NCERT solutions for class 11 maths chapter 8 Binomial Theorem

Question:1 Expand the expression. $(1-2x)^5$

Answer:

Given,

The Expression:

$(1-2x)^5$

the expansion of this Expression is,

$(1-2x)^5 =$

${}^5C_0(1)^5 - {}^5C_1(1)^4(2x) + {}^5C_2(1)^3(2x)^2 - {}^5C_3(1)^2(2x)^3 + {}^5C_4(1)^1(2x)^4 - {}^5C_5(2x)^5$

$1-5(2x)+10(4x^2)-10(8x^3)+5(16x^4)-(32x^5)$

$1-10x+40x^2-80x^3+80x^4-32x^5$

Question:2 Expand the expression. $\left(\frac{2}{x}-\frac{x}{2}\right)^5$

Answer:

Given,

The Expression:

$\left(\frac{2}{x}-\frac{x}{2}\right)^5$

the expansion of this Expression is,

$$\left(\frac{2}{x}-\frac{x}{2}\right)^5 \Rightarrow$$

$$+5\left(\frac{2}{x}\right)\left(\frac{x^4}{16}\right)-\frac{x^5}{32}$$

$$\Rightarrow \frac{32}{x^5}-\frac{40}{x^3}+\frac{20}{x}-5x+\frac{5x^2}{8}-\frac{x^3}{32}$$

Question:3 Expand the expression. $(2x-3)^6$

Answer:

Given,

The Expression:

$(2x-3)^6$

the expansion of this Expression is,

$(2x-3)^6 =$

$\Rightarrow {}^6C_0(2x)^6 -{}^6C_1(2x)^5(3) +{}^6C_2(2x)^4(3)^2 -{}^6C_3(2x)^3(3)^3+$

${}^6C_4(2x)^2(3)^4 -{}^6C_5(2x)(3)^5 +{}^6C_6(3)^6$

$\Rightarrow 64x^6-6(32x^5)(3)+15(16x^4)(9)-20(8x^3)(27)+15(4x^2)(81)-6(2x)(243)$

$+729$

$\Rightarrow 64x^6-576x^5+2160x^4-4320x^3+4860x^2-2916x+729$

Question:4 Expand the expression. $\left(\frac{x}{3}+\frac{1}{x}\right)^5$

Answer:

Given,

The Expression:

$$\left(\frac{x}{3}+\frac{1}{x}\right)^5$$

the expansion of this Expression is,

$$\left(\frac{x}{3}+\frac{1}{x}\right)^5 \Rightarrow$$

$$+5\left(\frac{x}{3}\right)\left(\frac{1}{x^4}\right)+\frac{1}{x^5}$$

$$\Rightarrow \frac{x^5}{243}+\frac{5x^3}{81}+\frac{10x}{27}+\frac{10}{9x}+\frac{5}{3x^2}+\frac{1}{x^5}$$

Question:5 Expand the expression. $\left(x+\frac{1}{x}\right)^6$

Answer:

Given,

The Expression:

$$\left(x+\frac{1}{x}\right)^{6}$$

the expansion of this Expression is,

$$\left(x+\frac{1}{x}\right)^{6}$$

$$+15(x^2)\left(\frac{1}{x^4}\right)+6(x)\left(\frac{1}{x^5}\right)+\frac{1}{x^6}$$

$$\Rightarrow x^6+6x^4+15x^2+20+\frac{15}{x^2}+\frac{6}{x^4}+\frac{1}{x^6}$$

Question:6 Using binomial theorem, evaluate the following: $(96)^3$

Answer:

As 96 can be written as (100-4);

$$\Rightarrow (96)^3$$
$$=(100-4)^3$$
$$={}^3C_0(100)^3-{}^3C_1(100)^2(4)+{}^3C_2(100)(4)^2-{}^3C_3(4)^3$$

$$=(100)^3-3(100)^2(4)+3(100)(4)^2-(4)^3$$

$$=1000000-120000+4800-64$$

$$=884736$$

Question:7 Using binomial theorem, evaluate the following: $(102)^5$

Answer:

As we can write 102 in the form 100+2

$\Rightarrow (102)^5$

$= (100+2)^5$

$={}^5C_0(100)^5 + {}^5C_1(100)^4(2) + {}^5C_2(100)^3(2)^2$

$+{}^5C_3(100)^2(2)^3 + {}^5C_4(100)^1(2)^4 + {}^5C_5(2)^5$

$= 10000000000 + 1000000000 + 40000000 + 800000 + 8000 + 32$

$= 11040808032$

Question:8 Using binomial theorem, evaluate the following:

$(101)^4$

Answer:

As we can write 101 in the form 100+1

$\Rightarrow (101)^4$

$= (100+1)^4$

$={}^4C_0(100)^4 + {}^4C_1(100)^3(1) + {}^4C_2(100)^2(1)^2 + {}^4C_3(100)^1(1)^3 + {}^4C_4(1)^4$

$= 100000000 + 4000000 + 60000 + 400 + 1$

$= 104060401$

Question:9 Using binomial theorem, evaluate the following: $(99)^5$

Answer:

As we can write 99 in the form 100-1

$\Rightarrow (99)^5$

$= (100-1)^5$

$= {}^5C_0(100)^5 - {}^5C_1(100)^4(1) + {}^5C_2(100)^3(1)^2$
$- {}^5C_3(100)^2(1)^3 + {}^5C_4(100)^1(1)^4 - {}^5C_5(1)^5$

$= 10000000000 - 500000000 + 10000000 - 100000 + 500 - 1$

$= 9509900499$

Question:10 Using Binomial Theorem, indicate which number is larger $(1.1)^{10000}$ or 1000.

Answer:

AS we can write 1.1 as 1 + 0.1,

$(1.1)^{10000} = (1+0.1)^{10000}$

$= {}^{10000}C_0 + {}^{10000}C_1(1.1) + Other\ positive\ terms$

$= 1 + 10000 \times 1.1 + \ Other\ positive\ term$

> 1000

Hence,

$(1.1)^{10000} > 1000$

Question:11 Find $(a+b)^4-(a-b)^4$. Hence, evaluate $(\sqrt{3}+\sqrt{2})^4-(\sqrt{3}-\sqrt{2})^4$.

Answer:

Using Binomial Theorem, the expressions $(a+b)^4$ and $(a-b)^4$ can be expressed as

$$(a+b)^4 =^4 C_0a^4 +^4 C_1a^3b +^4 C_2a^2b^2 +^4 C_3ab^3 +^4 C_4b^4$$

$$(a-b)^4 =^4 C_0a^4 -^4 C_1a^3b +^4 C_2a^2b^2 -^4 C_3ab^3 +^4 C_4b^4$$

From Here,

$$(a+b)^4-(a-b)^4 =^4 C_0a^4 +^4 C_1a^3b +^4 C_2a^2b^2 +^4 C_3ab^3 +^4 C_4b^4$$
$$-^4C_0a^4 +^4 C_1a^3b -^4 C_2a^2b^2 +^4 C_3ab^3 -^4 C_4b^4$$

$$(a+b)^4-(a-b)^4 = 2\times(^4C_1a^3b +^4 C_3ab^3)$$

$$(a+b)^4-(a-b)^4 = 8ab(a^2+b^2)$$

Now, Using this, we get

$$(\sqrt{3}+\sqrt{2})^4-(\sqrt{3}-\sqrt{2})^4 = 8(\sqrt{3})(\sqrt{2})(3+2) = 8\times\sqrt{6}\times 5 = 40\sqrt{6}$$

Question:12 Find $(x+1)^6+(x-1)^6$. Hence or otherwise evaluate $(\sqrt{2}+1)^6+(\sqrt{2}-1)^6$.

Answer:

Using Binomial Theorem, the expressions $(x+1)^4$ and $(x-1)^4$ can be expressed as ,

$$(x+1)^6 =^6 C_0x^6 +^6 C_1x^51 +^6 C_2x^41^2 +^4 C_3x^31^3 +^6 C_4x^21^4 +^6 C_5x1^5 +^6 C_61^6$$

$$(x-1)^6 =^6 C_0x^6 -^6 C_1x^51 +^6 C_2x^41^2 -^4 C_3x^31^3 +^6 C_4x^21^4 -^6 C_5x1^5 +^6 C_61^6$$

From Here,

$(x+1)^6-(x-1)^6={}^6C_0x^6+{}^6C_1x^51+{}^6C_2x^41^2+{}^4C_3x^31^3+$
${}^6C_4x^21^4+{}^6C_5x1^5+{}^6C_61^6$
$+{}^6C_0x^6-{}^6C_1x^51+{}^6C_2x^41^2-{}^4C_3x^31^3+{}^6C_4x^21^4-{}^6C_5x1^5+{}^6C_61^6$

$(x+1)^6+(x-1)^6=2({}^6C_0x^6+{}^6C_2x^41^2+{}^6C_4x^21^4+{}^6C_61^6)$

$(x+1)^6+(x-1)^6=2(x^6+15x^4+15x^2+1)$

Now, Using this, we get

$(\sqrt{2}+1)^6+(\sqrt{2}-1)^6=2((\sqrt{2})^6+15(\sqrt{2})^4+15(\sqrt{2})^2+1)$

$(\sqrt{2}+1)^6+(\sqrt{2}-1)^6=2(8+60+30+1)=2(99)=198$

Question:13 Show that $9^{n+1}-8n-9$ is divisible by 64, whenever *n* is a positive integer.

Answer:

If we want to prove that $9^{n+1}-8n-9$ is divisible by 64, then we have to prove that $9^{n+1}-8n-9=64k$

As we know, from binomial theorem,

$(1+x)^m={}^mC_0+{}^mC_1x+{}^mC_2x^2+{}^mC_3x^3+\ldots{}^mC_mx^m$

Here putting x = 8 and replacing m by n+1, we get,

$9^{n+1}={}^{n+1}C_0+{}^{n+1}C_18+{}^{n+1}C_28^2+\ldots\ldots+{}^{n+1}C_{n+1}8^{n+1}$

$$9^{n+1} = 1 + 8(n+1) + 8^2({}^{n+1}C_2 + {}^{n+1}C_3 8 + {}^{n+1}C_4 8^2 + ... + {}^{n+1}C_{n+1} 8^{n-1})$$

$$9^{n+1} = 1 + 8n + 8 + 64(k)$$

Now, Using This,

$$9^{n+1} - 8n - 9 = 9 + 8n + 64k - 9 - 8n = 64k$$

Hence

$9^{n+1} - 8n - 9$ is divisible by 64.

Question:14 Prove that $\sum_{r=0}^{n} 3^r \, {}^nC_r = 4^n$

Answer:

As we know from Binomial Theorem,

$$\sum_{r=0}^{n} a^r \, {}^nC_r = (1+a)^n$$

Here putting a = 3, we get,

$$\sum_{r=0}^{n} 3^r \, {}^nC_r = (1+3)^n$$

$$\sum_{r=0}^{n} 3^r \, {}^nC_r = 4^n$$

Hence Proved.

NCERT solutions for class 11 maths chapter 8 binomial theorem-Exercise: 8.2

Question:1 Find the coefficient of

x^5 in $(x+3)^8$

Answer:

As we know that the $(r+1)^{th}$ term T_{r+1} in the binomial expansion of $(a+b)^n$ is given by

$T_{r+1} = ^n C_r a^{n-r} b^r$

Now let's assume x^5 happens in the $(r+1)^{th}$ term of the binomial expansion of $(x+3)^8$

So,

$T_{r+1} = ^8 C_r x^{8-r} 3^r$

On comparing the indices of x we get,

$r = 3$

Hence the coefficient of the x^5 in $(x+3)^8$ is

$$^8C_3 \times 3^3 = \frac{8!}{5!3!} \times 9 = \frac{8 \times 7 \times 6}{3 \times 2} \times 9 = 1512$$

Question:2 Find the coefficient of $a^5 b^7$ in $(a-2b)^{12}$

Answer:

As we know that the $(r+1)^{th}$ term T_{r+1} in the binomial expansion of $(a+b)^n$ is given by

$$T_{r+1} = ^n C_r a^{n-r} b^r$$

Now let's assume a^5b^7 happens in the $(r+1)^{th}$ term of the binomial expansion of $(a-2b)^{12}$

So,

$$T_{r+1} = ^{12} C_r x^{12-r}(-2b)^r$$

On comparing the indices of x we get,

$$r = 7$$

Hence the coefficient of the a^5b^7 in $(a-2b)^{12}$ is

Question:3 Write the general term in the expansion of

$$(x^2-y)^6$$

Answer:

As we know that the general $(r+1)^{th}$ term T_{r+1} in the binomial expansion of $(a+b)^n$ is given by

$$T_{r+1} = ^n C_r a^{n-r} b^r$$

So the general term of the expansion of $(x^2-y)^6$:

$T_{r+1} = {}^6C_r (x^2)^{6-r}(-y)^r = (-1)^r \times {}^6C_r x^{12-2r} y^r$.

Question:4 Write the general term in the expansion of

$(x^2 - xy)^{12},\ x \neq 0$

Answer:

As we know that the general $(r+1)^{th}$ term T_{r+1} in the binomial expansion of $(a+b)^n$ is given by

$T_{r+1} = {}^nC_r a^{n-r} b^r$

So the general term of the expansion of $(x^2 - xy)^{12}$, is

.

Question:5 Find the 4 th term in the expansion of $(x-2y)^{12}$.

Answer:

As we know that the general $(r+1)^{th}$ term T_{r+1} in the binomial expansion of $(a+b)^n$ is given by

$T_{r+1} = {}^nC_r a^{n-r} b^r$

So the 4^{th} term of the expansion of $(x-2y)^{12}$ is

$$
\begin{aligned}
&= -8 \times \frac{12 \times 11 \times 10}{3 \times 2} \times x^9 y^3 \\
&= -8 \times 220 \times x^9 y^3 \\
&= -1760 x^9 y^3
\end{aligned}
$$

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Question:6 Find the 13 th term in the expansion of $\left(9x-\frac{1}{3\sqrt{x}}\right)^{18}$, $x \neq 0$

Answer:

As we know that the general $(r+1)^{th}$ term T_{r+1} in the binomial expansion of $(a+b)^n$ is given by

$T_{r+1} = ^nC_r a^{n-r} b^r$

So the 13^{th} term of the expansion of $\left(9x-\frac{1}{3\sqrt{x}}\right)^{18}$ is

$$= \frac{18 \times 17 \times 16 \times 15 \times 14 \times 13}{6 \times 5 \times 4 \times 3 \times 2} \times 9^6 \left(\frac{1}{3^{12}}\right)$$

$= 18564$

Question:7 Find the middle terms in the expansion of $\left(3-\frac{x^3}{6}\right)^7$

Answer:

As we know that the middle terms in the expansion of $(a+b)^n$ when n is odd are,

$$\left(\frac{n+1}{2}\right)^{th} term\ and\ \left(\frac{n+1}{2}+1\right)^{th} term$$

Hence the middle term of the expansion $\left(3-\frac{x^3}{6}\right)^7$ are

$$\left(\frac{7+1}{2}\right)^{th} term\ and\ \left(\frac{7+1}{2}+1\right)^{th} term$$

Which are $4^{th}\ term\ and\ \ 5^{th}\ term$

Now,

As we know that the general $(r+1)^{th}$ term T_{r+1} in the binomial expansion of $(a+b)^n$ is given by

$T_{r+1} = ^n C_r a^{n-r} b^r$

So the 4^{th} term of the expansion of $\left(3-\frac{x^3}{6}\right)^7$ is

$= -\frac{105}{8}x^9$

And the 5^{th} Term of the expansion of $\left(3-\frac{x^3}{6}\right)^7$ is,

$= \frac{35}{48}x^{12}$

Hence the middle terms of the expansion of given expression are

$-\frac{105}{8}x^9\ and\ \frac{35}{48}x^{12}.$

Question:8 Find the middle terms in the expansion of $\left(\frac{x}{3}+9y\right)^{10}$

Answer:

As we know that the middle term in the expansion of $(a+b)^n$ when n is even is,

$\left(\frac{n}{2}+1\right)^{th}$ $term$,

Hence the middle term of the expansion $\left(\frac{x}{3}+9y\right)^{10}$ is,

$$\left(\frac{10}{2}+1\right)^{th} \ term$$

Which is $6^{th}\ term$

Now,

As we know that the general $(r+1)^{th}$ term T_{r+1} in the binomial expansion of $(a+b)^{n}$ is given by

$$T_{r+1} = {}^{n}C_{r}a^{n-r}b^{r}$$

So the 6^{th} term of the expansion of $\left(\frac{x}{3}+9y\right)^{10}$ is

$$\Rightarrow T_{6} = T_{5+1}$$
$$= {}^{10}C_{5}\left(\frac{x}{3}\right)^{10-5}(9y)^{5}$$

$$= \left(\frac{1}{3}\right)^{5} \times 9^{5} \times {}^{10}C_{5} \times x^{5}y^{5}$$

$$= \left(\frac{1}{3}\right)^{5} \times 9^{5} \times \left(\frac{10!}{5!5!}\right) \times x^{5}y^{5}$$

$$= 61236x^{5}y^{5}$$

Hence the middle term of the expansion of $\left(\frac{x}{3}+9y\right)^{10}$ is **nbsp;** $61236x^{5}y^{5}$.

Question:9 In the expansion of $(1+a)^{m+n}$, prove that coefficients of a^m and a^n are equal

Answer:

As we know that the general $(r+1)^{th}$ term T_{r+1} in the binomial expansion of $(a+b)^n$ is given by

$$T_{r+1} = ^n C_r a^{n-r} b^r$$

So, the general $(r+1)^{th}$ term T_{r+1} in the binomial expansion of $(1+a)^{m+n}$ is given by

$$T_{r+1} = ^{m+n} C_r 1^{m+n-r} a^r = ^{m+n} C_r a^r$$

Now, as we can see a^m will come when $r = m$ and a^n will come when $r = n$

So,

Coefficient of a^m :

$$K_{a^m} = ^{m+n} C_m = \frac{(m+n)!}{m!n!}$$

CoeficientCoefficient of a^n :

$$K_{a^n} = ^{m+n} C_n = \frac{(m+n)!}{m!n!}$$

As we can see $K_{a^m} = K_{a^n}$.

Hence it is proved that the coefficients of a^m and a^n are equal.

Question:10 The coefficients of the $(r-1)$ th, *r* th and $(r+1)$ th terms in the expansion of $(x+1)^n$ are in the ratio 1 : 3 : 5. Find *n* and *r* .

Answer:

As we know that the general $(r+1)^{th}$ term T_{r+1} in the binomial expansion of $(a+b)^n$ is given by

$$T_{r+1} = {}^nC_r a^{n-r} b^r$$

So,

$(r+1)^{th}$ Term in the expansion of $(x+1)^n$:

$$T_{r+1} = {}^nC_r x^{n-r} 1^r = {}^nC_r x^{n-r}$$

r^{th} Term in the expansion of $(x+1)^n$:

$$T_r = {}^nC_{r-1} x^{n-r+1} 1^{r-1} = {}^nC_{r-1} x^{n-r+1}$$

$(r-1)^{th}$ Term in the expansion of $(x+1)^n$:

$$T_{r-1} = {}^nC_{r-2} x^{n-r+2} 1^{r-2} = {}^nC_{r-2} x^{n-r+2}$$

Now, As given in the question,

$$T_{r-1} : T_r : T_{r+1} = 1 : 3 : 5$$

$${}^nC_{r-2} : {}^nC_{r-1} : {}^nC_r = 1 : 3 : 5$$

$$\frac{n!}{(r-2)!(n-r+2)!} : \frac{n!}{(r-1)!(n-r+1)!} : \frac{n!}{r!(n-r)!} = 1 : 3 : 5$$

From here, we get ,

$$\frac{r-1}{n-r+2} = \frac{1}{3} \ and \ \frac{r}{n-r+1} = \frac{3}{5}$$

Which can be written as

$n - 4r + 5 = 0 \ and \ 3n - 8r + 3 = 0$

From these equations we get,

$n = 7 \ and \ r = 3$

Question:11 Prove that the coefficient of x^n in the expansion of $(1 + x)^{2n}$ is twice the coefficient of x^n in the expansion of $(1 + x)^{2n-1}$.

Answer:

As we know that the general $(r + 1)^{th}$ term T_{r+1} in the binomial expansion of $(a + b)^n$ is given by

$T_{r+1} =^n C_r a^{n-r} b^r$

So, general $(r + 1)^{th}$ term T_{r+1} in the binomial expansion of $(1 + x)^{2n}$ is,

$T_{r+1} =^{2n} C_r 1^{2n-r} x^r$

x^n will come when $r = n$,

So, Coefficient of x^n in the binomial expansion of $(1 + x)^{2n}$ is,

$K_{1x^n} =^{2n} C_n$

Now,

the general $(r + 1)^{th}$ term T_{r+1} in the binomial expansion of $(1 + x)^{2n-1}$ is,

$T_{r+1} =^{2n-1} C_r 1^{2n-1-r} x^r$

Here also x^n will come when $r = n$,

So, Coefficient of x^n in the binomial expansion of $(1+x)^{2n-1}$ is,

$$K_{2x^n} = {}^{2n-1}C_n$$

Now, As we can see

$$^{2n-1}C_n = \frac{1}{2} \times {}^{2n}C_n$$

$$2 \times {}^{2n-1}C_n = {}^{2n}C_n$$

$$2 \times K_{2x^n} = K_{1x^n}$$

Hence, the coefficient of x^n in the expansion of $(1+x)^{2n}$ is twice the coefficient of x^n in the expansion of $(1+x)^{2n-1}$.

Question:12 Find a positive value of *m* for which the coefficient of x^2 in the expansion $(1+x)^m$ is 6.

Answer:

As we know that the general $(r+1)^{th}$ term T_{r+1} in the binomial expansion of $(a+b)^n$ is given by

$$T_{r+1} = {}^nC_r a^{n-r} b^r$$

So, the general $(r+1)^{th}$ term T_{r+1} in the binomial expansion of $(1+x)^m$ is

$$T_{r+1} = {}^mC_r 1^{m-r} x^r = {}^mC_r x^r$$

x^2 will come when $r=2$. So,

The coeficient of x^2 in the binomial expansion of $(1+x)^m$ = 6

$$\Rightarrow {}^m C_2 = 6$$

$$\Rightarrow \frac{m!}{2!(m-2)!} = 6$$

$$\Rightarrow \frac{m(m-1)}{2} = 6$$

$$\Rightarrow m(m-1) = 12$$

$$\Rightarrow m^2 - m - 12 = 0$$

$$\Rightarrow (m+3)(m-4) = 0$$

$$\Rightarrow m = 4\, or\, -3$$

Hence the positive value of *m* for which the coefficient of x^2 in the expansion $(1+x)^m$ is 6, is 4.

NCERT solutions for class 11 maths chapter 8 binomial theorem-Miscellaneous Exercise

Question:1 Find *a* , *b* and *n* in the expansion of $(a+b)^n$ if the first three terms of the expansion are 729, 7290 and 30375, respectively.

Answer:

As we know the Binomial expansion of $(a+b)^n$ is given by

$$(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + ...{}^nC_nb^n$$

Given in the question,

$${}^nC_0a^n = 729.......(1)$$

$${}^nC_1a^{n-1}b = 7290.......(2)$$

$${}^nC_2a^{n-2}b^2 = 30375.......(3)$$

Now, dividing (1) by (2) we get,

$$\Rightarrow \frac{{}^nC_0a^n}{{}^nC_1a^{n-1}b} = \frac{729}{7290}$$

$$\Rightarrow \frac{\frac{n!}{n!0!}}{\frac{n!}{1!(n-1)!}} \times \frac{a}{b} = \frac{729}{7290}$$

$$\Rightarrow \frac{(n-1)!}{n!} \times \frac{a}{b} = \frac{1}{10}$$

$$\Rightarrow \frac{1}{n} \times \frac{a}{b} = \frac{1}{10}$$

$$10a = nb......(4)$$

Now, Dividing (2) by (3) we get,

$$\Rightarrow \frac{{}^nC_1a^{n-1}b}{{}^nC_2a^{n-2}b^2} = \frac{7290}{30375}$$

$$\Rightarrow \frac{\frac{n!}{1!(n-1)!}}{\frac{n!}{2!(n-2)!}} \times \frac{a}{b} = \frac{7290}{30375}$$

$$\Rightarrow \frac{2(n-2)!}{(n-1)!} \times \frac{a}{b} = \frac{7290}{30375}$$

$$\Rightarrow \frac{2}{(n-1)} \times \frac{a}{b} = \frac{7290}{30375}$$

$$\Rightarrow 2 \times 30375 \times a = 7290 \times b \times (n-1)$$

$$\Rightarrow 60750a = 7290b(n-1).......(5)$$

Now, From (4) and (5), we get,

$n = 6, a = 3\ and\ b = 5$

Question:2 Find *a* if the coefficients of x^2 and x^3 in the expansion of $(3 + ax)^9$ are equal.

Answer:

As we know that the general $(r+1)^{th}$ term T_{r+1} in the binomial expansion of $(a+b)^n$ is given by

$$T_{r+1} = ^n C_r a^{n-r} b^r$$

So, the general $(r+1)^{th}$ term T_{r+1} in the binomial expansion of $(3+ax)^9$ is

$$T_{r+1} = ^n C_r 3^{n-r} (ax)^r = ^n C_r 3^{n-r} a^r x^r$$

Now, x^2 will come when $r = 2$ and x^3 will come when $r = 3$

So, the coefficient of x^2 is

$$K_{x^2} = ^n C_2 3^{9-2} a^2 = ^n C_2 3^7 a^2$$

And the coefficient of x^3 is

$K_{x^3} = {}^9C_3 3^{9-3} a^2 = {}^9C_3 3^6 a^3$

Now, Given in the question,

$K_{x^2} = K_{x^3}$

${}^9C_2 3^7 a^2 = {}^9C_3 3^6 a^3$

$\frac{9!}{2!7!} \times 3 = \frac{9!}{3!6!} \times a$

$a = \frac{18}{14} = \frac{9}{7}$

Hence the value of a is 9/7.

Question:3 Find the coefficient of x^5 in the product $(1+2x)^6(1-x)^7$ using binomial theorem.

Answer:

First, lets expand both expressions individually,

So,

$(1+2x)^6 = {}^6C_0 + {}^6C_1(2x) + {}^6C_2(2x)^2 + {}^6C_3(2x)^3 + {}^6C_4(2x)^4 + {}^6C_5(2x)^5 +$
${}^6C_6(2x)^6$

$(1+2x)^6 = {}^6C_0 + 2\times{}^6C_1 x + 4\times{}^6C_2 x^2 + 8\times{}^6C_3 x^3 + 16\times{}^6C_4 x^4 + 32\times{}^6C_5 x^5 +$
$64\times{}^6C_6 x^6$

$(1+2x)^6 = 1 + 12x + 60x^2 + 160x^3 + 240x^4 + 192x^5 + 64x^6$

And

$$(1-x)^7 = {}^7C_0 - {}^7C_1x + {}^7C_2x^2 - {}^7C_3x^3 + {}^7C_4x^4 - {}^7C_5x^5 + {}^7C_6x^6 - {}^7C_7x^7$$

$$(1-x)^7 = 1 - 7x + 21x^2 - 35x^3 + 35x^4 - 21x^5 + 7x^6 - x^7$$

Now,

$$(1+2x)^6(1-x)^7 = (1 + 12x + 60x^2 + 160x^3 + 240x^4 + 192x^5 + 64x^6)$$
$$(1 - 7x + 21x^2 - 35x^3 + 35x^4 - 21x^5 + 7x^6 - x^7)$$

Now, for the coefficient of x^5 , we multiply and add those terms whose product gives x^5 .So,

The term which contain x^5 are,

$$\Rightarrow (1)(-21x^5)+(12x)(35x^4)+(60x^2)(-35x^3)+(160x^3)(21x^2)+(240x^4)(-7x)$$
$$+(192x^5)(1)$$

$$\Rightarrow 171x^5$$

Hence the coefficient of x^5 is 171.

Question:4 If a and b are distinct integers, prove that $a-b$ is a factor of $a^n - b^n$, whenever n is a positive integer.
[**Hint:** write $a^n = (a-b+b)^n$ and expand]

Answer:

we need to prove,

$a^n - b^n = k(a-b)$ where k is some natural number.

Now let's add and subtract b from a so that we can prove the above result,

$a = a - b + b$

$a^n = (a - b + b)^n = [(a - b) + b]^n$

$= ^n C_0(a - b)^n + ^n C_1(a - b)^{n-1}b + ... ^n C_n b^n$

$= (a - b)^n + ^n C_1(a - b)^{n-1}b + ... ^n C_{n-1}(a - b)b^{n-1} + b^n$
$\Rightarrow a^n - b^n = (a - b)[(a - b)^{n-1} + ^n C_2(a - b)^{n-2} + ... + ^n C_{n-1}b^{n-1}]$

$\Rightarrow a^n - b^n = k(a - b)$

Hence, $a - b$ is a factor of $a^n - b^n$.

Question:5 Evaluate $\left(\sqrt{3} + \sqrt{2}\right)^6 - \left(\sqrt{3} - \sqrt{2}\right)^6$.

Answer:

First let's simplify the expression $(a + b)^6 - (a - b)^6$ using binomial theorem,

So,

$(a + b)^6 = ^6 C_0 a^6 + ^6 C_1 a^5 b + ^6 C_2 a^4 b^2 + ^6 C_3 a^3 b^3 + ^6 C_4 a^2 b^4 + ^6 C_5 a b^5 + ^6 C_6 b^6$
$(a + b)^6 = a^6 + 6a^5 b + 15a^4 b^2 + 20a^3 b^3 + 15a^2 b^4 + 6ab^5 + b^6$

And

$(a - b)^6 = ^6 C_0 a^6 - ^6 C_1 a^5 b + ^6 C_2 a^4 b^2 - ^6 C_3 a^3 b^3 + ^6 C_4 a^2 b^4 - ^6 C_5 a b^5 + ^6 C_6 b^6$

$(a + b)^6 = a^6 - 6a^5 b + 15a^4 b^2 - 20a^3 b^3 + 15a^2 b^4 - 6ab^5 + b^6$

Now,

$$(a+b)^6-(a-b)^6=a^6+6a^5b+15a^4b^2+20a^3b^3+15a^2b^4+6ab^5+b^6$$

$$-a^6+6a^5b-15a^4b^2+20a^3b^3-15a^2b^4+6ab^5-b^6$$

$$(a+b)^6-(a-b)^6=2[6a^5b+20a^3b^3+6ab^5]$$

Now, Putting $a=\sqrt{3}\, and\, b=\sqrt{2}$, we get

$$\left(\sqrt{3}+\sqrt{2}\right)^6-\left(\sqrt{3}-\sqrt{2}\right)^6=2[54\sqrt{6}+120\sqrt{6}+24\sqrt{6}]$$

$$\left(\sqrt{3}+\sqrt{2}\right)^6-\left(\sqrt{3}-\sqrt{2}\right)^6=2\times 198\sqrt{6}$$

$$\left(\sqrt{3}+\sqrt{2}\right)^6-\left(\sqrt{3}-\sqrt{2}\right)^6=396\sqrt{6}$$

Question:6 Find the value of $\left(a^2+\sqrt{a^2-1}\right)^4+\left(a^2-\sqrt{a^2-1}\right)^4$

Answer:

First, lets simplify the expression $(x+y)^4-(x-y)^4$ using binomial expansion,

$$(x+y)^4={}^4C_0x^4+{}^4C_1x^3y+{}^4C_2x^2y^2+{}^4C_3xy^3+{}^4C_4y^4$$

$$(x+y)^4=x^4+4x^3y+6x^2y^2+4xy^3+y^4$$

And

$$(x-y)^4={}^4C_0x^4-{}^4C_1x^3y+{}^4C_2x^2y^2-{}^4C_3xy^3+{}^4C_4y^4$$

$$(x-y)^4=x^4-4x^3y+6x^2y^2-4xy^3+y^4$$

Now,

$(x+y)^4-(x-y)^4=x^4+4x^3y+6x^2y^2+4xy^3+y^4-$
$x^4+4x^3y-6x^2y^2+4xy^3-y^4$

$(x+y)^4-(x-y)^4=2(x^4+6x^2y^2+y^4)$

Now, Putting $x=a^2\ and\, y=\sqrt{a^2-1}$ we get,

$$\left(a^2+\sqrt{a^2-1}\right)^4+\left(a^2-\sqrt{a^2-1}\right)^4=2[a^8+6a^4(a^2-1)+(a^2-1)^2]$$

$$\left(a^2+\sqrt{a^2-1}\right)^4+\left(a^2-\sqrt{a^2-1}\right)^4=2a^8+12a^6-12a^4+2a^4-4a^2+2$$

$$\left(a^2+\sqrt{a^2-1}\right)^4+\left(a^2-\sqrt{a^2-1}\right)^4=2a^8+12a^6-10a^4-4a^2+2$$

Question:7 Find an approximation of $(0.99)^5$ using the first three terms of its expansion.

Answer:

As we can write 0.99 as 1-0.01,

$$(0.99)^5=(1-0.001)^5={}^5C_0(1)^5-{}^5C_1(1)^4(0.01)+{}^5C_2(1)^3(0.01)^2$$
$$+\,other\ negligible\ terms$$

$$\Rightarrow (0.99)^5=1-5(0.01)+10(0.01)^2$$

$$\Rightarrow (0.99)^5=1-0.05+0.001$$

$$\Rightarrow (0.99)^5=0.951$$

Hence the value of $(0.99)^5$ is 0.951 approximately.

Question:8 Find n, if the ratio of the fifth term from the beginning to the fifth term from the end in the expansion of $\left(\sqrt[4]{2}+\frac{1}{\sqrt[4]{3}}\right)^n$ is $\sqrt{6}:1$

Answer:

Given, the expression

$$\left(\sqrt[4]{2}+\frac{1}{\sqrt[4]{3}}\right)^n$$

Fifth term from the beginning is

$$T_5 =^n C_4(\sqrt[4]{2})^{n-4}\left(\frac{1}{\sqrt[4]{3}}\right)^4$$

$$T_5 =^n C_4\frac{(\sqrt[4]{2})^n}{(\sqrt[4]{2})^4}\times\frac{1}{3}$$

$$T_5 = \frac{n!}{4!(n-4)!}\times\frac{(\sqrt[4]{2})^n}{2}\times\frac{1}{3}$$

And Fifth term from the end is,

$$T_{n-5} =^n C_{n-4}(\sqrt[4]{2})^4\left(\frac{1}{\sqrt[4]{3}}\right)^{n-4}$$

$$T_{n-5} =^n C_{n-4}(\sqrt[4]{2})^4\left(\frac{(\sqrt[4]{3})^4}{(\sqrt[4]{3})^n}\right)$$

$$T_{n-5} = \frac{n!}{4!(n-4)!}\times 2\times\left(\frac{3}{(\sqrt[4]{3})^n}\right)$$

Now, As given in the question,

$$T_5 : T_{n-5} = \sqrt{6}:1$$

So,

From Here ,

$$\frac{(\sqrt[4]{2})^n}{6} : \frac{6}{(\sqrt[4]{3})^n} = \sqrt{6} : 1$$

$$\frac{(\sqrt[4]{2})^n(\sqrt[4]{3})^n}{6 \times 6} = \sqrt{6}$$

$$(\sqrt[4]{6})^n = 36\sqrt{6}$$

$$6^{\frac{n}{4}} = 6^{\frac{5}{2}}$$

From here,

$$\frac{n}{4} = \frac{5}{2}$$

$$n = 10$$

Hence the value of n is 10.

Question:9 Expand using Binomial Theorem $\left(1 + \frac{x}{2} - \frac{2}{x}\right)^4,\ x \neq 0$

Answer:

Given the expression,

$$\left(1 + \frac{x}{2} - \frac{2}{x}\right)^4,\ x \neq 0$$

Binomial expansion of this expression is

$$^4C_2\left(1+\frac{x}{2}\right)^2\left(\frac{2}{x}\right)^2 - ^4C_3\left(1+\frac{x}{2}\right)\left(\frac{2}{x}\right)^3 + ^4C_4\left(\frac{2}{x}\right)^4$$

$$\Rightarrow \left(1+\frac{x}{2}\right)^4 - \frac{8}{x}\left(1+\frac{x}{2}\right)^3 + \frac{24}{x^2} + \frac{24}{x} + 6 - \frac{32}{x^3} + \frac{16}{x^4}.........(1)$$

Now Applying Binomial Theorem again,

$$+^4C_4\left(\frac{x}{2}\right)^4$$

$$= 1 + 2x + \frac{3x^2}{2} + \frac{x^3}{3} + \frac{x^4}{16}.............(2)$$

And

$$\left(1+\frac{x}{2}\right)^3 = 1 + \frac{3x}{2} + \frac{3x^2}{4} + \frac{x^3}{8}.........(3)$$

Now, From (1), (2) and (3) we get,

$$+\frac{8}{x^2} + \frac{24}{x} + 6 - \frac{32}{x^3} + \frac{16}{x^4}$$

$$+\frac{8}{x^2} + \frac{24}{x} + 6 - \frac{32}{x^3} + \frac{16}{x^4}$$

$$= \frac{16}{x} + \frac{8}{x^2} - \frac{32}{x^3} + \frac{16}{x^4} - 4x + \frac{x^2}{2} + \frac{x^3}{2} + \frac{x^4}{16} - 5$$

Question:10 Find the expansion of $(3x^2 - 2ax + 3a^2)^3$ using binomial theorem .

Answer:

Given $(3x^2 - 2ax + 3a^2)^3$

By Binomial Theorem It can also be written as

$$(3x^2 - 2ax + 3a^2)^3 = ((3x^2 - 2ax) + 3a^2)^3$$

$$= ^3C_0(3x^2-2ax)^3 + ^3C_1(3x^2-2ax)^2(3a^2) + ^3C_2(3x^2-2ax)(3a^2)^2 + ^3C_3(3a^2)^3$$

$$= (3x^2 - 2ax)^3 + 3(3x^2 - 2ax)^2(3a^2) + 3(3x^2 - 2ax)(3a^2)^2 + (3a^2)^3$$

$$= (3x^2 - 2ax)^3 + 81a^2x^4 - 108a^3x^3 + 36a^4x^2 + 81a^4x^2 - 54a^5x + 27a^6$$

$$= (3x^2 - 2ax)^3 + 81a^2x^4 - 108a^3x^3 + 117a^4x^2 - 54a^5x + 27a^6 \text{...........}(1)$$

Now, Again By Binomial Theorem,

$$(3x^2 - 2ax)^3 = ^3C_0(3x^2)^3 - ^3C_1(3x^2)^2(2ax) + ^3C_2(3x^2)(2ax)^2 - ^3C_3(2ax)^3$$

$$(3x^2 - 2ax)^3 = 27x^6 - 3(9x^4)(2ax) + 3(3x^2)(4a^2x^2) - 8a^2x^3$$

$$(3x^2 - 2ax)^3 = 27x^6 - 54x^5 + 36a^2x^4 - 8a^3x^3 \text{............}(2)$$

From (1) and (2) we get,

$$(3x^2 - 2ax + 3a^2)^3 = 27x^6 - 54x^5 + 36a^2x^3 + 81a^2x^4 - 108a^3x^3 + 117a^4x^2$$
$$-54a^5x + 27a^6$$

$$(3x^2 - 2ax + 3a^2)^3 = 27x^6 - 54x^5 + 117a^2x^3 - 116a^3x^3 + 117a^4x^2$$
$$-54a^5x + 27a^6$$