = (14 + x)^2$

$\Rightarrow 42x + 3x^2 = 196 + x^2 + 28x$

$\Rightarrow 2x^2 + 14x - 196 = 0$

$\Rightarrow x^2 + 7x - 98 = 0$

$\Rightarrow x^2 + 14x - 7x - 98 = 0$

$\Rightarrow x(x + 14) - 7(x + 14) = 0$

$\Rightarrow (x + 14)(x - 7) = 0$

$\Rightarrow x = -14$ or 7

But $x = -14$ is not possible as length of sides will be negative.

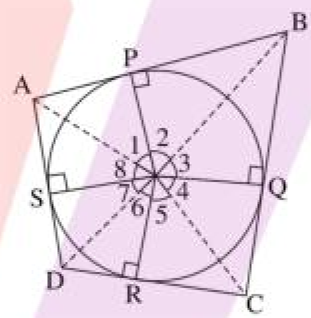
So, $x = 7$

Hence AB $= x + 8 = 7 + 8 = 15$ cm

CA $= 6 + x = 6 + 7 = 13$ cm

13. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Solution:



Let ABCD be a quadrilateral circumscribing a circle with centre O. Also, the sides AB, BC, CD and DA touches the circle at point P, Q, R, and S respectively.

Now join the vertices of the quadrilateral ABCD to the center O of the circle.

In triangles ΔOAP and ΔOAS

AP = AS (length of tangents from same point)

OP = OS (radius of the same circle)

OA = OA (common side)

Hence Δ OAP ≅ Δ OAS (by SSS congruency)

$\Rightarrow \angle POA = \angle SOA$ (by CPCT)

$\Rightarrow \angle 1 = \angle 8$ ………..(i)

Similarly, we can prove

$\angle 2 = \angle 3$ ………..(ii)

$\angle 4 = \angle 5$ ………..(iii) and

$\angle 6 = \angle 7$ ………..(iv)

On adding (i), (ii), (iii) and (iv), we get

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^o$

$\Rightarrow (\angle 1 + \angle 8) + (\angle 2 + \angle 3) + (\angle 4 + \angle 5) + (\angle 6 + \angle 7) = 360^o$

$\Rightarrow 2\angle 1 + 2\angle 2 + 2\angle 5 + 2\angle 6 = 360^o$

$\Rightarrow 2(\angle 1 + \angle 2) + 2(\angle 5 + \angle 6) = 360^o$

$\Rightarrow (\angle 1 + \angle 2) + (\angle 5 + \angle 6) = 180^o$

$\Rightarrow \angle AOB + \angle COD = 180^o$

Similarly, $\angle BOC + \angle DOA = 180^o$

Hence opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

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