

CBSE NCERT Solutions for Class 9 Mathematics Chapter 8

Back of Chapter Questions

Exercise: 8.1

1. The angles of a quadrilateral are in the ratio 3: 5: 9: 13. Find all the angles of the quadrilateral.

Solution:

The ratio of the angles = 3: 5: 9: 13

Let the angles be $3x$, $5x$, $9x$ and $13x$

$3x + 5x + 9x + 13x = 360^\circ$ (\because sum of all the angles of a quadrilateral equals 360°)

$$30x = 360^\circ$$

$$x = 12^\circ$$

$$\therefore 3x = 3 \times 12^\circ = 36^\circ$$

$$5x = 5 \times 12^\circ = 60^\circ$$

$$9x = 9 \times 12^\circ = 108^\circ$$

$$13x = 13 \times 12^\circ = 156^\circ$$

Therefore, the angles of a quadrilateral are 36° , 60° , 108° and 156°

2. If the diagonals of a parallelogram are equal, then show that it is a rectangle.

Solution:

ABCD is a parallelogram in which $AC = BD$ in $\triangle ABC$ and $\triangle BCD$

$$AC = BD \text{ (given)}$$

$$BC = BC \text{ (common side)}$$

$$AB = DC \text{ (opposite sides of a parallelogram are equal)}$$

$$\Rightarrow \triangle ABC = \triangle BCD \text{ (SSS criteria)}$$

\therefore Their corresponding parts are equal.

$$\Rightarrow \angle ABC = \angle DCB \dots (1)$$

$\because AB \parallel DC$, this implies BC is a transversal. [\because ABCD is a parallelogram]

$$\therefore \angle ABC + \angle DCB = 180^\circ \dots (2)$$

From (1) and (2), we have

$$\angle ABC = \angle DCB = 90^\circ$$

i.e. ABCD is a parallelogram having an angle equal to 90° .

Therefore, ABCD is a rectangle

3. Show that if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.

Solution:

In Quadrilateral ABCD, diagonals AC and BD bisect each other at right angles at O

\therefore In $\triangle AOB$ and $\triangle AOD$, we have

$$AO = AO \text{ (Common)}$$

$$OB = OD \text{ (since O is the midpoint of BD)}$$

$$\angle AOB = \angle AOD \text{ (right angles, i.e., } 90^\circ \text{)}$$

$$\triangle AOB \cong \triangle AOD \text{ (SAS criteria)}$$

\Rightarrow Their corresponding parts are equal.

$$AB = AD \dots (1)$$

$$\text{Similarly, } AB = BC \dots (2)$$

$$BC = CD \dots (3)$$

$$CD = AD \dots (4)$$

\therefore From (1), (2), (3) and (4), we get $AB = BC = CD = DA$

Therefore, the quadrilateral ABCD is a rhombus.

4. Show that the diagonals of a square are equal and bisect each other at right angles.

Solution:

ABCD is a square with its diagonals AC and BD intersecting at O.

(a) To prove that the diagonals are equal, i.e. $AC = BD$

In $\triangle ABC$ and $\triangle BAD$, $AB = BA$ (common side)

$BC = AD$ (opposite sides of the square ABCD)

$\angle ABC = \angle BAD$ (angles of a square are equal to 90°)

$\therefore \triangle ABC \cong \triangle BAD$ (SAS criteria)

\Rightarrow Their corresponding parts are equal.

$$\Rightarrow AC = BD \dots (1)$$

(b) To prove that 'O' is the midpoint of AC and BD.

$\therefore AD \parallel BC$ and AC is a transversal. (\because opposite sides of a square are parallel)

$\therefore \angle 1 = \angle 3$ (interior alternate angles)

Similarly, $\angle 2 = \angle 4$ (interior alternate angles)

Now, in $\triangle OAD$ and $\triangle OCB$, $AD = CB$ (opposite sides of the square)

$\angle 1 = \angle 3$ (proved)

$\angle 2 = \angle 4$ (proved)

$\triangle OAD \cong \triangle OCB$ [ASA criteria]

\therefore Their corresponding parts are equal.

$\Rightarrow OA = OC$ and $OD = OB$

$\Rightarrow O$ is the midpoint of AC and BD, i.e. the diagonals AC and BD bisect each other at O...(2)

(c) To prove that $AC \perp BD$

In $\triangle OBA$ and $\triangle ODA$, $OB = OD$ (proved)

$BA = DA$ (opposite sides of the square)

$OA = OA$ (common)

$\therefore \triangle OBA \cong \triangle ODA$ (SSS criteria)

\Rightarrow Their corresponding parts are equal.

$\Rightarrow \angle AOB = \angle AOD$

And $\angle AOB$ and $\angle AOD$ form a linear pair.

$\angle AOB + \angle AOD = 180^\circ$

$\angle AOB = \angle AOD = 90^\circ$

$\Rightarrow AC \perp BD$...(3)

Therefore, from (1), (2) and (3), we get AC and BD are equal and bisect each other at right angles.

5. Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.

Solution:

Quadrilateral ABCD has O as the midpoint of AC and BD.

$\Rightarrow AC \perp BD$

In $\triangle AOD$ and $\triangle AOB$, $AO = AO$ (Common)

$OD = OB$ (\because O is the midpoint of BD)

$\angle AOD = \angle AOB$ (right angles)

$\therefore \triangle AOD \cong \triangle AOB$ (SAS criteria)

\therefore Their corresponding parts are equal

$\Rightarrow AD = AB \dots (1)$

Similarly, $AB = BC \dots (2)$

$BC = CD \dots (3)$

$CD = DA \dots (4)$

From (1), (2), (3) and (4) we have: $AB = BC = CD = DA$

Therefore, Quadrilateral ABCD has all sides equal.

In $\triangle AOD$ and $\triangle COB$, $AO = CO$ (given)

$OD = OB$ (given)

$\angle AOD = \angle COB$ (vertically opposite angles)

$\therefore \triangle AOD \cong \triangle COB$ (SAS criteria)

\Rightarrow Their corresponding parts are equal.

$\Rightarrow \angle 1 = \angle 2$ (form a pair of interior alternate angles)

$\therefore AD \parallel BC$

Similarly, $AB \parallel DC \therefore$ ABCD is a parallelogram.

We know that parallelogram with all sides equal is a rhombus.

\therefore ABCD is a rhombus.

In $\triangle ABC$ and $\triangle BAD$, $AC = BD$ (Given)

$BC = AD$ (as proved)

$AB = BA$ (Common)

$\therefore \triangle ABC \cong \triangle BAD$ (SSS criteria)

\Rightarrow Their corresponding angles are equal.

$\angle ABC = \angle BAD$

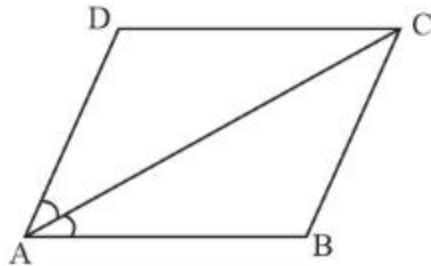
Now that $AD \parallel BC$ and AB is a transversal.

$\angle ABC + \angle BAD = 180^\circ$ (interior opposite angles are supplementary)

i.e. The rhombus ABCD has an angle equal to 90° .

Therefore, ABCD is a square.

6. Diagonal AC of a parallelogram ABCD bisects $\angle A$ (see Fig). Show that



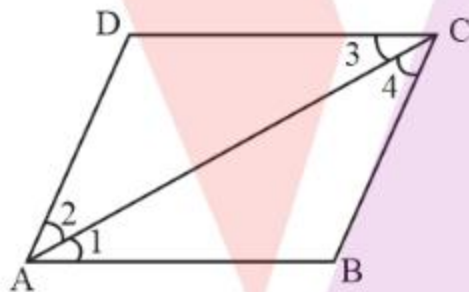
- (i) it bisects $\angle C$ also,
- (ii) ABCD is a rhombus.

Solution:

In parallelogram ABCD, diagonal AC bisects $\angle A$.

$$\Rightarrow \angle DAC = \angle BAC$$

- (i) To prove that AC bisects $\angle C$.



ABCD is a parallelogram,

$\Rightarrow AB \parallel DC$ and AC is a transversal.

$$\therefore \angle 1 = \angle 3 \text{ (interior alternate angles) ... (1)}$$

$\Rightarrow BC \parallel AD$ and AC is a transversal.

$$\therefore \angle 2 = \angle 4 \text{ (interior alternate angles) ... (2)}$$

We know, AC bisects $\angle A$. (given)

$$\therefore \angle 1 = \angle 2 \text{ ... (3)}$$

From (1), (2) and (3), we get

$$\angle 3 = \angle 4$$

Therefore, this proves AC bisects $\angle C$.

(ii) To prove ABCD is a rhombus.

In ΔABC , $\angle 1 = \angle 4$ [$\because \angle 1 = \angle 2 = \angle 4$]

$\Rightarrow BC = AB$ (sides opposite to equal angles) ... (4)

Similarly, $AD = DC$... (5)

Since it is given that ABCD is a parallelogram

$AB = DC$ (opposite sides of a parallelogram).... (6)

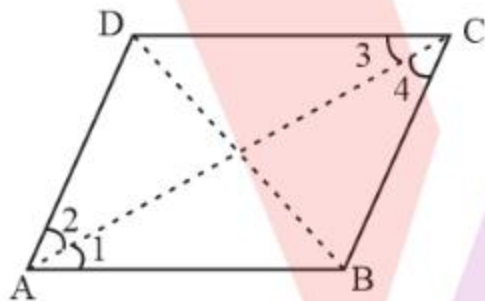
From (4), (5) and (6), we get $AB = BC = CD = DA$

Therefore, ABCD is a rhombus.

7. ABCD is a rhombus. Show that diagonal AC bisects $\angle A$ as well as $\angle C$ and diagonal BD bisects $\angle B$ as well as $\angle D$

Solution:

ABCD is a rhombus.



$\therefore AB = BC = CD = AD$

Then $AB \parallel CD$ and $AD \parallel BC$

Now, $AD = CD$

$\Rightarrow \angle 1 = \angle 2$ (1) (angles opposite to equal sides are equal]

Also, $CD \parallel AB$ (opposite sides of the parallelogram]

$\therefore \angle 1 = \angle 3$ (2)

And AC is transversal.

$\therefore \angle 1 = \angle 4$ (3)

From (1), (2) and (3), we have

$\angle 2 = \angle 3$ and $\angle 1 = \angle 4$

This shows that AC bisects $\angle C$ as well as $\angle A$.

Similarly, it is proved that BD bisects $\angle B$ as well as $\angle D$.

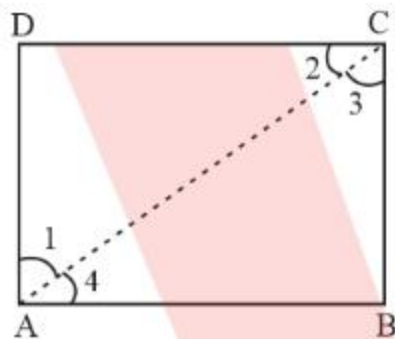
8. ABCD is a rectangle in which diagonal AC bisects $\angle A$ as well as $\angle C$. Show that:

- (i) ABCD is a square
- (ii) Diagonal BD bisects $\angle B$ as well as $\angle D$.

Solution:

In rectangle ABCD, AC bisects $\angle A$ and $\angle C$.

$$\Rightarrow \angle 1 = \angle 4 \text{ and } \angle 2 = \angle 3 \dots (1)$$



(i) ABCD is a parallelogram. (rectangle is a parallelogram)

$\Rightarrow AB \parallel CD$ and AC is a transversal.

$$\therefore \angle 2 = \angle 4 \dots (2) \quad (\text{alternate interior angles})$$

From (1) and (2), we get $\angle 3 = \angle 4$

$\Rightarrow AB = BC$ (sides opposite to equal angles in ΔABC are equal.)

$$AB = BC = CD = AD$$

\Rightarrow ABCD is a rectangle having with all sides equal.

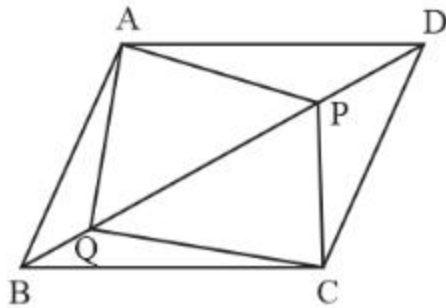
Therefore it is proved that ABCD is a square.

(ii) ABCD is a square, and diagonals of a square bisect the opposite angles.

\therefore BD bisects $\angle B$ as well as $\angle D$

9. In parallelogram ABCD, two points P and Q are taken on diagonal BD such that $DP = BQ$

(see fig) Show that:



- (i) $\triangle APD \cong \triangle CQB$
- (ii) $AP = CQ$
- (iii) $\triangle AQB \cong \triangle CPD$
- (iv) $AQ = CP$
- (v) APCQ is a parallelogram

Solution:

- (i) To prove that $\triangle APD \cong \triangle CQB$
 $AD \parallel BC$ and BD is a transversal. (\because ABCD is a parallelogram)
 $\therefore \angle ADB = \angle CBD$ (interior alternate angles)
 $\Rightarrow \angle ADP = \angle CBQ$
 In $\triangle APD$ and $\triangle CQB$,
 $AD = CB$ (opposite sides of the parallelogram)
 $DP = BQ$ (given)
 $\angle CBQ = \angle ADP$ (already proved)
 Therefore, $\triangle APD \cong \triangle CQB$ (SAS criteria)
- (ii) To prove that $AP = CQ$
 Since $\triangle APD \cong \triangle CQB$ (as proved)
 \therefore Their corresponding parts are equal.
 Therefore $AP = CQ$
- (iii) To prove that $\triangle AQB \cong \triangle CPD$.
 BD is a transversal
 $\Rightarrow AB \parallel CD$ (\because ABCD is a parallelogram)
 $\therefore \angle ABD = \angle CDB$

$$\Rightarrow \angle ABQ = \angle CDP$$

So, in $\triangle AQB$ and $\triangle CPD$, $QB = PD$ (given)

$$\angle ABQ = \angle CDP \text{ (as proved)}$$

$AB = CD$ (opposite sides of parallelogram $ABCD$)

Therefore $\triangle AQB \cong \triangle CPD$ (SAS criteria)

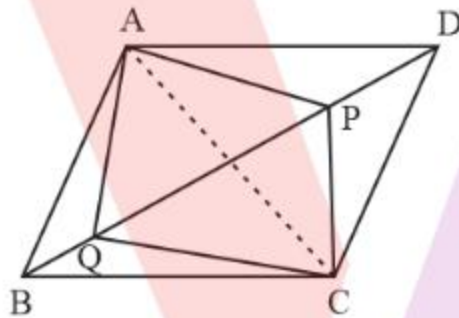
(iv) To prove that $AQ = CP$.

Now we have proved that $\triangle AQB \cong \triangle CPD$

\therefore Their corresponding parts are equal.

\Rightarrow this proves that $AQ = CP$.

(v) To prove that $APCQ$ is a parallelogram.



By joining AC , the diagonals of a parallelogram bisect each other

$$\Rightarrow AO = CO$$

And $BO = DO$

$$\Rightarrow (BO - BQ) = (DO - DP) \text{ (}\because BQ = DP\text{)}$$

$$\Rightarrow QO = PO$$

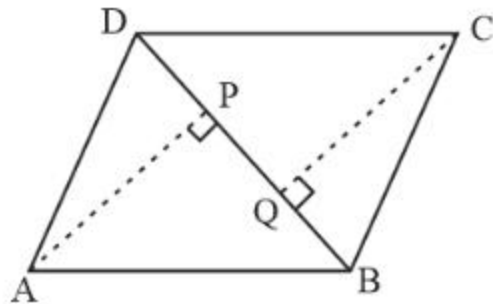
Now, in quadrilateral $APCQ$, we get

$$AO = CO \text{ and } QO = PO$$

$\Rightarrow AC$ and QP bisect each other at O .

Therefore, $APCQ$ is a parallelogram.

10. $ABCD$ is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD (see Fig.). Show that

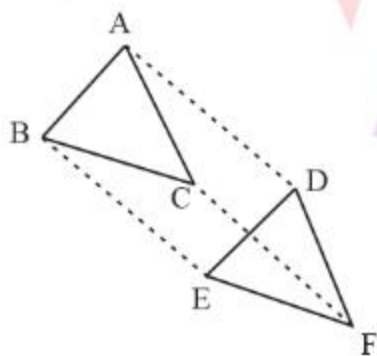


- (i) $\triangle APB \cong \triangle CQD$
- (ii) $AP = CQ$

Solution:

- (i) To prove $\triangle APB \cong \triangle CQD$
 In $\triangle APB$ and $\triangle CQD$, $\angle APB = \angle CQD$ (90° each)
 $AB = CD$ (opposite sides of parallelogram ABCD)
 $\angle ABP = \angle CDQ$
 Therefore, $\triangle APB \cong \triangle CQD$ (AAS criteria)
- (ii) To prove $AP = CQ$
 Now that $\triangle APB \cong \triangle CQD$
 \therefore Their corresponding parts are equal.
 Therefore, $AP = CQ$

11. In $\triangle ABC$ and $\triangle DEF$, $AB = DE$, $AB \parallel DE$, $BC = EF$ and $BC \parallel EF$. Vertices A, B and C are joined to vertices D, E and F respectively (see Fig.) Show that



- (i) quadrilateral ABED is a parallelogram
- (ii) quadrilateral BEFC is a parallelogram
- (iii) $AD \parallel CF$ and $AD = CF$

- (iv) quadrilateral ACFD is a parallelogram
- (v) $AC = DF$
- (vi) $\triangle ABC \cong \triangle DEF$.

Solution:

- (i) To prove that ABED is a parallelogram.

We know that "A quadrilateral is a parallelogram if a pair of opposite sides are of equal length."

Here, $AB = DE$ (given)

$AB \parallel DE$ (given)

Therefore, a pair of opposite sides of quadrilateral ABED is of equal length.

And this proves that ABED is a parallelogram.

To prove that ABED is a parallelogram.

- (ii) To prove that BEFC is a parallelogram.

$BC = EF$ (given)

And $BC \parallel EF$ (given)

Therefore, BECF is a quadrilateral in which a pair of opposite sides (BC and EF) is parallel and of equal length.

This proves that BECF is a parallelogram.

- (iii) To prove that $AD \parallel CF$ and $AD = CF$

ABED is a parallelogram. (proved)

\therefore Its opposite sides are parallel and equal.

$\Rightarrow AD \parallel BE$ and $AD = BE \dots(1)$

Also BEFC is a parallelogram. (proved)

$\therefore BE \parallel CF$ and $BE = CF \dots(2)$ (opposite sides of a parallelogram are parallel and equal)

From (1) and (2), it is proved $AD \parallel CF$ and $AD = CF$

- (iv) To prove that ACFD is a parallelogram.

$AD \parallel CF$ (proved)

and $AD = CF$ (proved)

Therefore, ACFD is a quadrilateral which has one pair of opposite sides (AD and CF) parallel and of equal length.

This proves that Quadrilateral ACFD is a parallelogram.

(v) To prove that $AC = DF$.

ACFD is a parallelogram (proved)

\Rightarrow Its opposite sides are parallel and of equal length.

Therefore $AC = DF$

(vi) To prove that $\triangle ABC = \triangle DEF$

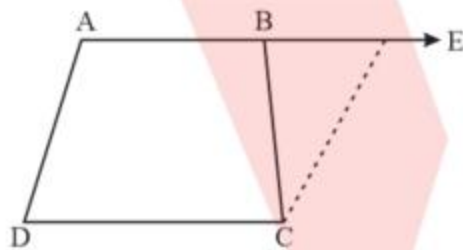
In $\triangle ABC$ and $\triangle DEF$, $AB = DE$ (opposite sides of a parallelogram)

$BC = EF$ (opposite sides of a parallelogram)

$AC = DF$ (proved)

Therefore $\triangle ABC \cong \triangle DEF$. (SSS criteria)

12. ABCD is a trapezium in which $AB \parallel CD$ and $AD = BC$ (see Fig.) Show that



(i) $\angle A = \angle B$

(ii) $\angle C = \angle D$

(iii) $\triangle ABC \cong \triangle BAD$

(iv) diagonal $AC =$ diagonal BD

Solution:

Given that $AB \parallel CD$ and $AD = BC$

(i) To prove that $\angle A = \angle B$.

Extend AB to E and draw $CE \parallel AD$.

$\therefore AB \parallel DC$

$\Rightarrow AE \parallel DC$ (given)

And $AD \parallel CE$

This shows that AECD is a parallelogram.

$\Rightarrow AD = CE$ (opposite sides of the parallelogram AECD)

But we know that $AD = BC$ (given)

$\therefore BC = CE$

In $\triangle BCE$, $BC = CE$

$\Rightarrow \angle CBE = \angle CEB \dots(1)$ (angles opposite to equal sides of a triangle are equal)

Also, $\angle ABC + \angle CBE = 180^\circ \dots(2)$ (linear pair)

and $\angle A + \angle CEB = 180^\circ \dots(3)$ (adjacent angles of a parallelogram are supplementary)

From (2) and (3), we get

$$\angle ABC + \angle CBE = \angle A + \angle CEB$$

We know, $\angle CBE = \angle CEB$

$$\therefore \angle ABC = \angle A$$

$$\text{or } \angle B = \angle A$$

$$\text{i.e., } \angle A = \angle B$$

(ii) To prove that $\angle C = \angle D$.

$AB \parallel CD$ and AD is a transversal.

$$\angle A + \angle D = 180^\circ \text{ (sum of interior opposite angles is } 180^\circ)$$

$$\text{Similarly, } \angle B + \angle C = 180^\circ$$

$$\Rightarrow \angle A + \angle D = \angle B + \angle C$$

We know, $\angle A = \angle B$ (as proved in (i))

$$\text{Therefore } \angle C = \angle D$$

(iii) To prove $\triangle ABC \cong \triangle BAD$

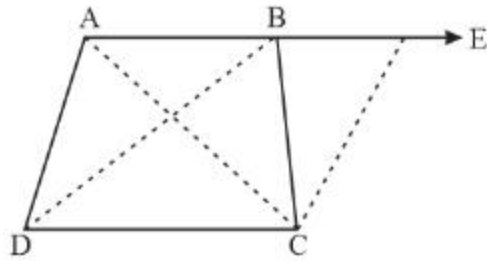
In $\triangle ABC$ and $\triangle BAD$, $AB = BA$ (common side)

$BC = AD$ (given)

$\angle ABC = \angle BAD$ (proved)

Therefore $\triangle ABC \cong \triangle BAD$ (SAS criteria)

(iv) To prove that diagonal $AC =$ diagonal BD



$\Delta ABC \cong \Delta BAD$ (as proved in (iii))

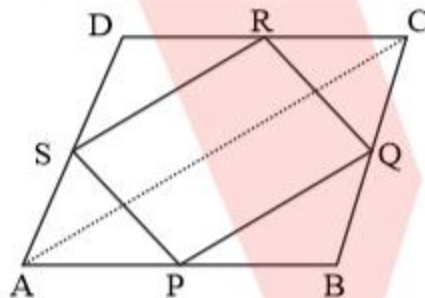
\therefore Their corresponding parts are equal.

This proves that diagonal $AC =$ diagonal BD .

Hint: Extend AB and draw a line through C parallel to DA intersecting AB produced at E .

Exercise: 8.2

13. $ABCD$ is a quadrilateral in which P, Q, R and S are mid-points of the sides AB, BC, CD and DA (see Fig.) AC is a diagonal. Show that:



- (i) $SR \parallel AC$ and $SR = \frac{1}{2}AC$
- (ii) $PQ = SR$
- (iii) $PQRS$ is a parallelogram.

Solution:

P is the midpoint of AB, Q is the midpoint of BC

- (i) To prove that $SR \parallel AC$ and $SR = \frac{1}{2}AC$ and $SR \parallel AC$

$ABCD$ is a quadrilateral with R as the midpoint of CD, S as the mid-point of DA , and AC as the diagonal of a quadrilateral $ABCD$.

In $\Delta ACD, S$ is the midpoint of $AD,$

R is the midpoint of $CD.$

\Rightarrow The line segment joining the mid-point of any two sides of a triangle is parallel to the third side and half of it.

$$SR = \frac{1}{2}AC \text{ and } SR \parallel AC$$

(ii) To prove that $PQ = SR$.

In $\triangle ABC$, we have

P is the midpoint of AB

Q is the midpoint of BC.

$$\therefore PQ = \frac{1}{2}AC \dots\dots(1)$$

$$\text{And, } SR = \frac{1}{2}AC \dots\dots(2) \quad (\text{as proved in (i)})$$

Therefore From (1) and (2), it is proved

$$PQ = SR$$

(iii) To prove that PQRS is a parallelogram.

In $\triangle ABC$, P is the midpoint of AB and Q is the midpoint of BC.

$$\therefore PQ = \frac{1}{2}AC \text{ and } PQ \parallel AC \dots\dots(3)$$

In $\triangle ACD$, S is the midpoint of DA and R is the midpoint of CD

$$\therefore SR = \frac{1}{2}AC \text{ and } SR \parallel AC \dots\dots(4)$$

From (3) and (4), we get

$$PQ = SR = \frac{1}{2}AC \text{ and } PQ \parallel AC \parallel SR$$

$$\Rightarrow PQ = SR \text{ and } PQ \parallel SR$$

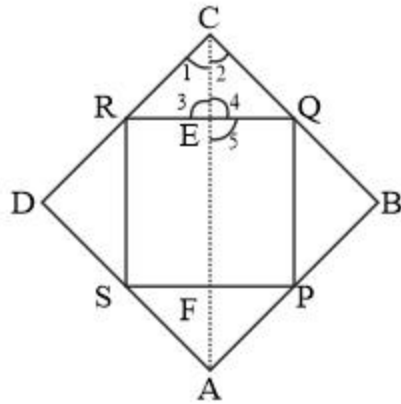
We know that one pair of opposite sides in a quadrilateral PQRS is equal and parallel.

Therefore, PQRS is a parallelogram.

14. ABCD is a rhombus and P, Q, R and S are the midpoints of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rectangle.

Solution:

To prove that PQRS is a rectangle



In Rhombus ABCD, P, Q, R, and S are the mid points of AB, BC, CD, and DA.

By joining AC, In ΔABC , P and Q are the mid-points of AB and BC.

$$\therefore PQ = \frac{1}{2}AC \text{ and } PQ \parallel AC \dots\dots(1)$$

In ΔADC , R and S are the mid-points of CD and DA.

$$\therefore SR = \frac{1}{2}AC \text{ and } SR \parallel AC \dots\dots(2)$$

From (1) and (2), we get

$$PQ = \frac{1}{2}AC = SR \text{ and } PQ \parallel AC \parallel SR$$

$$\Rightarrow PQ = SR \text{ and } PQ \parallel SR$$

We know that, one pair of opposite sides of quadrilateral PQRS is equal and parallel.

Therefore PQRS is a parallelogram.

Now, in ΔERC and ΔEQC ,

$$\angle 1 = \angle 2 \text{ (the diagonal of a rhombus bisects the opposite angles)}$$

$$CR = CQ \text{ (each is equal to } \frac{1}{2} \text{ of a side of rhombus)}$$

$$CE = CE \text{ (common)}$$

$$\Rightarrow \Delta ERC \cong \Delta EQC \text{ (SAS criteria)}$$

So we have $\angle 3 = \angle 4$ (corresponding parts of congruent triangles)

$$\text{But } \angle 3 + \angle 4 = 180^\circ \text{ (linear pair)}$$

$$\Rightarrow \angle 3 = \angle 4 = 90^\circ$$

But $\angle 5 = \angle 3$ (vertically opposite angles)

$$\angle 5 = 90^\circ$$

This gives $PQ \parallel AC \Rightarrow PQ \parallel EF$

$\therefore PQEF$ is a quadrilateral having a pair of opposite sides parallel and one of the angles is 90° .

Therefore $PQEF$ is a rectangle.

$\Rightarrow \angle RQP = 90^\circ$

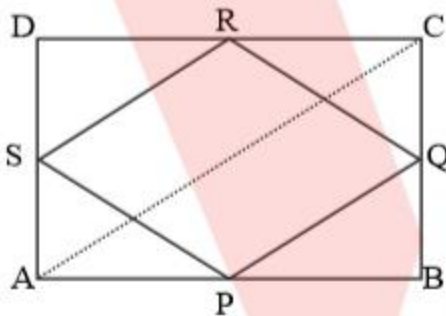
Now that one angle of parallelogram $PQRS$ is 90° .

It is proved that $PQRS$ is a rectangle.

15. $ABCD$ is a rectangle and P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral $PQRS$ is a rhombus.

Solution:

$ABCD$ is a rectangle which has P, Q, R and S as the midpoints of AB, BC, CD and DA, AC is the diagonal.



To prove that $PQRS$ is a rhombus

By joining AC , in $\triangle ABC$, $PQ = \frac{1}{2}AC$ and $PQ \parallel AC$...(1) (midpoint theorem)

And in $\triangle ACD$, $SR = \frac{1}{2}AC$ and $SR \parallel AC$...(2) (midpoint theorem)

From (1) and (2), we get

$PQ = SR$ and $PQ \parallel SR$

Similarly, by joining BD , we'll get

$PS = QR$ and $PS \parallel QR$

\Rightarrow Both pairs of opposite sides of quadrilateral $PQRS$ are equal and parallel.

$\therefore PQRS$ is a parallelogram.

Now, in $\triangle PAS$ and $\triangle PBQ$,

$\angle A = \angle B$ (each angle = 90°)

$AP = BP$ (each side = $\frac{1}{2}AB$)

$AS = BQ$ (each side = $\frac{1}{2}$ of opposite sides of a rectangle)

$\Rightarrow \Delta PAS \cong \Delta PBQ$ (SAS criteria)

\therefore Their corresponding parts are equal.

$\Rightarrow PS = PQ$

$PS = QR$ (proved already)

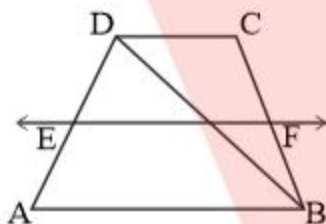
And $PQ = SR$ (proved already)

So, $PQ = QR = RS = SP$

i.e., PQRS is a parallelogram having all sides equal.

This proves that PQRS is a rhombus.

16. ABCD is a trapezium in which $AB \parallel DC$, BD is a diagonal and E is the midpoint of AD. A line is drawn through E parallel to AB intersecting BC at F (see Fig). Show that F is the midpoint of BC.



Solution:

In trapezium ABCD, $AB \parallel DC$. E is the midpoint of AD. EF is drawn parallel to AB.

To prove that F is the midpoint of BC.

Joining BD, In ΔDAB , E is the midpoint of AD

$EG \parallel AB$

So we get that G is the midpoint BD (converse of midpoint theorem)

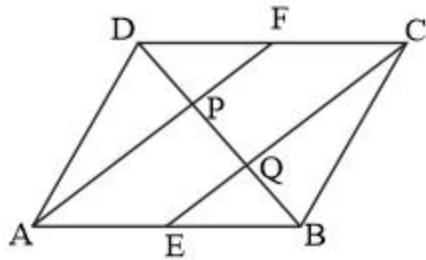
Now in ΔBDC , since G is the midpoint of BD (as proved already)

$GF \parallel DC$ ($\because AB \parallel DC$ and $EF \parallel AB$ and GF is a part of EF)

Therefore, by the converse of the mid-point theorem,

It is proved that F is the midpoint of BC

17. In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively (see Fig) Show that the line segments AF and EC trisect the diagonal BD.



Solution:

ABCD is a parallelogram with E and F as the midpoints of AB and CD.

To prove that the line segments AF and EC trisect the diagonal BD.

By joining the opposite vertices B and D.

We know that the opposite sides of a parallelogram are parallel and equal.

$$\therefore AB \parallel DC \Rightarrow AE \parallel FC \dots (1)$$

And also $AB = DC$

$$\text{i.e., } \frac{1}{2}AB = \frac{1}{2}DC \Rightarrow AE = FC \dots (2)$$

From (1) and (2), we get that AECF is a quadrilateral with a pair of opposite sides parallel and equal.

\therefore we can say AEFC is a parallelogram.

$$\Rightarrow AE \parallel CF$$

Now, in $\triangle DBC$, F is the midpoint of DC (given)

$FP \parallel CQ$ (since $AF \parallel CE$)

$\Rightarrow P$ is the mid-point of DQ (converse of midpoint theorem)

$$\Rightarrow DP = PQ \dots (3)$$

Similarly, in $\triangle BAP$, $BQ = PQ \dots (4)$

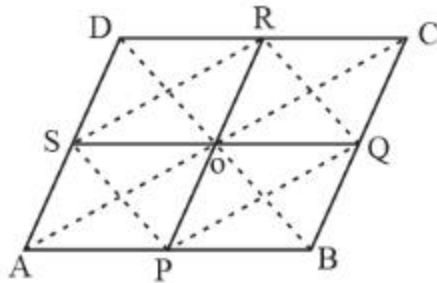
Therefore from (3) and (4), we have $DP = PQ = BQ$

This proves that the line segments AF and EC trisect the diagonal BD.

18. Show that the line segments joining the midpoints of the opposite sides of a quadrilateral bisect each other.

Solution:

ABCD is a quadrilateral with P, Q, R and S as the midpoints of AB, BC, CD and DA respectively,



To prove that the diagonals of PQRS bisect the sides at O.

By joining PQ, QR, RS, SP and also PR and SQ,

In ΔABC , P and Q are the midpoints of its sides AB and BC respectively.

$$\therefore PQ \parallel AC \text{ and } PQ = \frac{1}{2}AC$$

$$\text{Similarly, } RS \parallel AC \text{ and } RS = \frac{1}{2}AC$$

\Rightarrow PQRS is a quadrilateral with $PQ = RS$

and $PQ \parallel RS$

\therefore PQRS is a parallelogram.

Now we know that the diagonals of a parallelogram bisect each other.

i.e., PR and SQ bisect each other.

Therefore, it is proved that the line segments joining the mid-points of opposite sides of a quadrilateral ABCD bisect each other.

19. ABC is a triangle right angled at C. A line through the midpoint M of hypotenuse AB and parallel to BC intersects AC at D. Show that

(i) D is the midpoint of AC

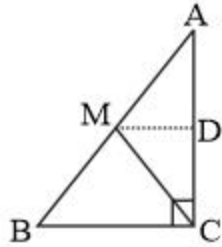
(ii) $MD \perp AC$

(iv) $CM = MA = \frac{1}{2}AB$

Solution:

ABC is a triangle, such that $\angle C = 90^\circ$

M is the mid-point of AB and $MD \parallel BC$.



- (i) To prove that D is the midpoint of AC.

In $\triangle ABC$, M is the midpoint of AB (given)

$MD \parallel BC$ (given)

Therefore, it is proved that D is the midpoint of AC. (by converse of midpoint theorem)

- (ii) To prove that $MD \perp AC$.

$MD \parallel BC$ (given)

and AC is a transversal.

$\angle MDA = \angle BCA$ (corresponding angles)

We know that $\angle BCA = 90^\circ$ (given)

$\Rightarrow \angle MDA = 90^\circ$

Therefore, it is proved $MD \perp AC$.

- (iii) To prove that $CM = MA = \frac{1}{2}AB$

In $\triangle ADM$ and $\triangle CDM$, $\angle ADM = \angle CDM$ (each angle = 90°)

$MD = MD$ (common)

$AD = CD$ (\because it is proved that M is the midpoint of AC)

$\therefore \triangle ADM \cong \triangle CDM$ (SAS criteria)

\therefore Their corresponding parts are equal

$\Rightarrow MA = MC \dots (1)$

\because M is the midpoint AB. (given)

\therefore we get $MA = \frac{1}{2}AB \dots (2)$

Therefore, from (1) and (2)

It is proved that $CM = MA = \frac{1}{2}AB$