CBSE NCERT Solutions for Class 9 Mathematics Chapter 1

Back of Chapter Questions

Exercise: 1.1

1. Is zero a rational number? Can you write it in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$?

Solution:

We know that a number 'r' is said to be a rational number if it can be represented in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

Now, we say that zero is a rational number if it can be represented in the above form. Further, we see that zero can be represented as $\frac{0}{1}, \frac{0}{2}, \frac{0}{3}$, etc.

Therefore, zero is a rational number.

2. Find six rational numbers between 3 and 4.

Solution:

An infinite number of rational numbers are possible between 3 and 4.

To get these numbers the criteria is that the denominator should be the sum of the 2 numbers i.e. $3 + 4 = 7$

Now, 3 and 4 can be represented in fractions by multiplying and dividing them by 7.

$3 = \frac{3 \times 7}{7} = \frac{21}{7}$ and $4 = \frac{4 \times 7}{7} = \frac{28}{7}$

The required rational numbers can be found by changing the numerators from 21 to 28. Therefore, the rational numbers are $\frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}, \frac{26}{7}, \frac{27}{7}$.

3. Find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

Solution:

There are infinite number of rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

We can find them by multiplying and dividing the numerator and denominator by a number. We are doing this so that the gap between the numerators of the two numbers increase and we can easily select the required numbers.

We can choose any number to multiply and divide but ideally, we choose the number that is more than the required number (here, 5).

So, let us choose 7. Now, $\frac{3}{5} = \frac{3\times7}{5\times7} = \frac{21}{35}$ and $\frac{4}{5} = \frac{4\times7}{5\times7} = \frac{28}{35}$.

Therefore, the required numbers are $\frac{22}{35}, \frac{23}{35}, \frac{24}{35}, \frac{25}{35}, \frac{26}{35}$.

4. State whether the following statements are true or false. Give reasons for your answers.

(i) Every natural number is a whole number.

(ii) Every integer is a whole number.

(iii) Every rational number is a whole number.

Solution:

(i) True, because we can say that whole numbers are nothing but natural numbers plus zero. Therefore, every natural number is a whole number but every whole number is not a natural number. As 0 is not a natural number.

(ii) False, because integers include both positive and negative numbers. The whole numbers include only positive numbers and negative numbers are not whole numbers. Therefore, every integer is a not a whole number.

(iii) False, because rational numbers can also be in the form of fractions and these fractional numbers are not whole numbers. For example, $\frac{2}{3}, \frac{3}{5}, \frac{4}{3}$ are not whole numbers but they are rational numbers.

Exercise: 1.2

1. State whether the following statements are true or false. Justify your answers.

(i) Every irrational number is a real number.

(ii) Every point on the number line is of the form $\sqrt{m}$, where m is a natural number.

(iii) Every real number is an irrational number.

Solution:

(i) True, since the real numbers are nothing but a combination of rational and irrational numbers.

(ii) False, because the negative numbers on the number line cannot be expressed in the form $\sqrt{m}$.

(iii) False, because real numbers contain both rational and irrational numbers. Therefore, every real number cannot be irrational.

2. Are the square roots of all positive integers irrational? If not, give an example of the square root of a number that is a rational number.

Solution:

No, the square root of all positive numbers need not be irrational

For example, $\sqrt{4} = 2$ and $\sqrt{9} = 3$.

Here, 2 and 3 are rational.

Therefore, the square roots of all positive integers are not irrational.

3. Show how $\sqrt{5}$ can be represented on the number line.

Solution:

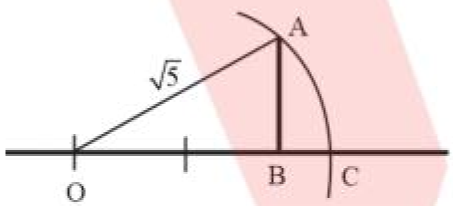
To represent $\sqrt{5}$ on the number line, take $OB = 2$ units and make a perpendicular at B so that $AB = 1$ unit.

By Pythagoras theorem, we get $OA^2 = OB^2 + AB^2$.

So, $OA^2 = 2^2 + 1^2$

$OA^2 = 5$

$OA = \sqrt{5}$



Now, taking O as center and OB as radius, draw an arc intersecting number line at C.

OC is the required distance that represents $\sqrt{5}$.

Exercise: 1.3

1. Write the following in decimal form and say what kind of decimal expansion each has:

(i) $\frac{36}{100}$

(ii) $\frac{1}{11}$

(iii) $4\frac{1}{8}$

(iv) $\frac{3}{13}$

(v) $\frac{2}{11}$

(vi) $\frac{329}{400}$

Solution:

(i) $\frac{36}{100} = 0.36\ldots\ldots$; Terminating.

(ii) $\frac{1}{11} = 0.90909\ldots\ldots$; Non terminating and recurring decimal.

(iii) $4\frac{1}{8} = \frac{33}{8} = 4.125\ldots\ldots$; Terminating.

(iv) $\frac{3}{13} = 0.230769230\ldots\ldots$; Non terminating and recurring decimal.

(v) $\frac{2}{11} = 0.1818181818\ldots\ldots$; Non terminating and recurring decimal.

(vi) $\frac{329}{400} = 0.8225$; Terminating.

2. You know that $\frac{1}{7} = 0.\overline{142857}$. Can you predict what the decimal expansions of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$ are, without actually doing the long division? If so, how?

Solution:

Yes, we can predict the expansions without actually doing the long division. It can be done as:

Given: $\frac{1}{7} = 0.\overline{142857}$

$$\frac{2}{7} = 2 \times \frac{1}{7} = 2 \times 0.142857 = 0.285714.$$

$$\frac{3}{7} = 3 \times \frac{1}{7} = 3 \times 0.142857 = 0.428571.$$

$$\frac{4}{7} = 4 \times \frac{1}{7} = 4 \times 0.142857 = 0.571428.$$

$$\frac{5}{7} = 5 \times \frac{1}{7} = 5 \times 0.142857 = 0.714285.$$

$$\frac{6}{7} = 6 \times \frac{1}{7} = 6 \times 0.142857 = 0.857142.$$

Hint:

Study the remainders while finding the value of $\frac{1}{7}$ carefully.

3. Express the following in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

(i) $0.\overline{6}$

(ii) $0.\overline{47}$

(iii) $\overline{0.001}$

Solution:

(i) $0.\overline{6} = 0.66666\ldots$

Let us consider, $x = 0.66666$ ......(i)

Multiplying 10 on both the sides of equation (i), we get,

$10x = 6.6666$ RHS can also be written as, $6 + 0.6666$

$10x = 6 + x$ [as, $x = 0.6666$]

$9x = 6$

$x = \frac{6}{9}$

$x = \frac{2}{3}$

(ii) $0.\overline{47} = 0.4777$

Let us consider,

$x = 0.4777\ldots\ldots$(i)

Multiplying 10 on both the sides of equation (i),

we get, $10x = 4.777$ …(ii)

Multiplying 10 on both the sides of equation (ii),

we get, $100\,x = 47.777$

RHS can also be written as, $43 + 4.777$

$100x = 43 + 10x$ [as, $10x = 4.777$]

$90x = 43$

$x = \frac{43}{90}$ (iii)

$0.\overline{001} = 0.001001001\ldots$

Let us consider,

$x = 0.001001001$ ......(i)

Multiplying 1000 on both the sides of equation (i),

we get, $1000\,x = 1.001001001$

RHS can also be written as, $1 + 0.0010001001$

$1000x = 1 + x$ [as, $10x = 4.777$]

$999x = 1$

$x = \frac{1}{999}$

4. Express 0.99999 ... ... in the form $\frac{p}{q}$. Are you surprised by your answer? With your teacher and classmates discuss why the answer makes sense.

Solution:

Let us consider,

$x = 0.99999 \ldots\ldots (i)$

Multiplying 10 on both the sides of equation (i),

we get, $10x = 9.9999$

RHS can also be written as, $9 + 0.9999$

$10x = 9 + x$ [as, $x = 0.9999$]

$9x = 9$

$x = \frac{9}{9}$

$x = \frac{1}{1} = 1.$

The answer does surprise us. But upon inspection we see that 0.9999 .... is very close to 1. Therefore, 0.99999 can be approximated to 1 and hence they are equal.

5. What can the maximum number of digits be in the repeating block of digits in the decimal expansion of $\frac{1}{17}$? Perform the division to check your answer.

Solution:

By performing the actual division operation, we see that, $\frac{1}{17} =$ 0.0588235294117647.

Therefore, the maximum number digits that can be in the repeating block of digits in the above expansion are 16.

6. Look at several examples of rational numbers in the form $\frac{p}{q}$ $(q \neq 0)$, where p and q are integers with no common factors other than 1 and having terminated decimal representations (expansions). Can you guess what property q must satisfy?

Solution:

Let us look at some examples:

$\frac{5}{4} = 1.25$

$\frac{21}{8} = 2.625$

$\frac{34}{5} = 6.8$

From the above examples we may generally conclude that, terminating decimal expansion will occur when denominator 'q' of rational number $\frac{p}{q}$ are in the form,

$2^a \times 5^b$, where 'a' and 'b' are integers.

7. Write three numbers whose decimal expansions are non-terminating non-recurring.

Solution:

There are infinite numbers of non-terminating and non-recurring decimals. Further, we observe that all irrational numbers are non-terminating. Some examples are,

(i) $0.645238456364\ldots$

(ii) $\sqrt{3} = 1.73205087\ldots$

(iii) $0.7235432436\ldots$

8. Find three different irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$.

Solution:

By performing long division, the numbers can be represented as

$\frac{5}{7} = 0.714285\ldots$ and $\frac{9}{11} = 0.81818181\ldots$

Now, any three numbers between these numbers will satisfy the given question. Therefore, the required numbers can be:

(i) $0.72534345029\ldots$

(ii) $0.7523028734\ldots$

(iii) $0.77623402347\ldots$

9. Classify the following numbers as rational or irrational:

(i) $\sqrt{23}$

(ii) $\sqrt{225}$

(iii) 0.3796

(iv) 7.478478

(v) 1.101001000100001

Solution:

(i) $\sqrt{23} = 4.79583152331\ldots$

In the above expansion, the number is non-terminating and non-recurring, therefore, it is an irrational number.

(ii) $\sqrt{225} = 15 = \frac{15}{1}$

As the above number can be represented in $\frac{p}{q}$ form, it is a rational number.

(iii) 0.3796

As the above number is terminating, it is a rational number.

(iv) $7.478478 \ldots = 7.478$

From the decimals it is a recurring number, but it is non-terminating. Therefore, it is a rational number.

(v) 1.101001000100001

We see that the decimal expansion of this number is non-repeating and non-recurring. Therefore, it is an irrational number.

Exercise: 1.4

1. Visualize 3.765 on the number line, using successive magnification.

Solution:

To visualize 3.765 on the number line, we must follow these steps:

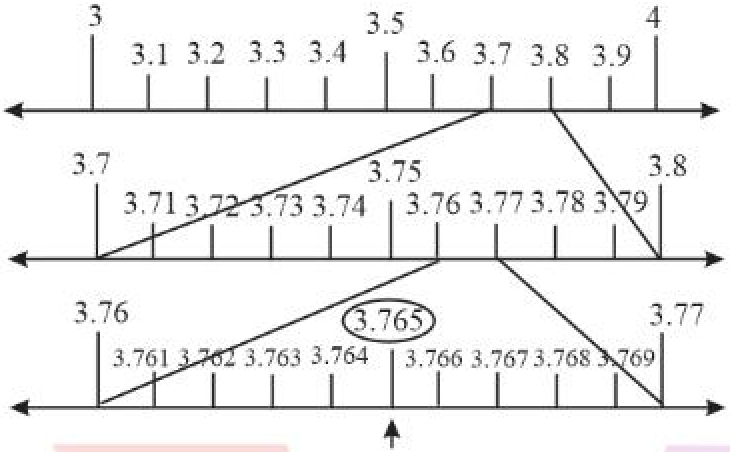
(i) First, we see that 3.765 lies between 3 and 4. Now, divide this portion into 10 equal parts.

(ii) Next, we locate 3.76. We observe that this lies between 3.7 and 3.8.

(iii) To get a more accurate visualization, we further divide this portion into 10 parts and locate it.

(iv) Further, we visualize 3.765 and observe that it lies between 3.76 and 3.77.

(v) To locate this, we divide the portion between 3.76 and 3.77 into 10 equal parts and hence locate 3.765



2. Visualize $4.\overline{26}$ on the number line, up to 4 decimal places.

Solution:

To visualize $4.\overline{26}$ on the number line, we must follow these steps:

(i) First, we see that 4.2 lies between 4 and 5.

(ii) Now, divide this portion into 10 equal parts.

(iii) Next, we locate 4.26. We observe that this lies between 4.2 and 4.3.

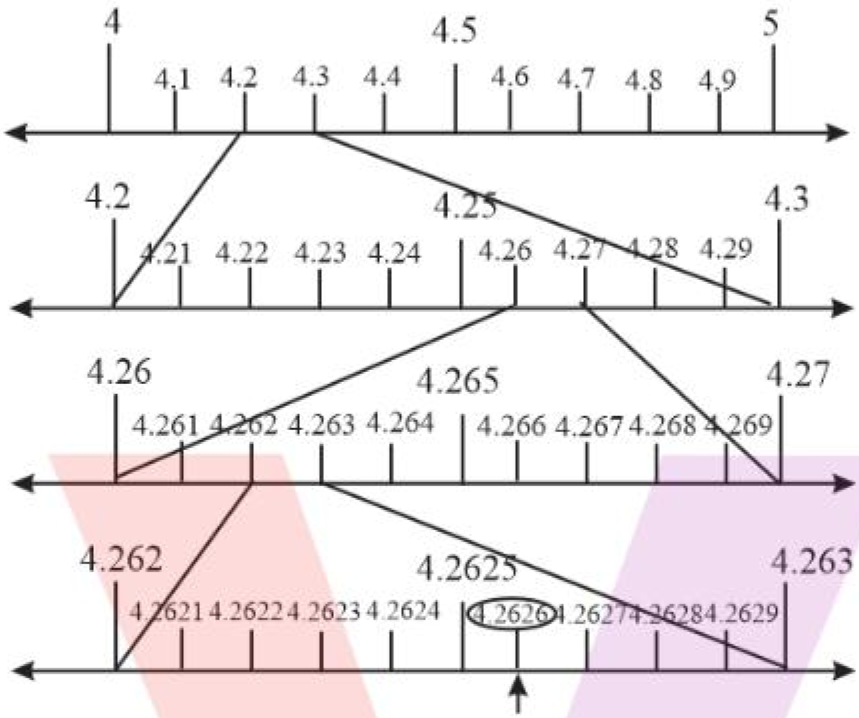
(iv) To get a more accurate visualization, we further divide this portion into 10 parts and locate it.

(v) Further, we visualize 4.262 and observe that it lies between 4.26 and 4.27.

(vi) To locate this, we divide the portion between 4.26 and 4.27 into 10 equal parts and locate 4.262

(vii) We observe that 4.2626 lies between 4.262 and 4.263.

(viii) To find this, we divide the portion further into 10 parts and hence locate 4.2626.



Exercise: 1.5

1. Classify the following numbers as rational or irrational:

(i) $2-\sqrt{5}$

(ii) $\left(3+\sqrt{23}\right)-\sqrt{23}$

(iii) $\frac{2\sqrt{7}}{7\sqrt{7}}$

(iv) $\frac{1}{\sqrt{2}}$

(v) 2π

Solution:

(i) $2-\sqrt{5}=2-2.2360679\ldots=-0.2360679\ldots$

We see that the decimal expansion of this expression is non-terminating and non-recurring. Therefore, it is an irrational number.

(ii) $\left(3+\sqrt{23}\right)-\sqrt{23}=3=\frac{3}{1}$

We see that the number can be represented in $\frac{p}{q}$form. Therefore, it is a rational number.

(iii) $\frac{2\sqrt{7}}{7\sqrt{7}}=\frac{2}{7}$

As it can be represented in $\frac{p}{q}$ form, therefore, it is a rational number.

(iv) $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} = 0.7071067811 \ldots$

We see that the decimal expansion of the above expression is non-terminating and non-recurring. Therefore, it is an irrational number.

(v) $2\pi = 2(3.1415 \ \ldots) = 6.2830 \ldots$

The decimal expansion of this expression is non-terminating and non-recurring. Therefore, it is an irrational number.

2. Simplify each of the following expressions:

(i) $(3+\sqrt{3})(2+\sqrt{2})$

(ii) $(3+\sqrt{3})(3-\sqrt{3})$

(iii) $(\sqrt{5}+\sqrt{2})^2$

(iv) $(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})$

Solution:

(i) $(3+\sqrt{3})(2+\sqrt{2}) = 3(2+\sqrt{2}) + \sqrt{3}(2+\sqrt{2})$

$= 6 + 3\sqrt{2} + 2\sqrt{3} + \sqrt{6}$

(ii) $(3+\sqrt{3})(3-\sqrt{3}) = (3)^2 - (\sqrt{3})^2$

$= 9 - 3 = 6$

(iii) $(\sqrt{5}+\sqrt{2})^2 = (5)^2 + (\sqrt{2})^2 + 2(\sqrt{5})(\sqrt{2})$

$= 5 + 2 + 2\sqrt{10} = 7 + 2\sqrt{10}$

(iv) $(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2}) = (\sqrt{5})^2 - (\sqrt{2})^2$

$= 5 - 2 = 3$

3. Recall, π is defined as the ratio of the circumference (say c) of a circle to its diameter (say d). That is, $\pi = \frac{c}{d}$. This seems to contradict the fact that π is irrational. How will you resolve this contradiction?

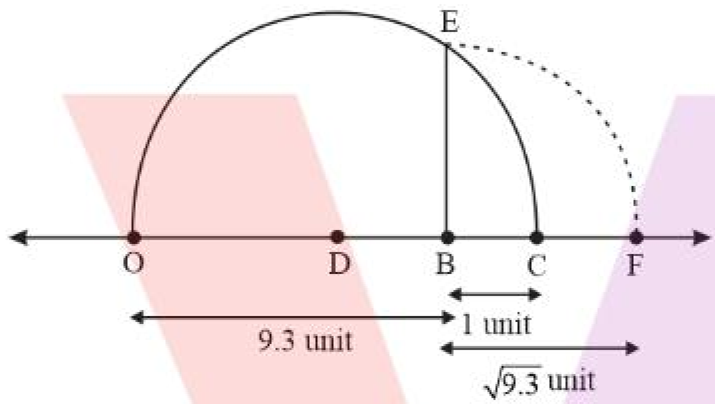
Solution:

There is no contradiction at all. When we measure a length with scale or any other instrument, we only obtain an approximate rational value which is rational. We never obtain an exact value. For this reason, we may not realize, that either c or d is irrational. Therefore, the fraction $\frac{c}{d}$ is irrational. Hence, π is irrational.

4. Represent $\sqrt{9.3}$ on the number line.

Solution:

To represent $\sqrt{9.3}$ on the number line, we first need to mark a line segment OB = 9.3 on number line. Now, take BC of 1 unit. Find the mid-point D of OC and draw a semi-circle on OC while taking D as its centre. Draw a perpendicular to line OC passing through point B. Let it intersect the semi-circle at E. Taking B as centre and BE as radius, draw an arc intersecting number line at F. BF is $\sqrt{9.3}$.



5. Rationalise the denominators of the following:

(i) $\frac{1}{\sqrt{7}}$

(ii) $\frac{1}{\sqrt{7}-\sqrt{6}}$

(iii) $\frac{1}{\sqrt{5}+\sqrt{2}}$

(iv) $\frac{1}{\sqrt{7}-2}$

Solution:

(i) $\frac{1}{\sqrt{7}} = \frac{1\times\sqrt{7}}{1\times\sqrt{7}} = \frac{\sqrt{7}}{7}$

(ii) $\frac{1}{\sqrt{7}-\sqrt{6}} = \frac{1(\sqrt{7}+\sqrt{6})}{(\sqrt{7}-\sqrt{6})(\sqrt{7}+\sqrt{6})}$

$= \frac{\sqrt{7}+\sqrt{6}}{(\sqrt{7})^2-(\sqrt{6})^2}$

$= \frac{\sqrt{7}+\sqrt{6}}{7-6} = \frac{\sqrt{7}+\sqrt{6}}{1} = \sqrt{7}+\sqrt{6}$

(iii) $\frac{1}{\sqrt{5}+\sqrt{2}} = \frac{1(\sqrt{5}-\sqrt{2})}{(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})}$

$= \frac{\sqrt{5}-\sqrt{2}}{(\sqrt{5})^2-(\sqrt{2})^2} = \frac{\sqrt{5}-\sqrt{2}}{5-2}$

$= \frac{\sqrt{5}-\sqrt{2}}{3}$

(iv) $\frac{1}{\sqrt{7}-2} = \frac{1(\sqrt{7}+2)}{(\sqrt{7}-2)(\sqrt{7}+2)}$

$= \frac{\sqrt{7}+2}{(\sqrt{7})^2-(2)^2}$

$= \frac{\sqrt{7}+2}{7-4}$

$= \frac{\sqrt{7}+2}{3}$

Exercise: 1.6

1. Find:

(i) $64^{\frac{1}{2}}$

(ii) $32^{\frac{1}{5}}$

(iii) $125^{\frac{1}{3}}$

Solution:

(i) $64^{\frac{1}{2}} = (2^6)^{\frac{1}{2}}$

$= 2^{6\times\frac{1}{2}}$ $[(a^m)^n = a^{mn}]$

$= 2^3 = 8$

(ii) $32^{\frac{1}{5}} = (2^5)^{\frac{1}{5}}$

$= (2)^{5\times\frac{1}{5}}$ $[(a^m)^n = a^{mn}]$

$= 2^1 = 2$

(iii) $(125)^{\frac{1}{3}} = (5^3)^{\frac{1}{3}}$

$= 5^{3\times\frac{1}{3}}$ $[(a^m)^n = a^{mn}]$

$= 5^1 = 5$

2. Find:

(i) $9^{\frac{3}{2}}$

(ii) $32^{\frac{2}{5}}$

(iii) $16^{\frac{3}{4}}$

(iv) $125^{\frac{-1}{3}}$

Solution:

(i) $9^{\frac{3}{2}} = (3^2)^{\frac{3}{2}}$

$= 3^{2\times\frac{3}{2}}$ $[(a^m)^n = a^{mn}]$

$= 3^3 = 27$

(ii) $(32)^{\frac{2}{5}} = (2^5)^{\frac{2}{5}}$

$= 2^{5\times\frac{2}{5}}$ $[(a^m)^n = a^{mn}]$

$= 2^2 = 4$

(iii) $(16)^{\frac{3}{4}} = (2^4)^{\frac{3}{4}}$

$= 2^{4\times\frac{3}{4}}$ $[(a^m)^n = a^{mn}]$

$= 2^3 = 8$

(iv) $(125)^{\frac{-1}{3}} = \frac{1}{(125)^{\frac{1}{3}}}$ $\left[a^{-m} = \frac{1}{a^m}\right]$

$= \frac{1}{(5^3)^{\frac{1}{3}}}$

$= \frac{1}{5^{3\times\frac{1}{3}}}$ $[(a^m)^n = a^{mn}]$

$= \frac{1}{5}$

3. Simplify:

(i) $2^{\frac{2}{3}}.2^{\frac{1}{5}}$

(ii) $\left(\frac{1}{3^3}\right)^7$

(iii) $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$

(iv) $7^{\frac{1}{2}}.8^{\frac{1}{2}}$

Solution:

(i) $2^{\frac{3}{2}}.2^{\frac{1}{5}} = 2^{\frac{2}{3}+\frac{1}{5}}$ $[a^m.a^n = a^{m+n}]$

$= 2^{\frac{10+3}{15}} = 2^{\frac{13}{15}}$

(ii) $\left(\frac{1}{3^3}\right)^7 = \frac{1}{3^{3\times7}}$ $[(a^m)^n = a^{mn}]$

$= \frac{1}{3^{21}}$

$= 3^{-21}$ $\left[\frac{1}{a^m} = a^{-m}\right]$

(iii) $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}} = 11^{\frac{1}{2}-\frac{1}{4}}$ $\left[\frac{a^m}{a^n} = a^{m-n}\right]$

$= 11^{\frac{2-1}{4}} = 11^{\frac{1}{4}}$

(iv) $7^{\frac{1}{2}}.8^{\frac{1}{2}} = (7 \times 8)^{\frac{1}{2}}$ $[a^m.b^m = (ab)^m]$

$= (56)^{\frac{1}{2}}$