NCERT solutions for class 11 maths chapter 7 permutation and combinations

Question:1(i) How many 3-digit numbers can be formed from the digits 1, 2, 3, 4 and 5 assuming that
repetition of the digits is allowed?

Answer:

The five digits are 1, 2, 3, 4 and 5

As we know that repetition of the digits is allowed,

so , unit place can be filled by any of five digits.

Similarly , tens and hundreds digits can also be filled by any of five digits.

$\therefore$ Number of 3-digit numbers can be formed $= 5 \times 5 \times 5 = 125$

Question:1(ii) How many 3-digit numbers can be formed from the digits 1, 2, 3, 4 and 5 assuming that
repetition of the digits is not allowed?

Answer:

The five digits are 1, 2, 3, 4 and 5

As we know that repetition of the digits is not allowed,

so, the unit place can be filled by any of five digits.

Tens place can be filled with any of the remaining four digits.

Hundreds place can be filled with any of the remaining three digits.

$\therefore$ Number of 3-digit numbers can be formed when repetition is not allowed $= 5 \times 4 \times 3 = 60$

Question:2 How many 3-digit even numbers can be formed from the digits 1, 2, 3, 4, 5, 6 if the digits can be repeated?

Answer:

The six digits are 1, 2, 3, 4 ,5 and 6

As we know that repetition of the digits is allowed,

so, the unit place can be filled by any of even digits i.e.2,4 or 6

Similarly, tens and hundreds of digits can also be filled by any of six digits.

$\therefore$ Number of 3-digit even numbers can be formed $= 3 \times 6 \times 6 = 108$

Question:3 How many 4-letter code can be formed using the first 10 letters of the English alphabet, if no letter can be repeated?

Answer:

There are 10 letters of English alphabets.

As we know that repetition of the letters is not allowed,

so, the first place can be filled by any of 10 letters.

Second place can be filled with any of the remaining 9 letters.

Third place can be filled with any of the remaining 8 letters.

The fourth place can be filled with any of the remaining 7 letters.

$\therefore$ Number of 4-letter code can be formed when the repetition of letters is not allowed $= 10 \times 9 \times 8 \times 7 = 5040$

Hence, 5040 4-letter codes can be formed using the first 10 letters of the English alphabet, if no letter can be repeated.

Question:4 How many 5-digit telephone numbers can be constructed using the digits 0 to 9 if each number starts with 67 and no digit appears more than once?

Answer:

It is given that 5-digit telephone numbers always start with 67.

First, two digits among 5-digit telephone numbers are fixed and rest 3 digits are variable.

$6, 7, -, -, -$

The 10 digits are from 0 to 9.

As we know that repetition of the digits is not allowed,

so, the first and second place is filled by two digits 67

Third place can be filled with any of the remaining 8 digits.

The fourth place can be filled with any of the remaining 7 digits.

The fifth place can be filled with any of the remaining 6 digits.

∴ Number of 5-digit telephone numbers can be formed when repetition is not allowed $= 8 \times 7 \times 6 = 336$

Question:5 A coin is tossed 3 times and the outcomes are recorded. How many possible outcomes are there?

Answer:

When a coin is tossed the number of outcomes is 2 i.e. head or tail.

When a coin is tossed 3 times then by multiplication principle,

the number of outcomes $= 2 \times 2 \times 2 = 8$

Question:6 Given 5 flags of different colours, how many different signals can be generated if each signal requires the use of 2 flags, one below the other?

Answer:

Each signal requires use of 2 flags.

There will be as many flags as there are ways of filling in 2 vacant places in succession by the given 5 flags of different colours.

The upper vacant place can be filled in 5 different ways with any of 5 flags and lower vacant place can be filled in 4 different ways by any of rest 4 flags.

Hence, by multiplication principle number of different signals that can be generated $= 5 \times 4 = 20$

NCERT solutions for class 11 maths chapter 7 permutation and combinations-Exercise: 7.2

Question:1(i) Evaluate $8!$

Answer:

Factorial can be given as :

$8! = 8 \times 7 \times 6 \times 5 \times \times 4 \times 3 \times 2 \times 1 = 40320$

Question:1(ii) Evaluate $4! - 3!$

Answer:

Factorial can be given as :

$(ii)\ 4! - 3!$

$= (4 \times 3 \times 2 \times 1) - (3 \times 2 \times 1)$

$= 24 - 6$

$= 18$

Question:2 Is $3! + 4! = 7!$?

Answer:

Factorial can be given as :

To prove : $3! + 4! = 7!$

R.H.S : $3! + 4!$

$= (3 \times 2 \times 1) + (4 \times 3 \times 2 \times 1)$

$= 6 + 24$

$= 30$

L.H.S : $7!$

$= 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$

$L.H.S \neq R.H.S$

Question:3 Compute $\frac{8!}{6! \times 2!}$

Answer:

To compute the factorial :

$\frac{8!}{6! \times 2!} = \frac{8 \times 7 \times 6!}{6! \times 2 \times 1}$

$= \frac{8 \times 7}{2}$

$= 4 \times 7 = 28$

So the answer is 28

Question:4 If $\frac{1}{6!} + \frac{1}{7!} = \frac{x}{8!}$, find x

Answer:

Factorial can be given as :

To find x : $\frac{1}{6!}+\frac{1}{7!}=\frac{x}{8!}$,

$R.H.S: \ \frac{1}{6!}+\frac{1}{7!}$

$=\frac{1}{6!}+\frac{1}{7\times 6!}$

$=\frac{1}{6!}(1+\frac{1}{7})$

$=\frac{1}{6!}(\frac{8}{7})$

$L.H.S: \ \frac{x}{8!}$,

$=\frac{x}{8\times 7\times 6!}$

Given : $L.H.S=R.H.S$

$\frac{1}{6!}\left(\frac{8}{7}\right)=\frac{x}{8\times 7\times 6!}$

$\Rightarrow \ 8=\frac{x}{8}$

$\Rightarrow \ x=8\times 8=64$

Question:5(i) Evaluate $\frac{n!}{(n-r)!}$, when

$n=6,r=2$

Answer:

To evaluate $\frac{n!}{(n-r)!}$,

Put $n=6, r=2$

$\frac{n!}{(n-r)!}=\frac{6!}{(6-2)!}$

$=\frac{6!}{4!}$

$=\frac{6\times5\times4!}{4!}$

$=6\times5=30$

Question:5(ii) Evaluate $\frac{n!}{(n-r)!}$, when

$n=9, r=5$

Answer:

To evaluate $\frac{n!}{(n-r)!}$,

Put $n=9, r=5$

$\frac{n!}{(n-r)!}=\frac{9!}{(9-5)!}$

$=\frac{9!}{4!}$

$=\frac{9\times8\times7\times6\times5\times4!}{4!}$

$=9\times8\times7\times6\times5=15120$

NCERT solutions for class 11 maths chapter 7 permutation and combinations-Exercise: 7.3

Question:1 How many 3-digit numbers can be formed by using the digits 1 to 9 if no digit is repeated?

Answer:

3-digit numbers have to be formed by using the digits 1 to 9.

Here, the order of digits matters.

Therefore, there will be as many 3-digit numbers as there are permutations of 9 different digits taken 3 at a time.

Therefore, the required number of 3-digit numbers $= {}^9P_3$

$= \frac{9!}{(9-3)!}$

$= \frac{9!}{6!}$

$= \frac{9 \times 8 \times 7 \times 6!}{6!}$

$= 9 \times 8 \times 7 = 504$

Question:2 How many 4-digit numbers are there with no digit repeated?

Answer:

The thousands place of 4-digit numbers has to be formed by using the digits 1 to 9(0 cannot be included).

Therefore, the number of ways in which thousands place can be filled is 9.

Hundreds,tens, unit place can be filled by any digits from 0 to 9.

The digit cannot be repeated in 4-digit numbers and thousand places is occupied with a digit.

Hundreds, tens, unit place can be filled by remaining any 9 digits.

Therefore, there will be as many 3-digit numbers as there are permutations of 9 different digits taken 3 at a time.

Therefore, the required number of 3-digit numbers $=^9 P_3$

$$= \frac{9!}{(9-3)!}$$

$$= \frac{9!}{6!}$$

$$= \frac{9 \times 8 \times 7 \times 6!}{6!}$$

$$= 9 \times 8 \times 7 = 504$$

Thus, by multiplication principle, required 4 -digit numbers is $9 \times 504 = 4536$

Question:3 How many 3-digit even numbers can be made using the digits 1, 2, 3, 4, 6, 7, if no digit is repeated?

Answer:

3-digit even numbers can be made using the digits 1, 2, 3, 4, 6, 7, if no digit is repeated.

The unit place can be filled in 3 ways by any digits from 2,4 or 6.

The digit cannot be repeated in 3-digit numbers and the unit place is occupied with a digit(2,4 or 6).

Hundreds, tens place can be filled by remaining any 5 digits.

Therefore, there will be as many 2-digit numbers as there are permutations of 5 different digits taken 2 at a time.

Therefore, the required number of 2-digit numbers $= {}^5P_2$

$$= \frac{5!}{(5-2)!}$$

$$= \frac{5!}{3!}$$

$$= \frac{5 \times 4 \times 3!}{3!}$$

$$= 5 \times 4 = 20$$

Thus, by multiplication principle, required 3 -digit numbers is $3 \times 20 = 60$

Question:4 Find the number of 4-digit numbers that can be formed using the digits 1, 2, 3, 4,5 if no digit is repeated. How many of these will be even?

Answer:

4-digit numbers that can be formed using the digits 1, 2, 3, 4,5.

Therefore, there will be as many 4-digit numbers as there are permutations of 5 different digits taken 4 at a time.

Therefore, the required number of 4-digit numbers $= {}^5P_4$

$$= \frac{5!}{(5-4)!}$$

$$= \frac{5!}{1!}$$

$$= 5 \times 4 \times 3 \times 2 \times 1 = 120$$

4-digit even numbers can be made using the digits 1, 2, 3, 4, 5 if no digit is repeated.

The unit place can be filled in 2 ways by any digits from 2 or 4.

The digit cannot be repeated in 4-digit numbers and the unit place is occupied with a digit(2 or 4).

Thousands, hundreds, tens place can be filled by remaining any 4 digits.

Therefore, there will be as many 3-digit numbers as there are permutations of 4 different digits taken 3 at a time.

Therefore, the required number of 3-digit numbers $= {}^4P_3$

$$= \frac{4!}{(4-3)!}$$

$$= \frac{4!}{1!}$$

$= 4 \times 3 \times 2 \times 1 = 24$

Thus, by multiplication principle, required 4 -digit numbers is $2 \times 24 = 48$

Question:5 From a committee of 8 persons, in how many ways can we choose a chairman and a vice chairman assuming one person cannot hold more than one position?

Answer:

From a committee of 8 persons, chairman and a vice chairman are to be chosen assuming one person can not hold more than one position.

Therefore,number of ways of choosing a chairman and a vice chairman is permutations of 8 different objects taken 2 at a time.

Therefore, required number of ways $=^8 P_2$

$$= \frac{8!}{(8-2)!}$$

$$= \frac{8!}{6!}$$

$$= \frac{8 \times 7 \times 6!}{6!}$$

$$= 8 \times 7 = 56$$

Question:6 Find n if $^{n-1}P_3 :^n P_4 = 1 : 9.$

Answer:

Given : $^{n-1}P_3 :^n P_4 = 1 : 9.$

To find the value of n

$^{n-1}P_3 :^n P_4 = 1 : 9.$

$\Rightarrow \frac{^{n-1}P_3}{^nP_4} = \frac{1}{9}$

$\Rightarrow \frac{\frac{(n-1)!}{(n-1-3)!}}{\frac{n!}{n-4!}} = \frac{1}{9}$

$\Rightarrow \frac{\frac{(n-1)!}{(n-4)!}}{\frac{n!}{(n-4)!}} = \frac{1}{9}$

$\Rightarrow \frac{(n-1)!}{n!} = \frac{1}{9}$

$\Rightarrow \frac{(n-1)!}{(n-1)! \times n} = \frac{1}{9}$

$\Rightarrow \frac{1}{n} = \frac{1}{9}$

$\Rightarrow n = 9.$

Question:7(i) Find r if

$^5P_r = 2\ ^6P_{r-1}$

Answer:

Given : $^5P_r = 2\ ^6P_{r-1}$

To find the value of r.

$^5P_r = 2\ ^6P_{r-1}$

$$\Rightarrow \frac{5!}{(5-r)!} = 2 \times \frac{6!}{(6-(r-1))!}$$

$$\Rightarrow \frac{5!}{(5-r)!} = 2 \times \frac{6!}{(6-r+1)!}$$

$$\Rightarrow \frac{5!}{(5-r)!} = 2 \times \frac{6 \times 5!}{(6-r+1)!}$$

$$\Rightarrow \frac{1}{(5-r)!} = 2 \times \frac{6}{(7-r)!}$$

$$\Rightarrow \frac{1}{(5-r)!} = \frac{12}{(7-r) \times (6-r) \times (5-r)!}$$

$$\Rightarrow \frac{1}{1} = \frac{12}{(7-r) \times (6-r)}$$

$$\Rightarrow (7-r) \times (6-r) = 12$$

$$\Rightarrow 42 - 6r - 7r + r^2 = 12$$

$$\Rightarrow r^2 - 13r + 30 = 0$$

$$\Rightarrow r^2 - 3r - 10r + 30 = 0$$

$$\Rightarrow r(r-3) - 10(r-3) = 0$$

$$\Rightarrow (r-3)(r-10) = 0$$

$$\Rightarrow r = 3, 10$$

We know that

$$^{n}P_r = \frac{n!}{(n-r)!}$$

where $0 \leq r \leq n$

$\therefore 0 \leq r \leq 5$

Thus the value of, $r = 3$

Question:7(ii) Find r if

${}^{5}P_{r} = {}^{6}P_{r-1}$

Answer:

Given : ${}^{5}P_{r} = {}^{6}P_{r-1}$

To find the value of r.

${}^{5}P_{r} = {}^{6}P_{r-1}$

$\Rightarrow \dfrac{5!}{(5-r)!} = \dfrac{6!}{(6-(r-1))!}$

$\Rightarrow \dfrac{5!}{(5-r)!} = \dfrac{6!}{(6-r+1)!}$

$\Rightarrow \dfrac{5!}{(5-r)!} = \dfrac{6 \times 5!}{(6-r+1)!}$

$\Rightarrow \dfrac{1}{(5-r)!} = \dfrac{6}{(7-r)!}$

$\Rightarrow \dfrac{1}{(5-r)!} = \dfrac{6}{(7-r) \times (6-r) \times (5-r)!}$

$\Rightarrow \dfrac{1}{1} = \dfrac{6}{(7-r) \times (6-r)}$

$\Rightarrow (7-r) \times (6-r) = 6$

$\Rightarrow 42 - 6r - 7r + r^2 = 6$

$\Rightarrow \ r^2 - 13r + 36 = 0$

$\Rightarrow \ r^2 - 4r - 9r + 36 = 0$

$\Rightarrow \ r(r-4) - 9(r-4) = 0$

$\Rightarrow \ (r-4)(r-9) = 0$

$\Rightarrow \ r = 4, 9$

We know that

$^nP_r = \dfrac{n!}{(n-r)!}$

where $0 \leq r \leq n$

$\therefore 0 \leq r \leq 5$

Thus, $r = 4$

Question:8 How many words, with or without meaning, can be formed using all the letters of the word EQUATION, using each letter exactly once?

Answer:

There are 8 different letters in word EQUATION.

Therefore, words, with or without meaning, can be formed using all the letters of the word EQUATION, using each letter exactly once

is permutations of 8 different letters taken 8 at a time, which is $^8P_8 = 8!$

Hence, the required number of words formed $= 8! = 40320$

Question:9(i) How many words, with or without meaning can be made from the letters of the word MONDAY, assuming that no letter is repeated, if.

4 letters are used at a time

Answer:

There are 6 letters in word MONDAY.

Therefore, words that can be formed using 4 letters of the word MONDAY.

Hence, the required number of words formed using 4 letters $=^6 P_4$

$$= \frac{6!}{(6-4)!}$$

$$= \frac{6!}{(2)!}$$

$$= \frac{6 \times 5 \times 4 \times 3 \times 2!}{(2)!}$$

$$= 6 \times 5 \times 4 \times 3$$

$$= 360$$

Question:9(ii) How many words, with or without meaning can be made from the letters of the word MONDAY, assuming that no letter is repeated, if.

all letters are used at a time

Answer:

There are 6 letters in word MONDAY.

Therefore, words that can be formed using all 6 letters of the word MONDAY.

Hence, the required number of words formed using 6 letters at a time $={}^6P_6$

$=\frac{6!}{(6-6)!}$

$=\frac{6!}{(0)!}$

$=6\times5\times4\times3\times2\times1$

$=720$

Question:9(iii) How many words, with or without meaning can be made from the letters of the word MONDAY, assuming that no letter is repeated, if.

all letters are used but first letter is a vowel?

Answer:

There are 6 letters and 2 vowels in word MONDAY.

Therefore, the right most position can be filled by any of these 2 vowels in 2 ways.

Remaining 5 places of the word can be filled using any of rest 5 letters of the word MONDAY.

Hence, the required number of words formed using 5 letters at a time $={}^5P_5$

$=5!$

$= 5 \times 4 \times 3 \times 2 \times 1$

$= 120$

Words formed starting from vowel using 6 letters $= 2 \times 120 = 240$

Question:10 In how many of the distinct permutations of the letters in MISSISSIPPI do the four I's not come together?

Answer:

In the given word MISSISSIPPI, I appears 4 times, S appears 4 times, M appears 1 time and P appear 2 times.

Therefore, the number of distinct permutations of letters of the given word is

$$= \frac{11!}{4!4!2!}$$

$$= \frac{11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4!}{4!4!2!}$$

$$= \frac{11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 2}$$

$= 34650$

There are 4 I's in the given word. When they occur together they are treated as a single object for the time being. This single object with the remaining 7 objects will be 8 objects.

These 8 objects in which there are 4Ss and 2Ps can be arranged in $\frac{8!}{4!2!} = 840$ ways.

The number of arrangement where all I's occur together = 840.

Hence, the distinct permutations of the letters in MISSISSIPPI in which the four I's do not come together $= 34650 - 840 = 33810$

Question:11(i) In how many ways can the letters of the word PERMUTATIONS be arranged if the

words start with P and end with S?

Answer:

There are 2 T's in word PERMUTATIONS, all other letters appear at once only.

If words start with P and ending with S i.e. P and S are fixed, then 10 letters are left.

The required number of arrangements are

$$= \frac{10!}{2!} = 1814400$$

Question:11(ii) In how many ways can the letters of the word PERMUTATIONS be arranged if the

vowels are all together?

Answer:

There are 5 vowels in word PERMUTATIONS and each appears once.

Since all 5 vowels are to occur together so can be treated as 1 object.

The single object with the remaining 7 objects will be 8 objects.

The 8 objects in which 2 T's repeat can be arranged as

$= \frac{8!}{2!}$ ways.

These 5 vowels can also be arranged in $5!$ ways.

Hence, using the multiplication principle, the required number of arrangements are

$= \frac{8!}{2!} \times 5! = 2419200$ ways.

Question:11(iii) In how many ways can the letters of the word PERMUTATIONS be arranged if the

there are always 4 letters between P and S?

Answer:

The letters of the word PERMUTATIONS be arranged in such a way that there are always 4 letters between P and S.

Therefore, in a way P and S are fixed. The remaining 10 letters in which 2 T's are present can be arranged in

$= \frac{10!}{2!}$ ways.

Also, P and S can be placed such that there are 4 letters between them in $2 \times 7 = 14$ ways.

Therefore, using the multiplication principle required arrangements

$$= \frac{10!}{2!}14 = 25401600$$

NCERT solutions for class 11 maths chapter 7 permutation and combinations-Exercise: 7.4

Question:1 If ${}^nC_8 = {}^nC_2$, find ${}^nC_2$

Answer:

Given : ${}^nC_8 = {}^nC_2$,

We know that ${}^nC_a = {}^nC_b \Rightarrow a = b \;\; or \; n = a + b$

$${}^nC_8 = {}^nC_2,$$

$$\Rightarrow n = 8 + 2$$

$$\Rightarrow n = 10$$

$${}^nC_2 = {}^{10}C_2$$

$$= \frac{10!}{(10-2)!2!}$$

$$= \frac{10!}{8!2!}$$

$$= \frac{10 \times 9 \times 8!}{8!2!}$$

$$= 5 \times 9 = 45$$

Thus the answer is 45

Question:2(i) Determine n if

$^{2n}C_3 :^n C_3 = 12 : 1$

Answer:

Given that : $(i)\ ^{2n}C_3 :^n C_3 = 12 : 1$

$$\Rightarrow \ \frac{^{2n}C_3}{^nC_3} = \frac{12}{1}$$

The ratio can be written as

$$\Rightarrow \ \frac{\frac{2n!}{(2n-3)!3!}}{\frac{n!}{(n-3)!3!}} = \frac{12}{1}$$

$$\Rightarrow \ \frac{2n!(n-3)!}{(2n-3)!n!} = \frac{12}{1}$$

$$\Rightarrow \ \frac{2n \times (2n-1) \times (2n-2)}{n \times (n-1) \times (n-2)} = \frac{12}{1}$$

$$\Rightarrow \ \frac{2 \times (2n-1) \times 2}{(n-2)} = \frac{12}{1}$$

$$\Rightarrow \ \frac{4 \times (2n-1)}{(n-2)} = \frac{12}{1}$$

$$\Rightarrow \ \frac{(2n-1)}{(n-2)} = \frac{3}{1}$$

$$\Rightarrow \ 2n - 1 = 3n - 6$$

$$\Rightarrow\ 6-1=3n-2n$$

$$\Rightarrow\ n=5$$

Question:2(ii) Determine n if

$$^{2n}C_3 :^{n} C_3=11:1$$

Answer:

Given that : $(ii)\ ^{2n}C_3 :^{n} C_3=11:1$

$$\Rightarrow\ \frac{^{2n}C_3}{^{n}C_3}=\frac{11}{1}$$

$$\Rightarrow\ \frac{\frac{2n!}{(2n-3)!3!}}{\frac{n!}{(n-3)!3!}}=\frac{11}{1}$$

$$\Rightarrow\ \frac{2n!(n-3)!}{(2n-3)!n!}=\frac{11}{1}$$

$$\Rightarrow\ \frac{2n\times(2n-1)\times(2n-2)}{n\times(n-1)\times(n-2)}=\frac{11}{1}$$

$$\Rightarrow\ \frac{2\times(2n-1)\times2}{(n-2)}=\frac{11}{1}$$

$$\Rightarrow\ \frac{4\times(2n-1)}{(n-2)}=\frac{11}{1}$$

$$\Rightarrow\ 8n-4=11n-22$$

$$\Rightarrow\ 22-4=11n-8n$$

$\Rightarrow\ 3n = 18$

$\Rightarrow\ n = 6$

Thus the value of n=6

Question:3 How many chords can be drawn through 21 points on a circle?

Answer:

To draw chords 2 points are required on the circle.

To know the number of chords on the circle , when points on the circle are 21.

Combinations =Number of chords $= {}^{21}C_2$

$$= \frac{21!}{(21-2)!2!}$$

$$= \frac{21!}{19!2!}$$

$$= \frac{21 \times 20}{2}$$

$$= 210$$

Question:4 In how many ways can a team of 3 boys and 3 girls be selected from 5 boys and 4 girls?

Answer:

A team of 3 boys and 3 girls be selected from 5 boys and 4 girls.

3 boys can be selected from 5 boys in ${}^{5}C_3$ ways.

3 girls can be selected from 4 boys in ${}^4C_3$ ways.

Therefore, by the multiplication principle, the number of ways in which a team of 3 boys and 3 girls can be selected $={}^5C_3 \times {}^4C_3$

$$=\frac{5!}{2!3!}\times\frac{4!}{1!3!}$$

$$=10\times 4=40$$

Question:5 Find the number of ways of selecting 9 balls from 6 red balls, 5 white balls and 5 blue balls if each selection consists of 3 balls of each colour.

Answer:

There are 6 red balls, 5 white balls and 5 blue balls.

9 balls have to be selected in such a way that consists of 3 balls of each colour.

3 balls are selected from 6 red balls in ${}^6C_3$.

3 balls are selected from 5 white balls in ${}^5C_3$

3 balls are selected from 5 blue balls in ${}^5C_3$.

Hence, by the multiplication principle, the number of ways of selecting 9 balls $={}^6C_3\times{}^5C_3\times{}^5C_3$

$$=\frac{6!}{3!3!}\times\frac{5!}{2!3!}\times\frac{5!}{2!3!}$$

$$=20\times 10\times 10=2000$$

Question:6 Determine the number of 5 card combinations out of a deck of 52 cards if there is exactly one ace in each combination.

Answer:

In a deck, there is 4 ace out of 52 cards.

A combination of 5 cards is to be selected containing exactly one ace.

Then, one ace can be selected in ${}^{4}C_{1}$ ways and other 4 cards can be selected in ${}^{48}C_{4}$ ways.

Hence, using the multiplication principle, required the number of 5 card combination $={}^{4}C_{1}\times{}^{48}C_{4}$

$$=\frac{4!}{1!3!}\times\frac{48!}{44!4!}$$

$$=4\times\frac{48\times47\times46\times45}{4\times3\times2}=778320$$

Question:7 In how many ways can one select a cricket team of eleven from 17 players in which only 5 players can bowl if each cricket team of 11 must include exactly 4 bowlers?

Answer:

Out off, 17 players, 5 are bowlers.

A cricket team of 11 is to be selected such that there are exactly 4 bowlers.

4 bowlers can be selected in ${}^{5}C_{4}$ ways and 7 players can be selected in ${}^{12}C_{7}$ ways.

Thus, using multiplication priciple, number of ways of selecting the team $= {}^5C_4 . {}^{12}C_7$

$$= \frac{5!}{1!4!} \times \frac{12!}{5!7!}$$

$$= 5 \times \frac{12 \times 11 \times 10 \times 9 \times 8}{5 \times 4 \times 3 \times 2}$$

$$= 3960$$

Question:8 A bag contains 5 black and 6 red balls. Determine the number of ways in which 2 black and 3 red balls can be selected.

Answer:

A bag contains 5 black and 6 red balls.

2 black balls can be selected in ${}^5C_2$ ways and 3 red balls can be selected in ${}^6C_3$ ways.

Thus, using multiplication priciple, number of ways of selecting 2 black and 3 red balls $= {}^5C_2 . {}^6C_3$

$$= \frac{5!}{2!3!} \times \frac{6!}{3!3!}$$

$$= 10 \times 20 = 200$$

Question:9 In how many ways can a student choose a programme of 5 courses if 9 courses are available and 2 specific courses are compulsory for every student?

Answer:

9 courses are available and 2 specific courses are compulsory for every student.

Therefore, every student has to select 3 courses out of the remaining 7 courses.

This can be selected in $^7C_3$ ways.

Thus, using multiplication priciple, number of ways of selecting courses $=^7 C_3$

$$= \frac{7!}{3!4!}$$

$$= \frac{7 \times 6 \times 5}{3 \times 2}$$

$$= 35$$

NCERT solutions for class 11 maths chapter 7 permutation and combinations-Miscellaneous Exercise

Question:1 How many words, with or without meaning, each of 2 vowels and 3 consonants can be formed from the letters of the word DAUGHTER?

Answer:

In the word DAUGHTER, we have

vowels = 3(A,E,U)

consonants = 5(D,G,H,T,R)

Number of ways of selecting 2 vowels $=^3 C_2$

Number of ways of selecting 3 consonants $=^5 C_3$

Therefore, the number of ways of selecting 2 vowels and 3 consonants $=^3 C_2 . ^5C_3$

$= 3 \times 10 = 30$

Each of these 30 combinations of 2 vowels and 3 consonants can be arranged in $5!$ ways.

Thus, the required number of different words $= 5! \times 30 = 120 \times 30 = 3600$

Question:2 How many words, with or without meaning, can be formed using all the letters of the word EQUATION at a time so that the vowels and consonants occur together?

Answer:

In the word EQUATION, we have

vowels = 5(A,E,I,O,U)

consonants = 3(Q,T,N)

Since all the vowels and consonants occur together so (AEIOU) and (QTN) can be assumed as single objects.

Then, permutations of these two objects taken at a time $=^2 P_2 = 2! = 2$

Corresponding to each of these permutations, there are $5!$ permutations for vowels and $3!$permutations for consonants.

Thus, by multiplication principle, required the number of different words $= 2 \times 5! \times 3! = 2 \times 120 \times 6 = 1440$

Question:3(i) A committee of 7 has to be formed from 9 boys and 4 girls. In how many ways can this be done when the committee consists of:

exactly 3 girls?

Answer:

There are 9 boys and 4 girls. A committee of 7 has to be formed.

Given : Girls =3, so boys in committee= 7-3=4

Thus, the required number of ways $=^4 C_3.^9C_4$

$$= \frac{4!}{3!1!} \times \frac{9!}{4!5!}$$

$$= 4 \times \frac{9 \times 8 \times 7 \times 6 \times 5!}{4 \times 3 \times 2 \times 5!}$$

$$= 9 \times 8 \times 7 = 504$$

Question:3(ii) A committee of 7 has to be formed from 9 boys and 4 girls. In how many ways can this be done when the committee consists of:

at least 3 girls?

Answer:

There are 9 boys and 4 girls. A committee of 7 has to be formed.

(ii) at least 3 girls, there can be two cases :

(a) Girls =3, so boys in committee= 7-3=4

Thus, the required number of ways $=^4 C_3.^9C_4$

$$= \frac{4!}{3!1!} \times \frac{9!}{4!5!}$$

$$= 4 \times \frac{9 \times 8 \times 7 \times 6 \times 5!}{4 \times 3 \times 2 \times 5!}$$

$$= 9 \times 8 \times 7 = 504$$

(b) Girls =4, so boys in committee= 7-4=3

Thus, the required number of ways $= ^4C_4 . ^9C_3$

$$= \frac{4!}{4!0!} \times \frac{9!}{3!6!}$$

$$= \frac{9 \times 8 \times 7 \times 6!}{3 \times 2 \times 6!}$$

$$= 84$$

Hence, in this case, the number of ways = 504+84=588

Question:3(iii) A committee of 7 has to be formed from 9 boys and 4 girls. In how many ways can this be done when the committee consists of:

atmost 3 girls?

Answer:

There are 9 boys and 4 girls. A committee of 7 has to be formed.

(ii) atmost 3 girls, there can be 4 cases :

(a) Girls =0, so boys in committee= 7-0=7

Thus, the required number of ways $= ^4C_0 . ^9C_7$

$$= \frac{4!}{4!0!} \times \frac{9!}{2!7!}$$

$$= 9 \times 4 = 36$$

(b) Girls =1, so boys in committee= 7-1=6

Thus, the required number of ways $= ^4C_1 . ^9C_6$

$$= \frac{4!}{3!1!} \times \frac{9!}{3!6!}$$

$$= 336$$

(c) Girls =2, so boys in committee= 7-2=5

Thus, the required number of ways $= ^4C_2 . ^9C_5$

$$= \frac{4!}{2!2!} \times \frac{9!}{4!5!}$$

$$= 756$$

(d) Girls =3, so boys in committee= 7-3=4

Thus, the required number of ways $= ^4C_3 . ^9C_4$

$$= \frac{4!}{3!1!} \times \frac{9!}{4!5!}$$

$$= 504$$

Hence, in this case, the number of ways = 36+336+756+504=1632

Question:4 If the different permutations of all the letter of the word EXAMINATION are listed as in a dictionary, how many words are there in this list before the first word starting with E?

Answer:

In the word EXAMINATION, we have 11 letters out of which A,I, N appear twice and all other letters appear once.

The word that will be listed before the first word starting with E will be words starting with A.

Therefore, to get the number of words starting with A, letter A is fixed at extreme left position, the remaining 10 letters can be arranged.

Since there are 2 I's and 2 N's in the remaining 10 letters.

Number of words starting with A $= \frac{10!}{2!2!} = 907200$

Thus, the required number of different words = 907200

Question:5 How many 6-digit numbers can be formed from the digits 0, 1, 3, 5, 7 and 9 which are divisible by 10 and no digit is repeated?

Answer:

For the number to be divisible by 10, unit digit should be 0.

Thus, 0 is fixed at a unit place.

Therefore, the remaining 5 places should be filled with 1,3,5,7,9.

The remaining 5 vacant places can be filled in $5!$ ways.

Hence, the required number of 6 digit numbers which are divisible by 10 $= 5! = 120$

Question:6 The English alphabet has 5 vowels and 21 consonants. How many words with two different vowels and 2 different consonants can be formed from the alphabet?

Answer:

Two different vowels and 2 different consonants are to be selected from the English alphabets.

Since there are 5 different vowels so the number of ways of selecting two different vowels = ${}^5C_2$

$$= \frac{5!}{2!3!} = 10$$

Since there are 21 different consonants so the number of ways of selecting two different consonants = ${}^{21}C_2$

$$= \frac{21!}{2!19!} = 210$$

Therefore, the number of combinations of 2 vowels and 2 consonants $= 10 \times 210 = 2100$

Each of these 2100 combinations has 4 letters and these 4 letters arrange among themselves in $4!$ways.

Hence, the required number of words $= 210 \times 4! = 50400$

Question:7 In an examination, a question paper consists of 12 questions divided into two parts i.e., Part I and Part II, containing 5 and 7 questions, respectively. A student is required to attempt 8 questions in all, selecting at least 3 from each part. In how many ways can a student select the questions?

Answer:

It is given that a question paper consists of 12 questions divided in two parts i.e., Part I and Part II, containing 5 and 7 questions, respectively.

A student is required to attempt 8 questions in all, selecting at least 3 from each part.

This can be done as follows:

(i) 3 questions from part I and 5 questions from part II

(ii) 4 questions from part I and 4 questions from part II

(iii) 5 questions from part I and 3 questions from part II

3 questions from part I and 5 questions from part II can be selected in ${}^5C_3 . {}^7C_5$ ways.

4 questions from part I and 4 questions from part II can be selected in ${}^5C_4 . {}^7C_4$ ways.

5 questions from part I and 3 questions from part II can be selected in ${}^5C_5 . {}^7C_3$ ways.

Hence, required number of ways of selecting questions :

$$={}^5C_3 . {}^7C_5 + {}^5C_4 . {}^7C_4 + {}^5C_5 . {}^7C_3$$

$= 210 + 175 + 35$

$= 420$

Question:8 Determine the number of 5-card combinations out of a deck of 52 cards if each selection of 5 cards has exactly one king.

Answer:

From a deck of 52 cards, 5 cards combinations have to be made in such a way that in each selection of 5 cards there is exactly 1 king.

Number of kings =4

Number of ways of selecting 1 king $= {}^4C_1$

4 cards from the remaining 48 cards are selected in ${}^{48}C_4$ ways.

Thus, the required number of 5 card combinations $= {}^4C_1 . {}^{48}C_4$

Question:9 It is required to seat 5 men and 4 women in a row so that the women occupy the even places. How many such arrangements are possible?

Answer:

It is required to seat 5 men and 4 women in a row so that the women occupy the even places.

The 5 men can be seated in $5!$ ways.

4 women can be seated at cross marked places (so that women occupy even places)

Therefore, women can be seated in $4!$ ways.

Thus, the possible arrangements $= 5! \times 4! = 120 \times 24 = 2880$

Question:10 From a class of 25 students, 10 are to be chosen for an excursion party. There are 3 students who decide that either all of them will join or none of them will join. In how many ways can the excursion party be chosen ?

Answer:

From a class of 25 students, 10 are to be chosen for an excursion party.

There are 3 students who decide that either all of them will join or none of them will join, there are two cases :

The case I: All 3 off them join.

Then, the remaining 7 students can be chosen from 22 students in ${}^{22}C_7$ ways.

Case II : All 3 of them do not join.

Then,10 students can be chosen from 22 students in ${}^{22}C_{10}$ ways.

Thus, the required number of ways for the excursion of party $= {}^{22}C_7 + {}^{22}C_{10}$

Question:11 In how many ways can the letters of the word ASSASSINATION be arranged so that all the S's are together?

Answer:

In the word ASSASSINATION, we have

number of S =4

number of A =3

number of I= 2

number of N =2

Rest of letters appear at once.

Since all words have to be arranged in such a way that all the S are together so we can assume SSSS as an object.

The single object SSSS with other 9 objects is counted as 10.

These 10 objects can be arranged in (we have 3 A's,2 I's,2 N's)

$=\frac{10!}{3!2!2!}$ ways.

Hence, requires the number of ways of arranging letters

$=\frac{10!}{3!2!2!}=151200$