NCERT solutions for class 11 maths chapter 4 principle of mathematical induction

Question:1 Prove the following by using the principle of mathematical induction for all $n \in N$: $1+3+3^2+...+3^{n-1}=\frac{(3^n-1)}{2}$

Answer:

Let the given statement be **p(n)** i.e.

$p(n): 1+3+3^2+...+3^{n-1}=\frac{(3^n-1)}{2}$

For **n = 1** we have

$p(1): 1=\frac{(3^1-1)}{2}=\frac{3-1}{2}=\frac{2}{2}=1$, which is true

For **n = k** we have

$p(k): 1+3+3^2+...+3^{k-1}=\frac{(3^k-1)}{2} \qquad -(i)$ **,** Let's assume that this statement is true

Now,

For **n = k + 1** we have

$p(k+1): 1+3+3^2+...+3^{k+1-1}=1+3+3^2+...+3^{k-1}+3^k$

$=(1+3+3^2+...+3^{k-1})+3^k$

$=\frac{3^k-1+2.3^k}{2}$

$=\frac{3^k(1+2)-1}{2}$

$=\frac{3.3^k-1}{2}$

$=\frac{3^{k+1}-1}{2}$

Thus, **p(k+1)** is true whenever **p(k)** is true

Hence, by the principle of mathematical induction, statement **p(n)** is true for all natural numbers **n**

Question:2 Prove the following by using the principle of mathematical induction for all $n \in N$: $1^3 + 2^3 + 3^3 + ... + n^3 = \left(\frac{n(n+1)}{2}\right)^2$

Answer:

Let the given statement be **p(n)** i.e.

$$p(n) : 1^3 + 2^3 + 3^3 + ... + n^3 = \left(\frac{n(n+1)}{2}\right)^2$$

For **n = 1** we have

$$p(1) : 1 = \left(\frac{1(1+1)}{2}\right)^2 = \left(\frac{1(2)}{2}\right)^2 = (1)^2 = 1$$, which is true

For **n = k** we have

$$p(k) : 1^3 + 2^3 + 3^3 + ... + k^3 = \left(\frac{k(k+1)}{2}\right)^2 \qquad -(i)$$ **, Let's assume that this statement is true**

Now,

For **n = k + 1** we have

$$p(k+1) : 1^3 + 2^3 + 3^3 + ... + (k+1)^3 = 1^3 + 2^3 + 3^3 + ... + k^3 + (k+1)^3$$
$$= (1^3 + 2^3 + 3^3 + ... + k^3) + (k+1)^3$$
$$= \left(\frac{k(k+1)}{2}\right)^2 + (k+1)^3 \qquad (using\ (i))$$
$$= \frac{k^2(k+1)^2 + 4(k+1)^3}{4}$$

$$= \frac{(k+1)^2(k^2+4(k+1))}{4}$$
$$= \frac{(k+1)^2(k^2+4k+4)}{4}$$
$$= \frac{(k+1)^2(k+2)^2}{4} \qquad (\because a^2+2ab+b^2=(a+b)^2)$$
$$= \left(\frac{(k+1)(k+2)}{2}\right)^2$$

Thus, **p(k+1)** is true whenever **p(k)** is true

Hence, by the principle of mathematical induction, statement **p(n)** is true for all natural numbers **n**

Question:3 Prove the following by using the principle of mathematical induction for all $n \in N$:

$$1+\frac{1}{(1+2)}+\frac{1}{(1+2+3)}+...+\frac{1}{(1+2+3+...+n)}=\frac{2n}{(n+1)}$$

Answer:

Let the given statement be **p(n)** i.e.

$$p(n): 1+\frac{1}{(1+2)}+\frac{1}{(1+2+3)}+...+\frac{1}{(1+2+3+...+n)}=\frac{2n}{(n+1)}$$

For **n = 1** we have

$$p(1): 1=\left(\frac{2(1)}{1+1}\right)=\left(\frac{2}{2}\right)=1$$, which is true

For **n = k** we have

$$p(k): 1+\frac{1}{(1+2)}+\frac{1}{(1+2+3)}+...+\frac{1}{(1+2+3+...+k)}=\frac{2k}{(k+1)} \quad -(i)$$, Let's assume that this statement is true

Now,

For **n = k + 1** we have

$$p(k+1): 1+\frac{1}{(1+2)}+\frac{1}{(1+2+3)}+...+\frac{1}{(1+2+3+...+k+1)}$$

$$=\left(1+\frac{1}{(1+2)}+\frac{1}{(1+2+3)}+...+\frac{1}{(1+2+3+...+k)}\right)+\frac{1}{(1+2+3+...+k+k+1)}$$

$$=\frac{2k}{k+1}+\frac{1}{(1+2+3+...+k+(k+1))} \qquad (using\ (i))$$

$$=\frac{2k}{k+1}+\frac{2}{(k+1)(k+2)}$$

$$=\frac{2}{k+1}\left(k+\frac{1}{k+2}\right)$$

$$=\frac{2}{k+1}\left(\frac{k^2+2k+1}{k+2}\right)$$

$$=\frac{2}{k+1}.\frac{(k+1)^2}{k+2}$$

$$=\frac{2(k+1)}{k+2}$$

Thus, **p(k+1)** is true whenever **p(k)** is true

Hence, by the principle of mathematical induction, statement **p(n)** is true for all natural numbers **n**

Question:4 Prove the following by using the principle of mathematical induction for all $n \in N$: $1.2.3+2.3.4+...+n(n+1)(n+2)=\frac{n(n+1)(n+2)(n+3)}{4}$

Answer:

Let the given statement be **p(n)** i.e.

$$p(n): 1.2.3+2.3.4+...+n(n+1)(n+2)=\frac{n(n+1)(n+2)(n+3)}{4}$$

For **n = 1** we have

$p(1) : 6 = \left(\frac{1(1+1)(1+2)(1+3)}{4}\right) = \left(\frac{1.2.3.4}{4}\right) = 6$, which is true

For **n = k** we have

$p(k) : 1.2.3 + 2.3.4 + ... + k(k+1)(k+2) = \frac{k(k+1)(k+2)(k+3)}{4} \quad -(i)$ **, Let's** assume that this statement is true

Now,

For **n = k + 1** we have

$$p(k+1) : 1.2.3 + 2.3.4 + ... + k(k+1)(k+2) + (k+1)(k+2)(k+3)$$
$$= (1.2.3 + 2.3.4 + ... + k(k+1)(k+2)) + (k+1)(k+2)(k+3)$$

$$= \frac{k(k+1)(k+2)(k+3)}{4} + (k+1)(k+2)(k+3) \qquad (using\ (i))$$
$$= \frac{k(k+1)(k+2)(k+3) + 4(k+1)(k+2)(k+3)}{4}$$
$$= \frac{(k+1)(k+2)(k+3)(k+4)}{4}$$

Thus, **p(k+1)** is true whenever **p(k)** is true

Hence, by the principle of mathematical induction, statement **p(n)** is true for all natural numbers **n**

Question:5 Prove the following by using the principle of mathematical induction for all $n \in \mathbb{N}$: $1.3 + 2.3^2 + 3.3^3 + ... + n.3^n = \frac{(2n-1)3^{n+1} + 3}{4}$

Answer:

Let the given statement be **p(n)** i.e.

$$p(n): 1.3 + 2.3^2 + 3.3^3 + ... + n.3^n = \frac{(2n-1)3^{n+1}+3}{4}$$

For **n = 1** we have

$p(1): 3 = \frac{(2(1)-1)3^{1+1}+3}{4} = \frac{(2-1)9+3}{4} = \frac{12}{4} = 3$, which is true

For **n = k** we have

$p(k): 1.3 + 2.3^2 + 3.3^3 + ... + k.3^k = \frac{(2k-1)3^{k+1}+3}{4} \qquad -(i)$, Let's assume that this statement is true

Now,

For **n = k + 1** we have

$$p(k+1): 1.3 + 2.3^2 + 3.3^3 + ... + (k+1).3^{(k+1)}$$
$$= 1.3 + 2.3^2 + 3.3^3 + ... + k.3^k + (k+1).3^{(k+1)}$$

$$= \frac{(2k-1)3^{k+1}+3}{4} + (k+1).3^{(k+1)}$$
$$= \frac{(2k-1)3^{k+1}+3+4(k+1).3^{(k+1)}}{4}$$
$$= \frac{3^{k+1}((2k-1)+4(k+1))+3}{4}$$
$$= \frac{3^{k+1}(6k+3)+3}{4}$$
$$= \frac{3^{k+1}.3(2k+1)+3}{4}$$
$$= \frac{(2k+1)3^{k+2}+3}{4}$$
$$= \frac{(2(k+1)-1)3^{(k+1)+1}+3}{4}$$

Thus, **p(k+1)** is true whenever **p(k)** is true

Hence, by the principle of mathematical induction, statement **p(n)** is true for all natural numbers **n**

Question:6 Prove the following by using the principle of mathematical induction for all $n \in \mathbb{N}$: $1.2 + 2.3 + 3.4 + ... + n.(n+1) = \left[\frac{n(n+1)(n+2)}{3}\right]$

Answer:

Let the given statement be **p(n)** i.e.

$$p(n) : 1.2 + 2.3 + 3.4 + ... + n.(n+1) = \left[\frac{n(n+1)(n+2)}{3}\right]$$

For **n = 1** we have

$p(1) : 2 = \left[\frac{1(1+1)(1+2)}{3}\right] = \frac{1.2.3}{3} = 2$, which is true

For **n = k** we have

$p(k) : 1.2 + 2.3 + 3.4 + ... + k.(k+1) = \left[\frac{k(k+1)(k+2)}{3}\right] \qquad -(i)$, Let's assume that this statement is true

Now,

For **n = k + 1** we have

$$p(k+1) : 1.2 + 2.3 + 3.4 + ... + (k+1).(k+2)$$

$$= 1.2 + 2.3 + 3.4 + ... + k(k+1) + (k+1).(k+2)$$

$$= \frac{k(k+1)(k+2)}{3} + (k+1).(k+2) \qquad (using\ (i))$$

$$= \frac{k(k+1)(k+2)+3(k+1).(k+2)}{3}$$
$$= \frac{(k+1)(k+2)(k+3)}{3}$$
$$= \frac{(k+1)(k+1+1)(k+1+2)}{3}$$

Thus, **p(k+1)** is true whenever **p(k)** is true

Hence, by principle of mathematical induction , statement **p(n)** is true for all natural numbers **n**

Question:7 Prove the following by using the principle of mathematical induction for all $n \in \mathbb{N}$: $1.3+3.5+5.7+...+(2n-1)(2n+1) = \frac{n(4n^2+6n-1)}{3}$

Answer:

Let the given statement be **p(n)** i.e.

$$p(n): 1.3+3.5+5.7+...+(2n-1)(2n+1) = \frac{n(4n^2+6n-1)}{3}$$

For **n = 1** we have

$p(1): 1.3 = 3 = \frac{1(4(1)^2+6(1)-1)}{3} = \frac{4+6-1}{3} = \frac{9}{3} = 3$, which is true

For **n = k** we have

$p(k): 1.3+3.5+5.7+...+(2k-1)(2k+1) = \frac{k(4k^2+6k-1)}{3} \quad -(i)$, Let's assume that this statement is true

Now,

For **n = k + 1** we have

$$p(k+1): 1.3+3.5+5.7+...+(2(k+1)-1)(2(k+1)+1)$$

$= 1.3 + 3.5 + 5.7 + ... + (2k-1)(2k+1) + (2(k+1)-1)(2(k+1)+1)$

$$= \frac{k(4k^2+6k-1)}{3} + (2k+1)(2k+3) \qquad (using\ (i))$$
$$= \frac{k(4k^2+6k-1)+3(2k+1)(2k+3)}{3}$$
$$= \frac{k(4k^2+6k-1)+3(4k^2+8k+3)}{3}$$
$$= \frac{(4k^3+6k^2-k+12k^2+28k+9)}{3}$$
$$= \frac{(4k^3+18k^2+23k+9)}{3}$$
$$= \frac{(4k^3+14k^2+9k+4k^2+14k+9)}{3}$$
$$= \frac{(k(4k^2+14k+9)+4k^2+14k+9)}{3}$$
$$= \frac{(4k^2+14k+9)(k+1)}{3}$$
$$= \frac{(k+1)(4k^2+8k+4+6k+6-1)}{3}$$
$$= \frac{(k+1)(4(k^2+2k+1)+6(k+1)-1)}{3}$$
$$= \frac{(k+1)(4(k+1)^2+6(k+1)-1)}{3}$$

Thus, **p(k+1)** is true whenever **p(k)** is true

Hence, by the principle of mathematical induction, statement **p(n)** is true for all natural numbers **n**

Question:8 Prove the following by using the principle of mathematical induction for all $n \in \mathbb{N}: 1.2 + 2.2^2 + 3.2^3 + ... + n.2^n = (n-1)2^{n+1} + 2$

Answer:

Let the given statement be **p(n)** i.e.

$p(n): 1.2 + 2.2^2 + 3.2^3 + ... + n.2^n = (n-1)2^{n+1} + 2$

For **n = 1** we have

$p(1): 1.2 = 2 = (1-1)2^{1+1} + 2 = 2$, which is true

For **n = k** we have

$p(k): 1.2 + 2.2^2 + 3.2^3 + ... + k.2^k = (k-1)2^{k+1} + 2 \qquad -(i)$ **,** Let's assume that this statement is true

Now,

For **n = k + 1** we have

$p(k+1): 1.2 + 2.2^2 + 3.2^3 + ... + (k+1).2^{k+1}$

$= 1.2 + 2.2^2 + 3.2^3 + ... + k.2^k + (k+1).2^{k+1}$

$= (k-1)2^{k+1} + 2 + (k+1).2^{k+1} \qquad (using\ (i))$

$= 2^{k+1}(k-1+k+1) + 2$

$= 2^{k+1}(2k) + 2$

$= k.2^{k+2} + 2$

$= (k+1-1).2^{k+1+1} + 2$

Thus, **p(k+1)** is true whenever **p(k)** is true

Hence, by the principle of mathematical induction, statement **p(n)** is true for all natural numbers **n**

Question:9 Prove the following by using the principle of mathematical induction for all $n \in \mathbb{N}: \frac{1}{2}+\frac{1}{4}+\frac{1}{8}+...+\frac{1}{2^n}=1-\frac{1}{2^n}$

Answer:

Let the given statement be **p(n)** i.e.
$p(n): \frac{1}{2}+\frac{1}{4}+\frac{1}{8}+...+\frac{1}{2^n}=1-\frac{1}{2^n}$

For **n = 1** we have
$p(1): \frac{1}{2}=1-\frac{1}{2^1}=1-\frac{1}{2}=\frac{1}{2}$, which is true

For **n = k** we have

$p(k): \frac{1}{2}+\frac{1}{4}+\frac{1}{8}+...+\frac{1}{2^k}=1-\frac{1}{2^k} \qquad -(i)$ **, Let's assume that this statement is true**

Now,

For **n = k + 1** we have
$p(k+1): \frac{1}{2}+\frac{1}{4}+\frac{1}{8}+...+\frac{1}{2^{k+1}}=\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+...+\frac{1}{2^k}+\frac{1}{2^{k+1}}$

$$=1-\frac{1}{2^k}+\frac{1}{2^{k+1}} \qquad (using\ (i))$$
$$=1-\frac{1}{2^k}\left(1-\frac{1}{2}\right)$$
$$=1-\frac{1}{2^k}\left(\frac{1}{2}\right)$$
$$=1-\frac{1}{2^{k+1}}$$

Thus, **p(k+1)** is true whenever **p(k)** is true

Hence, by the principle of mathematical induction, statement **p(n)** is true for all natural numbers **n**

Question:10 Prove the following by using the principle of mathematical induction for all $n \in \mathbb{N}$: $\frac{1}{2.5}+\frac{1}{5.8}+\frac{1}{8.11}+...+\frac{1}{(3n-1)(3n+2)}=\frac{n}{(6n+4)}$

Answer:

Let the given statement be **p(n)** i.e.

$$p(n): \frac{1}{2.5}+\frac{1}{5.8}+\frac{1}{8.11}+...+\frac{1}{(3n-1)(3n+2)}=\frac{n}{(6n+4)}$$

For **n = 1** we have

$p(1): \frac{1}{2.5}=\frac{1}{10}=\frac{1}{(6(1)+4)}=\frac{1}{10}$, which is true

For **n = k** we have

, Let's assume that this statement is true

Now,

For **n = k + 1** we have

$$p(k+1): \frac{1}{2.5}+\frac{1}{5.8}+\frac{1}{8.11}+...+\frac{1}{(3(k+1)-1)(3(k+1)+2)}$$

$$=\frac{1}{2.5}+\frac{1}{5.8}+\frac{1}{8.11}+...+\frac{1}{(3k-1)(3k+2)}+\frac{1}{(3(k+1)-1)(3(k+1)+2)}$$

$$=\frac{k}{6k+4}+\frac{1}{(3k+2)(3k+5)} \qquad (using\ (i))$$

$$=\frac{1}{3k+2}\left(\frac{k}{2}+\frac{1}{3k+5}\right)$$

$$= \frac{1}{3k+2}\left(\frac{k(3k+5)+2}{2(3k+5)}\right)$$
$$= \frac{1}{3k+2}\left(\frac{3k^2+5k+2}{2(3k+5)}\right)$$
$$= \frac{1}{3k+2}\left(\frac{3k^2+3k+2k+2}{2(3k+5)}\right)$$
$$= \frac{1}{3k+2}\left(\frac{(3k+2)(k+1)}{2(3k+5)}\right)$$
$$= \frac{(k+1)}{6k+10}$$
$$= \frac{(k+1)}{6(k+1)+4}$$

Thus, **p(k+1)** is true whenever **p(k)** is true

Hence, by the principle of mathematical induction, statement **p(n)** is true for all natural numbers **n**

Question:11 Prove the following by using the principle of mathematical induction for all $n \in \mathbb{N}$: $\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}+...+\frac{1}{n(n+1)(n+2)}=\frac{n(n+3)}{4(n+1)(n+2)}$

Answer:

Let the given statement be **p(n)** i.e.

$$p(n): \frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}+...+\frac{1}{n(n+1)(n+2)}=\frac{n(n+3)}{4(n+1)(n+2)}$$

For **n = 1** we have

$p(1): \frac{1}{1.2.3}=\frac{1}{6}=\frac{1(1+3)}{4(1+1)(1+2)}=\frac{4}{4.2.3}=\frac{1}{6}$, which is true

For **n = k** we have

, Let's assume that this statement is true

Now,

For **n = k + 1** we have

$$p(k+1):\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}+...+\frac{1}{(k+1)(k+2)(k+3)}$$
$$=\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}+...+\frac{1}{k(k+1)(k+2)}+\frac{1}{(k+1)(k+2)(k+3)}$$

$$=\frac{k(k+3)}{4(k+1)(k+2)}+\frac{1}{(k+1)(k+2)(k+3)} \qquad (using\ (i))$$
$$=\frac{1}{(k+1)(k+2)}\left(\frac{k(k+3)}{4}+\frac{1}{k+3}\right)$$
$$=\frac{1}{(k+1)(k+2)}\left(\frac{k(k+3)^2+4}{4(k+3)}\right)$$
$$=\frac{1}{(k+1)(k+2)}\left(\frac{k(k^2+9+6k)+4}{4(k+3)}\right)$$
$$=\frac{1}{(k+1)(k+2)}\left(\frac{k^3+9k+6k^2+4}{4(k+3)}\right)$$
$$=\frac{1}{(k+1)(k+2)}\left(\frac{k^3+2k^2+k+8k+4k^2+4}{4(k+3)}\right)$$
$$=\frac{1}{(k+1)(k+2)}\left(\frac{k(k^2+2k+1)+4(k^2+2k+1)}{4(k+3)}\right)$$
$$=\frac{1}{(k+1)(k+2)}\left(\frac{(k+1)^2(k+4)}{4(k+3)}\right)$$
$$=\frac{(k+1)((k+1)+3)}{4(k+1+1)(k+1+2)}$$

Thus, **p(k+1)** is true whenever **p(k)** is true

Hence, by the principle of mathematical induction, statement **p(n)** is true for all natural numbers **n**

Question:12 Prove the following by using the principle of mathematical induction for all $n \in \mathbb{N}$: $a + ar + ar^2 + ... + ar^{n-1} = \frac{a(r^n - 1)}{r - 1}$

Answer:

Let the given statement be **p(n)** i.e.

$p(n) : a + ar + ar^2 + ... + ar^{n-1} = \frac{a(r^n - 1)}{r - 1}$

For **n = 1** we have

$p(1) : a = \frac{a(r^1 - 1)}{r - 1} = \frac{r - 1}{r - 1} = 1$, which is true

For **n = k** we have

$p(k) : a + ar + ar^2 + ... + ar^{k-1} = \frac{a(r^k - 1)}{r - 1} \qquad -(i)$, Let's assume that this statement is true

Now,

For **n = k + 1** we have

$p(k + 1) : a + ar + ar^2 + ... + ar^k = a + ar + ar^2 + ... + ar^{k-1} + ar^k$

$= a.\frac{r^k - 1}{r - 1} + ar^k \qquad (using\ (i))$

$= \frac{a(r^k - 1) + (r - 1)ar^k}{r - 1}$

$= \frac{ar^k(1 + r - 1) - a}{r - 1}$

$= \frac{ar^k.r - a}{r - 1}$

$= \frac{a(r^{k+1} - 1)}{r - 1}$

Thus, **p(k+1)** is true whenever **p(k)** is true

Hence, by the principle of mathematical induction, statement **p(n)** is true for all natural numbers **n**

Question:13 Prove the following by using the principle of mathematical induction for all $n \in \mathbb{N}$:

Answer:

Let the given statement be **p(n)** i.e.

For **n = 1** we have

$$p(1) : \left(1 + \frac{3}{1}\right) = 4 = (1+1)^2 = 2^2 = 4$$, which is true

For **n = k** we have

, Let's assume that this statement is true

Now,

For **n = k + 1** we have

$$= (k+1)^2\left(1 + \frac{(2(k+1)+1)}{(k+1)^2}\right) \qquad (using\ (i))$$

$$= (k+1)^2\left(\frac{(k+1)^2 + (2(k+1)+1)}{(k+1)^2}\right)$$

$= (k^2 + 1 + 2k + 2k + 2 + 1)$
$= (k^2 + 4k + 4)$
$= (k + 2)^2$
$= (k + 1 + 1)^2$

Thus, **p(k+1)** is true whenever **p(k)** is true

Hence, by the principle of mathematical induction, statement **p(n)** is true for all natural numbers **n**

Question:14 Prove the following by using the principle of mathematical induction for all $n \in \mathbb{N}$:

Answer:

Let the given statement be **p(n)** i.e.

For **n = 1** we have
$p(1) : \left(1 + \frac{1}{1}\right) = 2 = (1 + 1) = 2$, which is true

For **n = k** we have

, Let's assume that this statement is true

Now,

For **n = k + 1** we have

&nbsnbsp;

$= (k+1)\left(1+\frac{1}{k+1}\right) \qquad (using\ (i))$

$= (k+1)\left(\frac{k+1+1}{k+1}\right)$

$= (k+2)$

$= (k+1+1)$

Thus, **p(k+1)** is true whenever **p(k)** is true

Hence, by the principle of mathematical induction, statement **p(n)** is true for all natural numbers **n**

Question:15 Prove the following by using the principle of mathematical induction for all $n \in \mathbb{N}$: $1^2+3^2+5^2+...+(2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$

Answer:

Let the given statement be **p(n)** i.e.

$p(n) : 1^2+3^2+5^2+...+(2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$

For **n = 1** we have

$p(1) : 1^2 = 1 = \frac{1(2(1)-1)(2(1)+1)}{3} = \frac{1.1.3}{3} = 1$, which is true

For **n = k** we have

$p(k) : 1^2+3^2+5^2+...+(2k-1)^2 = \frac{k(2k-1)(2k+1)}{3} \qquad -(i)$ **, Let's** assume that this statement is true

Now,

For **n = k + 1** we have

$$p(k+1) : 1^2 + 3^2 + 5^2 + ... + (2(k+1)-1)^2$$
$$= 1^2 + 3^2 + 5^2 + ... + (2k-1)^2 + (2(k+1)-1)^2$$

$$= \frac{k(2k-1)(2k+1)}{3} + (2(k+1)-1)^2 \qquad (using\ (i))$$
$$= \frac{k(2k-1)(2k+1) + 3(2(k+1)-1)^2}{3}$$
$$= \frac{k(2k-1)(2k+1) + 3(2k+1)^2}{3}$$
$$= \frac{(2k+1)(k(2k-1) + 3(2k+1))}{3}$$
$$= \frac{(2k+1)(2k^2 - k + 6k + 3)}{3}$$
$$= \frac{(2k+1)(2k^2 + 5k + 3)}{3}$$
$$= \frac{(2k+1)(2k^2 + 2k + 3k + 3)}{3}$$
$$= \frac{(2k+1)(2k+3)(k+1)}{3}$$
$$= \frac{(k+1)(2(k+1)-1)(2(k+1)+1)}{3}$$

Thus, **p(k+1)** is true whenever **p(k)** is true

Hence, by the principle of mathematical induction, statement **p(n)** is true for all natural numbers **n**

Question:16 Prove the following by using the principle of mathematical induction for all $n \in \mathbb{N}$: $\frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + ... + \frac{1}{(3n-2)(3n+1)} = \frac{n}{(3n+1)}$

Answer:

Let the given statement be **p(n)** i.e.

$$p(n): \frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+...+\frac{1}{(3n-2)(3n+1)}=\frac{n}{(3n+1)}$$

For **n = 1** we have

$p(1): \frac{1}{1.4}=\frac{1}{4}=\frac{1}{(3(1)+1)}=\frac{1}{3+1}=\frac{1}{4}$, which is true

For **n = k** we have

, Let's assume that this statement is true

Now,

For **n = k + 1** we have

$$p(k+1): \frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+...+\frac{1}{(3(k+1)-2)(3(k+1)+1)}$$

$$=\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+...+\frac{1}{(3k-2)(3k+1)}+\frac{1}{(3(k+1)-2)(3(k+1)+1)}$$

$$=\frac{k}{3k+1}+\frac{1}{(3k+1)(3k+4)} \qquad (using\ (i))$$

$$=\frac{1}{3k+1}\left(k+\frac{1}{3k+4}\right)$$

$$=\frac{1}{3k+1}\left(\frac{k(3k+4)+1}{3k+4}\right)$$

$$=\frac{1}{3k+1}\left(\frac{3k^2+4k+1}{3k+4}\right)$$

$$=\frac{1}{3k+1}\left(\frac{3k^2+3k+k+1}{3k+4}\right)$$

$$=\frac{1}{3k+1}\left(\frac{(3k+1)(k+1)}{3k+4}\right)$$

$$=\frac{(k+1)}{3k+4}$$

$$= \frac{(k+1)}{3(k+1)+1}$$

Thus, **p(k+1)** is true whenever **p(k)** is true

Hence, by the principle of mathematical induction, statement **p(n)** is true for all natural numbers **n**

Question:17 Prove the following by using the principle of mathematical induction for all $n \in N$: $\frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + ... + \frac{1}{(2n+1)(2n+3)} = \frac{n}{3(2n+3)}$

Answer:

Let the given statement be **p(n)** i.e.

$$p(n) : \frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + ... + \frac{1}{(2n+1)(2n+3)} = \frac{n}{3(2n+3)}$$

For **n = 1** we have

$p(1) : \frac{1}{3.5} = \frac{1}{15} = \frac{1}{3(2(1)+3)} = \frac{1}{3.5} = \frac{1}{15}$, which is true

For **n = k** we have

, Let's assume that this statement is true

Now,

For **n = k + 1** we have

$$p(k+1) : \frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + ... + \frac{1}{(2(k+1)+1)(2(k+1)+3)}$$

$$= \frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + ... + \frac{1}{(2k+1)(2k+3)} + \frac{1}{(2(k+1)+1)(2(k+1)+3)}$$

$$= \frac{k}{3(2k+3)} + \frac{1}{(2k+3)(2k+5)} \qquad (using\ (i))$$
$$= \frac{1}{2k+3}\left(\frac{k}{3} + \frac{1}{2k+5}\right)$$
$$= \frac{1}{2k+3}\left(\frac{k(2k+5)+3}{3(2k+5)}\right)$$
$$= \frac{1}{2k+3}\left(\frac{2k^2+5k+3}{3(2k+5)}\right)$$
$$= \frac{1}{2k+3}\left(\frac{2k^2+2k+3k+3}{3(2k+5)}\right)$$
$$= \frac{1}{2k+3}\left(\frac{(2k+3)(k+1)}{3(2k+5)}\right)$$
$$= \frac{(k+1)}{3(2k+5)}$$
$$= \frac{(k+1)}{3(2(k+1)+3)}$$

Thus, **p(k+1)** is true whenever **p(k)** is true

Hence, by the principle of mathematical induction, statement **p(n)** is true for all natural numbers **n**

Question:18 Prove the following by using the principle of mathematical induction for all $n \in \mathbb{N}$: $1+2+3+...+n < \frac{1}{8}(2n+1)^2.$

Answer:

Let the given statement be **p(n)** i.e.
$p(n): 1+2+3+...+n < \frac{1}{8}(2n+1)^2.$

For **n = 1** we have

$p(1) : 1 < \frac{1}{8}(2(1)+1)^2 = \frac{1}{8}(3)^2 = \frac{9}{8}$, which is true

For **n = k** we have

$p(k) : 1+2+3+...+k < \frac{1}{8}(2k+1)^2 \qquad -(i)$ **,** Let's assume that this statement is true

Now,

For **n = k + 1** we have

$p(k+1) : 1+2+3+...+k+1 = 1+2+3+...+k+k+1$

$$< \frac{1}{8}(2k+1)^2 + (k+1) \qquad (using\ (i))$$
$$< \frac{1}{8}\left((2k+1)^2 + 8(k+1)\right)$$
$$< \frac{1}{8}\left(4k^2+4k+1+8k+8\right)$$
$$< \frac{1}{8}\left(4k^2+12k+9\right)$$
$$< \frac{1}{8}(2k+3)^2$$
$$< \frac{1}{8}(2(k+1)+1)^2$$

Thus, **p(k+1)** is true whenever **p(k)** is true

Hence, by the principle of mathematical induction, statement **p(n)** is true for all natural numbers **n**

Question:19 Prove the following by using the principle of mathematical induction for all $n \in \mathbb{N}$: $n(n+1)(n+5)$ is a multiple of 3.

Answer:

Let the given statement be **p(n)** i.e.

$p(n) : n(n+1)(n+5)$

For **n = 1** we have

$p(1) : 1(1+1)(1+5) = 1.2.6 = 12$, which is multiple of 3, hence true

For **n = k** we have

$p(k) : k(k+1)(k+5) \qquad -(i)$ **,** Let's assume that this is multiple of 3 = 3m

Now,

For **n = k + 1** we have

$p(k+1) : (k+1)((k+1)+1)((k+1)+5) = (k+1)(k+2)((k+5)+1)$

$= (k+1)(k+2)(k+5) + (k+1)(k+2)$

$= k(k+1)(k+5) + 2(k+1)(k+5) + (k+1)(k+2)$

$= 3m + 2(k+1)(k+5) + (k+1)(k+2) \qquad (using\ (i))$

$= 3m + (k+1)(2(k+5) + (k+2)) \qquad (using\ (i))$

$= 3m + (k+1)(2k+10+k+2)$

$= 3m + (k+1)(3k+12)$

$= 3m + 3(k+1)(k+4)$

$= 3(m + (k+1)(k+4))$

$= 3l$ **Where** $(l = (m + (k+1)(k+4)))$ **some natural number**

Thus, **p(k+1)** is true whenever **p(k)** is true

Hence, by the principle of mathematical induction, statement **p(n)** is multiple of 3 for all natural numbers **n**

Question:20 Prove the following by using the principle of mathematical induction for all $n \in N : 10^{2n-1} + 1$ is a divisible by $11.$

Answer:

Let the given statement be **p(n)** i.e.

$p(n) : 10^{2n-1} + 1$

For **n = 1** we have

$p(1) : 10^{2(1)-1} + 1 = 10^{2-1} + 1 = 10^1 + 1 = 11$, which is divisible by 11, hence true

For **n = k** we have

$p(k) : 10^{2k-1} + 1 \qquad - (i)$ **,** Let's assume that this is divisible by 11 = 11m

Now,

For **n = k + 1** we have

$p(k+1) : 10^{2(k+1)-1} + 1 = 10^{2k+2-1} + 1$

nbsp;

$= 10^{2k+1} + 1$

$= 10^2(10^{2k-1} + 1 - 1) + 1$

$= 10^2(10^{2k-1} + 1) - 10^2 + 1$

$= 10^2(11m) - 100 + 1 \qquad (using\ (i))$

$= 100(11m) - 99$

$= 11(100m - 9)$

$= 11l$ **Where** $l = (100m - 9)$ **some natural number**

Thus, **p(k+1)** is true whenever **p(k)** is true

Hence, by the principle of mathematical induction, statement **p(n)** is divisible by 11 for all natural numbers **n**

Question:21 Prove the following by using the principle of mathematical induction for all $n \in \mathbb{N}$: $x^{2n} - y^{2n}$ is divisible by $x + y$.

Answer:

Let the given statement be **p(n)** i.e.

$p(n) : x^{2n} - y^{2n}$

For **n = 1** we have

$p(1) : x^{2(1)} - y^{2(1)} = x^2 - y^2 = (x - y)(x + y)$, which is divisible by $(x + y)$, hence true $(using\ a^2 - b^2 = (a + b)(a - b))$

For **n = k** we have

$p(k) : x^{2k} - y^{2k} \qquad - (i)$ **,** Let's assume that this is divisible by $(x + y) = (x + y)m$

Now,

For **n = k + 1** we have

$p(k + 1) : x^{2(k+1)} - y^{2(k+1)} = x^{2k}.x^2 - y^{2k}.y^2$

$= x^2(x^{2k} + y^{2k} - y^{2k}) - y^{2k}.y^2$

$= x^2(x^{2k} - y^{2k}) + x^2.y^{2k} - y^{2k}.y^2$
$= x^2(x+y)m + (x^2 - y^2)y^{2k} \qquad (using\ (i))$
$= x^2(x+y)m + ((x-y)(x+y))y^{2k} \qquad (using\ a^2 - b^2 = (a+b)(a-b))$
$= (x+y)\left(x^2.m + (x-y).y^{2k}\right)$
$= (x+y)l$ **where** $l = (x^2.m + (x-y).y^{2k})$ **some natural number**

Thus, **p(k+1)** is true whenever **p(k)** is true

Hence, by the principle of mathematical induction, statement **p(n)** is divisible by $(x+y)$ for all natural numbers **n**

Question:22 Prove the following by using the principle of mathematical induction for all $n \in \mathbb{N}$: $3^{2n+2} - 8n - 9$ is divisible by 8.

Answer:

Let the given statement be **p(n)** i.e.
$p(n) : 3^{2n+2} - 8n - 9$

For **n = 1** we have
$p(1) : 3^{2(1)+2} - 8(1) - 9 = 3^4 - 8 - 9 = 81 - 17 = 64 = 8 \times 8$, which is divisible by 8, hence true

For **n = k** we have

$p(k) : 3^{2k+2} - 8k - 9 \qquad -(i)$, Let's assume that this is divisible by **8 = 8m**

Now,

For **n = k + 1** we have
$p(k+1) : 3^{2(k+1)+2} - 8(k+1) - 9 = 3^{2k+2+2} - 8(k+1) - 9$

$= 3^{2k+2}.3^2 - 8k - 8 - 9$

$= 3^2(3^{2k+2} - 8k - 9 + 8k + 9) - 8k - 17$

$= 3^2(3^{2k+2} - 8k - 9) + 3^2(8k + 9) - 8k - 17$

$= 9 \times 8m + 9(8k + 9) - 8k - 17 \qquad (using\ (i))$

$= 9 \times 8m + 72k + 81 - 8k - 17$

$= 9 \times 8m + 80k - 64$

$= 9 \times 8m + 8(10k - 8)$

$= 8(9m + 10k - 8)$

$= 8l$ **where** $l = 9m + 10k - 8$ **some natural number**

Thus, **p(k+1)** is true whenever **p(k)** is true

Hence, by the principle of mathematical induction, statement **p(n)** is divisible by **8** for all natural numbers **n**

Question:23 Prove the following by using the principle of mathematical induction for all $n \in \mathbb{N} : 41^n - 14^n$ is a multiple of 27.

Answer:

Let the given statement be **p(n)** i.e.

$p(n) : 41^n - 14^n$

For **n = 1** we have

$p(1) : 41^1 - 14^1 = 41 - 14 = 27$, which is divisible by 27, hence true

For **n = k** we have

$p(k) : 41^k - 14^k \qquad - (i)$ **,** Let's assume that this is divisible by **27 = 27m**

Now,

For **n = k + 1** we have

$p(k+1) : 41^{k+1} - 14^{k+1} = 41^k.41 - 14^k.14$

$= 41(41^k - 14^k + 14^k) - 14^k.14$

$= 41(41^k - 14^k) + 14^k.41 - 14^k.14$

$= 41(27m) + 14^k(41 - 14) \qquad (using\ (i))$

$= 41(27m) + 14^k.27$

$= 27(41m + 14^k)$

$= 27l$ **where** $l = 41m + 14^k$ **some natural number**

Thus, **p(k+1)** is true whenever **p(k)** is true

Hence, by the principle of mathematical induction, statement **p(n)** is divisible by **27** for all natural numbers **n**