

NCERT solutions for class 11 maths chapter 16 Probability

Question:1 Describe the sample space for the indicated experiment.

A coin is tossed three times.

Answer:

Let H denote Heads and T denote Tails.

For each toss, there are two possible outcomes = H or T

The required sample space is:

$$S = \{HHH, HHT, HTH, THH, TTH, HTT, THT, TTT\}$$

Question:2 . Describe the sample space for the indicated experiment.

A die is thrown two times.

Answer:

When a die is thrown, the possible outcomes are = $\{1, 2, 3, 4, 5, 6\}$

The required sample space is:

$$S = \{(x, y) : x, y = 1, 2, 3, 4, 5, 6\}$$

$$\text{or } S = \{(1,1), (1,2), (1,3), \dots, (1,6), (2,1), (2,2), \dots, (2,6), \dots, (6, 1), (6, 2), \dots, (6,6)\}$$

Question:3 Describe the sample space for the indicated experiment.

A coin is tossed four times.

Answer:

Let H denote Heads and T denote Tails.

For each toss, there are two possible outcomes = H or T

The required sample space is:

$S = \{HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTHT, HTTH, THHT, THTH, TTTH, THTT, TTHT, TTTT\}$

Question:4 Describe the sample space for the indicated experiment.

A coin is tossed and a die is thrown.

Answer:

Let H denote Heads and T denote Tails.

For each toss, there are two possible outcomes = H or T

And,

When a die is thrown, the possible outcomes are = $\{1, 2, 3, 4, 5, 6\}$

The required sample space is:

$S = \{H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6\}$

Question:5 Describe the sample space for the indicated experiment.

A coin is tossed and then a die is rolled only in case a head is shown on the coin.

Answer:

Let H denote Heads and T denote Tails.

For each toss, there are two possible outcomes = H or T

For H, when a die is thrown, the possible outcomes are = {1, 2, 3, 4, 5, 6}

The required sample space is:

$$S = \{H1, H2, H3, H4, H5, H6, T\}$$

Question:6 Describe the sample space for the indicated experiment.

2 boys and 2 girls are in Room X, and 1 boy and 3 girls in Room Y. Specify the sample space for the experiment in which a room is selected and then a person.

Answer:

Let X denote the event Room X is selected, Y denote the event Room Y is selected.

B1, B2 denote the event a boy is selected and G1, G2 denote the event a girl is selected from room X.

B3 denotes the event a boy is selected and G3, G4, G5 denote the event a girl is selected from Room Y.

The required sample space is:

$$S = \{XB1, XB2, XG1, XG2, YB3, YG3, YG4, YG5\}$$

Question:7. Describe the sample space for the indicated experiment.

One die of red colour, one of white colour and one of blue colour are placed in a bag. One die is selected at random and rolled, its colour and the number on its uppermost face is noted. Describe the sample space.

Answer:

Let, R denote the event the red die comes out,

W denote the event the white die comes out,

B denote the event the Blue die is chosen

When a die is thrown, the possible outcomes are = {1, 2, 3, 4, 5, 6}

The required sample space is:

$S = \{R1, R2, R3, R4, R5, R6, W1, W2, W3, W4, W5, W6, B1, B2, B3, B4, B5, B6\}$

Question:8(i) An experiment consists of recording boy–girl composition of families with 2 children.

What is the sample space if we are interested in knowing whether it is a boy or girl in the order of their births?

Answer:

Let B denote the event a boy is born,

G denote the event a girl is born

The required sample space with a boy or girl in the order of their births is:

$$S = \{BB, BG, GB, GG\}$$

Question: 8(ii). An experiment consists of recording boy–girl composition of families with 2 children.

What is the sample space if we are interested in the number of girls in the family?

Answer:

(ii) In a family with two child, there can be only three possible cases:

no girl child, 1 girl child or 2 girl child

The required sample is:

$$S = \{0, 1, 2\}$$

Question:9 A box contains 1 red and 3 identical white balls. Two balls are drawn at random in succession without replacement. Write the sample space for this experiment.

Answer:

Given, Number of red balls =1

Number of white balls = 3

Let R denote the event that the red ball is drawn.

And W denotes the event that a white ball is drawn.

Since two balls are drawn at random in succession without replacement,

if the first ball is red, the second ball will be white. And if the first ball is white, second can be either of red and white

The required sample space is:

$$S = \{RW, WR, WW\}$$

Question:10 An experiment consists of tossing a coin and then throwing it second time if a head occurs. If a tail occurs on the first toss, then a die is rolled once. Find the sample space.

Answer:

Let H denote the event that Head occurs and T denote the event that Tail occurs.

For T in first toss, the possible outcomes when a die is thrown = $\{1, 2, 3, 4, 5, 6\}$

The required sample space is :

$$S = \{HH, HT, T1, T2, T3, T4, T5, T6\}$$

Question:11. Suppose 3 bulbs are selected at random from a lot. Each bulb is tested and classified as defective (D) or non – defective(N). Write the sample space of this experiment.

Answer:

Let D denote the event the bulb is defective and N denote the event the bulb is non-defective

The required sample space is:

$$S = \{DDD, DDN, DND, NDD, DNN, NDN, NND, NNN\}$$

Question:12 A coin is tossed. If the out come is a head, a die is thrown. If the die shows up an even number, the die is thrown again. What is the sample space for the experiment?

Answer:

Possible outcomes when a coin is tossed = {H,T}

Possible outcomes when a die is thrown = {1,2,3,4,5,6}

When T occurs, experiment is finished. $S_1 = \{T\}$

When H occurs, a die is thrown.

If the outcome is odd ({1,3,5}), $S_2 = \{H1, H3, H5\}$

If the outcome is even({2,4,6}), the die is thrown again.,

$S_3 = \{H21, H22, H23, H24, H25, H26, H41, H42, H43, H44, H45, H46, H61, H62, H63, H64, H65, H66\}$

The required sample space is:

$S = \{T, H1, H3, H5, H21, H22, H23, H24, H25, H26, H41, H42, H43, H44, H45, H46, H61, H62, H63, H64, H65, H66\}$

Question:13. The numbers 1, 2, 3 and 4 are written separately on four slips of paper. The slips are put in a box and mixed thoroughly. A person draws two slips from the box, one after the other, without replacement. Describe the sample space for the experiment.

Answer:

Given, two slips are drawn from the box, one after the other, without replacement.

Let 1, 2, 3, 4 denote the event that 1, 2, 3, 4 numbered slip is drawn respectively.

When two slips are drawn without replacement, the first event has 4 possible outcomes and the second event has 3 possible outcomes

$$S = \{(1,2), (1,3), (1,4), (2,1), (2,3), (2,4), (3,1), (3,2), (3,4), (4,1), (4,2), (4,3)\}$$

Question:14. An experiment consists of rolling a die and then tossing a coin once if the number on the die is even. If the number on the die is odd, the coin is tossed twice.

Write the sample space for this experiment.

Answer:

Possible outcomes when a die is thrown = $\{1,2,3,4,5,6\}$

Possible outcomes when a coin is tossed = $\{H,T\}$

If the number on the die is even $\{2,4,6\}$, the coin is tossed once.

$$S_1 = \{2H, 2T, 4H, 4T, 6H, 6T\}$$

If the number on the die is odd $\{1,3,5\}$, the coin is tossed twice.

$$S_2 = \{1HH, 1HT, 1TH, 1TT, 3HH, 3HT, 3TH, 3TT, 5HH, 5HT, 5TH, 5TT\}$$

The required sample space is:

$$S = \{1HH, 1HT, 1TH, 1TT, 2H, 2T, 3HH, 3HT, 3TH, 3TT, 4H, 4T, 5HH, 5HT, 5TH, 5TT, 6H, 6T\}$$

Question:15 A coin is tossed. If it shows a tail, we draw a ball from a box which contains 2 red and 3 black balls. If it shows head, we throw a die. Find the sample space for this experiment.

Answer:

Possible outcomes when a coin is tossed = {H,T}

Possible outcomes when a die is thrown = {1,2,3,4,5,6}

Let R1 and R2 denote the event that a red ball is drawn
and B1, B2, B3 denote the event that a blue ball is drawn

If H occurs, a die is thrown.

$S_1 = \{H_1, H_2, H_3, H_4, H_5, H_6\}$

If T occurs, a ball from a box which contains 2 red and 3 black balls is drawn.

$S_2 = \{TR_1, TR_2, TB_1, TB_2, TB_3\}$

The required sample space is:

$S = \{TR_1, TR_2, TB_1, TB_2, TB_3, H_1, H_2, H_3, H_4, H_5, H_6\}$

Question:16 A die is thrown repeatedly until a six comes up. What is the sample space for this experiment?

Answer:

Given, a die is thrown repeatedly until a six comes up.

Possible outcomes when a die is thrown = $\{1,2,3,4,5,6\}$

In the experiment 6 may come up on the first throw, or the 2nd throw, or the 3rd throw and so on till 6 is obtained.

The required sample space is:

$S = \{6, (1,6), (2,6), (3,6), (4,6), (5,6), (1,1,6), (1,2,6), \dots, (1,5,6), (2,1,6), (2,2,6), \dots, (2,5,6), \dots, (5,1,6), (5,2,6), \dots\}$

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NCERT solutions for class 11 maths chapter 16 probability-Exercise: 16.2

Question:1. A die is rolled. Let E be the event "die shows 4 " and F be the event "die shows even number". Are E and F mutually exclusive?

Answer:

When a die is rolled, the sample space of possible outcomes:

$S = \{1, 2, 3, 4, 5, 6\}$

Now,

E = event that the die shows 4 = $\{4\}$

F = event that the die shows even number = $\{2, 4, 6\}$

$$E \cap F = \{4\} \cap \{2, 4, 6\}$$

$$= \{4\} \neq \phi$$

Hence E and F are not mutually exclusive event.

Question:2(i) A die is thrown. Describe the following events:

A : a number less than 7

Answer:

When a die is rolled, the sample space of possible outcomes:

$$S = \{1, 2, 3, 4, 5, 6\} \text{ or } \{x : x \in \mathbb{N}, x < 7\}$$

Given, A : a number less than 7

As every number on a die is less than 7

$$A = \{1, 2, 3, 4, 5, 6\} = S$$

Question:2(ii) A die is thrown. Describe the following events:

B : a number greater than 7

Answer:

When a die is rolled, the sample space of possible outcomes:

$$S = \{1, 2, 3, 4, 5, 6\} \text{ or } \{x : x \in \mathbb{N}, x < 7\}$$

Given, B: a number greater than 7

As no number on the die is greater than 7

$$B = \phi$$

Question:2(iii) A die is thrown. Describe the following events:

C: a multiple of 3.

Answer:

When a die is rolled, the sample space of possible outcomes:

$$S = \{1, 2, 3, 4, 5, 6\} \text{ or } \{x : x \in \mathbb{N}, x < 7\}$$

Given, C : a multiple of 3

$$C = \{3, 6\}$$

Question:2(iv) A die is thrown. Describe the following events:

D: a number less than 4

Answer:

When a die is rolled, the sample space of possible outcomes:

$$S = \{1, 2, 3, 4, 5, 6\} \text{ or } \{x : x \in \mathbb{N}, x < 7\}$$

Given, D : a number less than 4

$$D = \{1, 2, 3\}$$

Question:2(v) A die is thrown. Describe the following events:

E: an even multiple greater than 4

Answer:

When a die is rolled, the sample space of possible outcomes:

$$S = \{1, 2, 3, 4, 5, 6\} \text{ or } \{x : x \in \mathbb{N}, x < 7\}$$

Given, E : an even number greater than 4

$$S_1 = \text{Subset of } S \text{ containing even numbers} = \{2, 4, 6\}$$

Therefore, E = {6}

Question:2(vi) A die is thrown. Describe the following events:

F: a number not less than 3

Answer:

When a die is rolled, the sample space of possible outcomes:

$$S = \{1, 2, 3, 4, 5, 6\} \text{ or } \{x : x \in \mathbb{N}, x < 7\}$$

Given, F : a number not less than 3

$$F = \{x : x \in S, x \geq 3\} = \{3, 4, 5, 6\}$$

Question:2(vi) A die is thrown. Describe the following events:

Also find (a) $A \cup B$

Answer:

$$A = \{1, 2, 3, 4, 5, 6\}$$

$$B = \phi$$

$$\therefore A \cup B = \{1, 2, 3, 4, 5, 6\} \cup \phi = \{1, 2, 3, 4, 5, 6\}$$

Question: 2.(vi) A die is thrown. Describe the following events:

Also find (b) $A \cap B$.

Answer:

$$A = \{1, 2, 3, 4, 5, 6\}$$

$$B = \phi$$

$$\therefore A \cap B = \{1, 2, 3, 4, 5, 6\} \cap \phi = \phi$$

Question: 2.(vi) A die is thrown. Describe the following events:

Also find (c) $B \cup C$

Answer:

$$B = \phi$$

$$C = \{3, 6\}$$

$$\therefore B \cup C = \phi \cup \{3, 6\} = \{3, 6\}$$

Question: A die is thrown. Describe the following events:

(d) Also find $E \cap F$

Answer:

$$E = \{6\}$$

$$F = \{3, 4, 5, 6\}$$

$$\therefore E \cap F = \{6\} \cap \{3, 4, 5, 6\} = \{6\}$$

Question: A die is thrown. Describe the following events:

Also find (e) $D \cap E$

Answer:

$$D = \{1, 2, 3\}$$

$$E = \{6\}$$

$$\therefore D \cap E = \{1, 2, 3\} \cap \{6\} = \phi \text{ (As nothing is common in these sets)}$$

Question: A die is thrown. Describe the following events:

Also find (f) $A - C$

Answer:

$$A = \{1, 2, 3, 4, 5, 6\}$$

$$C = \{3, 6\}$$

$$\therefore A - C = \{1, 2, 3, 4, 5, 6\} - \{3, 6\} = \{1, 2, 4, 5\}$$

Question: 2.(vi) A die is thrown. Describe the following events:

Also find (g) $D - E$

Answer:

$$D = \{1, 2, 3\}$$

$$E = \{6\}$$

$$\therefore D - E = \{1, 2, 3\} - \{6\} = \{1, 2, 3\}$$

Question: 2.(vi) A die is thrown. Describe the following events:

Also find (h) $E \cap F'$

Answer:

$$E = \{6\}$$

$$F = \{3, 4, 5, 6\}$$

$$\therefore F' = \{3, 4, 5, 6\}' = S - F = \{1, 2\}$$

$$\therefore E \cap F' = \{6\} \cap \{1, 2\} = \phi$$

Question:2.(vi) A die is thrown. Describe the following events:

Also find (i) F'

Answer:

$$F = \{3, 4, 5, 6\}$$

$$\therefore F' = \{3, 4, 5, 6\}' = S - F = \{1, 2\}$$

Question:3(a) An experiment involves rolling a pair of dice and recording the numbers that come up. Describe the following events:

the sum is greater than 8

Answer:

Sample space when a die is rolled:

$$S = \{1, 2, 3, 4, 5, 6\}$$

Let E = Event of rolling a pair of dice (= Event that a die is rolled twice!) [$6 \times 6 = 36$ possible outcomes]

$$E = [\{ (x,y) : x,y \in S \}] = \{ (1,1), (1,2), \dots, (1,6), (2,1), \dots, (6,5), (6,6) \}$$

Now,

A : the sum is greater than 8

Possible sum greater than 8 are 9, 10, 11 and 12

$A = [\{ (a,b) : (a,b) \in E, a+b > 8 \}] = \{ (3,6), (4,5), (5, 4), (6,3), (4,6), (5,5), (6,4), (5,6), (6,5), (6,6) \}$

Question:3(b) An experiment involves rolling a pair of dice and recording the numbers that come up. Describe the following events:

2 occurs on either die

Answer:

Sample space when a die is rolled:

$S = \{1, 2, 3, 4, 5, 6\}$

Let E = Event of rolling a pair of dice (= Event that a die is rolled twice!) [$6 \times 6 = 36$ possible outcomes]

$E = [\{ (x,y) : x,y \in S \}] = \{ (1,1), (1,2) \dots (1,6), (2,1) \dots (6,5), (6,6) \}$

Now,

B: 2 occurs on either die

Hence the number 2 can come on first die or second die or on both the die simultaneously.

$B = [\{ (a,b) : (a,b) \in E, a \text{ or } b = 2 \}] = \{ (1,2), (2,2), (3, 2), (4,2), (5,2), (6,2), (2,1), (2,3), (2,4), (2,5), (2,6) \}$

Question:3(c). An experiment involves rolling a pair of dice and recording the numbers that come up. Describe the following events:

the sum is at least 7 and a multiple of 3

Answer:

Sample space when a die is rolled:

$$S = \{1, 2, 3, 4, 5, 6\}$$

Let E = Event of rolling a pair of dice (= Event that a die is rolled twice!) [6x6 = 36 possible outcomes]

$$E = [\{(x,y): x,y \in S \}] = \{(1,1), (1,2)\dots(1,6),(2,1)\dots(6,5),(6,6)\}$$

Now,

C: the sum is at least 7 and a multiple of 3

The sum can only be 9 or 12.

$$C = [\{(a,b): (a,b) \in E, a+b > 6 \ \& \ a+b = 3k, k \in I \}] = \{(3,6), (6,3), (5, 4), (4,5), (6,6)\}$$

Question: An experiment involves rolling a pair of dice and recording the numbers that come up. Describe the following events:

Which pairs of these events are mutually exclusive?

Answer:

For two elements to be mutually exclusive, there should not be any common element amongst them.

$$\text{Also, } A = \{ (3,6) , (4,5), (5, 4), (6,3) , (4,6), (5,5), (6,4), (5,6), (6,5), (6,6) \}$$

$$B = \{(1,2), (2,2), (3, 2), (4,2), (5,2), (6,2), (2,1), (2,3), (2,4), (2,5), (2,6)\}$$

$$C = \{ (3,6), (6,3), (5, 4), (4,5), (6,6) \}$$

Now, $A \cap B = \phi$ (no common element in A and B)

Hence, A and B are mutually exclusive

Again, $B \cap C = \phi$ (no common element in B and C)

Hence, B and C are mutually exclusive

$$\text{Again, } C \cap A = \{(3,6), (6,3), (5, 4), (4,5), (6,6)\}$$

Therefore,

A and B, B and C are mutually exclusive.

Question: 4(i) Three coins are tossed once. Let A denote the event 'three heads show', B denote the event "two heads and one tail show", C denote the event" three tails show and D denote the event 'a head shows on the first coin". Which events are

mutually exclusive?

Answer:

Sample space when three coins are tossed = [Sample space when a coin is tossed thrice!]

$$S = \{HHH, HHT, HTH, HTT, THH, TTH, THT, TTT\}$$

Now,

A = Event that three heads show up = {HHH}

B = Event that two heads and one tail show up = {HHT, HTH, THH}

C = Event that three tails show up = {TTT}

D = Event that a head shows on the first coin = {HHH, HHT, HTH, HTT}

(i). For two elements X and Y to be mutually exclusive, $X \cap Y = \phi$

$A \cap B = \{HHH\} \cap \{HHT, HTH, THH\} = \phi$; Hence A and B are mutually exclusive.

$B \cap C = \{HHT, HTH, THH\} \cap \{TTT\} = \phi$; Hence B and C are mutually exclusive.

$C \cap D = \{TTT\} \cap \{HHH, HHT, HTH, HTT\} = \phi$; Hence C and D are mutually exclusive.

$D \cap A = \{HHH, HHT, HTH, HTT\} \cap \{HHH\} = \{HHH\}$; Hence D and A are **not** mutually exclusive.

$A \cap C = \{HHH\} \cap \{TTT\} = \phi$; Hence A and C are mutually exclusive.

$B \cap D = \{HHT, HTH, THH\} \cap \{HHH, HHT, HTH, HTT\} = \{HHT, HTH\}$; Hence B and D are **not** mutually exclusive.

Question:4.(ii) Three coins are tossed once. Let A denote the event 'three heads show', B denote the event "two heads and one tail show", C denote the event" three tails show and D denote the event 'a head shows on the first coin". Which events are simple?

Answer:

Sample space when three coins are tossed = [Sample space when a coin is tossed thrice!]

$S = \{HHH, HHT, HTH, HTT, THH, TTH, THT, TTT\}$

Now,

$A = \text{Event that three heads show up} = \{HHH\}$

$B = \text{Event that two heads and one tail show up} = \{HHT, HTH, THH\}$

$C = \text{Event that three tails show up} = \{TTT\}$

$D = \text{Event that a head shows on the first coin} = \{HHH, HHT, HTH, HTT\}$

(ii). If an event X has only one sample point of a sample space, it is called a simple event.

$A = \{HHH\}$ and $C = \{TTT\}$

Hence, A and C are simple events.

Question:4.(iii) Three coins are tossed once. Let A denote the event 'three heads show', B denote the event "two heads and one tail show", C denote the event "three tails show" and D denote the event 'a head shows on the first coin'. Which events are

Compound?

Answer:

Sample space when three coins are tossed = [Sample space when a coin is tossed thrice!]

$S = \{HHH, HHT, HTH, THH, TTH, TTT\}$

Now,

$A = \text{Event that three heads show up} = \{HHH\}$

$B = \text{Event that two heads and one tail show up} = \{HHT, HTH, THH\}$

$C = \text{Event that three tails show up} = \{TTT\}$

$D = \text{Event that a head shows on the first coin} = \{HHH, HHT, HTH, HTT\}$

(iv). If an event has more than one sample point, it is called a Compound event.

$B = \{HHT, HTH, THH\}$ and $D = \{HHH, HHT, HTH, HTT\}$

Hence, B and D are compound events.

Question:5(i) Three coins are tossed. Describe

Two events which are mutually exclusive.

Answer:

Sample space when three coins are tossed = [Sample space when a coin is tossed thrice!]

$S = \{HHH, HHT, HTH, THH, TTH, TTT\}$

(i)

$A =$ Event that three heads show up = $\{HHH\}$

$B =$ Event that three tails show up = $\{TTT\}$

$A \cap B = \{HHH\} \cap \{TTT\} = \phi$; Hence A and B are mutually exclusive.

Question: 5(ii) Three coins are tossed. Describe

Three events which are mutually exclusive and exhaustive.

Answer:

Sample space when three coins are tossed = [Sample space when a coin is tossed thrice!]

$S = \{HHH, HHT, HTH, HTT, THH, TTH, THT, TTT\}$

Let ,

$A =$ Getting no tails = $\{HHH\}$

$B =$ Getting exactly one tail = $\{HHT, HTH, THH\}$

$C =$ Getting at least two tails = $\{HTT, THT, TTH\}$

Clearly, $A \cap B = \phi$; $B \cap C = \phi$; $C \cap A = \phi$

Since (A and B), (B and C) and (A and C) are mutually exclusive

Therefore A, B and C are mutually exclusive.

Also,

$$A \cup B \cup C = S$$

Hence A, B and C are exhaustive events.

Hence, A, B and C are three events which are mutually exclusive and exhaustive.

Question: Three coins are tossed. Describe

Two events, which are not mutually exclusive.

Answer:

Sample space when three coins are tossed = [Sample space when a coin is tossed thrice!]

$$S = \{HHH, HHT, HTH, HTT, THH, TTH, THT, TTT\}$$

Let ,

$$A = \text{Getting at least one head} = \{HHH, HHT, HTH, THH, TTH\}$$

$$B = \text{Getting at most one head} = \{TTH, TTT\}$$

$$\text{Clearly, } A \cap B = \{TTH\} \neq \phi$$

Hence, A and B are two events which are not mutually exclusive.

Question: Three coins are tossed. Describe

Two events which are mutually exclusive but not exhaustive.

Answer:

Sample space when three coins are tossed = [Sample space when a coin is tossed thrice!]

$S = \{HHH, HHT, HTH, HTT, THH, TTH, THT, TTT\}$

Let ,

$A = \text{Getting exactly one head} = \{HTT, THT, TTH\}$

$B = \text{Getting exactly one tail} = \{HHT, HTH, THH\}$

Clearly, $A \cap B = \phi$

Hence, A and B are mutually exclusive.

Also, $A \cup B \neq S$

Hence, A and B are not exhaustive.

Question: Three coins are tossed. Describe

Three events which are mutually exclusive but not exhaustive

Answer:

Sample space when three coins are tossed = [Sample space when a coin is tossed thrice!]

$S = \{HHH, HHT, HTH, HTT, THH, TTH, THT, TTT\}$

Let ,

$A = \text{Getting exactly one tail} = \{HHT, HTH, THH\}$

$B =$ Getting exactly two tails $= \{HTT, TTH, THT\}$

$C =$ Getting exactly three tails $= \{TTT\}$

Clearly, $A \cap B = \phi$; $B \cap C = \phi$; $C \cap A = \phi$

Since $(A$ and $B)$, $(B$ and $C)$ and $(A$ and $C)$ are mutually exclusive

Therefore A , B and C are mutually exclusive.

Also,

$A \cup B \cup C = \{HHT, HTH, THH, HTT, TTH, THT, TTT\} \neq S$

Hence A , B and C are not exhaustive events.

Question:6.(i) Two dice are thrown. The events A , B and C are as follows:

A : getting an even number on the first die.

B : getting an odd number on the first die.

C : getting the sum of the numbers on the dice ≤ 5 .

Describe the events

A'

Answer:

Sample space when two dice are thrown:

$S = \{(x,y): 1 \leq x,y \leq 6\}$

A: getting an even number on the first die = $\{(a,b): a \in \{2,4,6\} \text{ and } 1 \leq b \leq 6\}$

= $\{(2, 1), (2,2), (2,3), (2,4), (2,5), (2,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$

(i) Therefore, $A' = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6)\}$

= B : getting an odd number on the first die.

Question:6.(ii) Two dice are thrown. The events A, B and C are as follows:

A: getting an even number on the first die.

B: getting an odd number on the first die.

C: getting the sum of the numbers on the dice ≤ 5 .

Describe the events

not B

Answer:

Sample space when two dice are thrown:

$S = \{(x,y): 1 \leq x,y \leq 6\}$

B: getting an odd number on the first die = $\{(a,b): a \in \{1,3,5\} \text{ and } 1 \leq b \leq 6\}$

= $\{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6)\}$

(ii) Therefore, $B' = \{(2, 1), (2,2), (2,3), (2,4), (2,5), (2,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$

= A : getting an even number on the first die.

Question: 6. (iii) Two dice are thrown. The events A, B and C are as follows:

A: getting an even number on the first die.

B: getting an odd number on the first die.

C: getting the sum of the numbers on the dice ≤ 5 .

Describe the events

A or B

Answer:

Sample space when two dice are thrown:

$S = \{(x,y): 1 \leq x,y \leq 6\}$

A: getting an even number on the first die = $\{(a,b): a \in \{2,4,6\} \text{ and } 1 \leq b \leq 6\}$

= $\{(2, 1), (2,2), (2,3), (2,4), (2,5), (2,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$

B: getting an odd number on the first die = $\{(a,b): a \in \{1,3,5\} \text{ and } 1 \leq b \leq 6\}$

$= \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6)\}$

(iii) $A \text{ or } B = A \cup B = \{(1,1), (1,2) \dots (1,6), (3,1), (3,2) \dots (3,6), (5,1), (5,2) \dots (5,6), (2,1), (2,2) \dots (2,6), (4,1), (4,2) \dots (4,6), (6,1), (6,2) \dots (6,6)\} = S$

Question:6.(iv) Two dice are thrown. The events A, B and C are as follows:

A: getting an even number on the first die.

B: getting an odd number on the first die.

C: getting the sum of the numbers on the dice ≤ 5

Describe the events

A and B

Answer:

Sample space when two dice are thrown:

$S = \{(x,y): 1 \leq x,y \leq 6\}$

A: getting an even number on the first die $= \{(a,b): a \in \{2,4,6\} \text{ and } 1 \leq b \leq 6\}$

$= \{(2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$

B: getting an odd number on the first die $= \{(a,b): a \in \{1,3,5\} \text{ and } 1 \leq b \leq 6\}$

$= \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6)\}$

(iii) $A \text{ and } B = A \cap B = A \cap A' = \phi$ (From (ii))

Question:6.(v) Two dice are thrown. The events A, B and C are as follows:

A: getting an even number on the first die.

B: getting an odd number on the first die.

C: getting the sum of the numbers on the dice ≤ 5

Describe the events

A but not C

Answer:

Sample space when two dice are thrown:

$S = \{(x,y): 1 \leq x,y \leq 6\}$

A: getting an even number on the first die $= \{(a,b): a \in \{2,4,6\} \text{ and } 1 \leq b \leq 6\}$

$= \{(2, 1), (2,2), (2,3), (2,4), (2,5), (2,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$

C: getting the sum of the numbers on the dice ≤ 5

The possible sum are 2,3,4,5

$C = \{(a,b): 2 \leq a + b \leq 5\} = \{(1, 1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (3,1), (3,2), (4,1)\}$

(v) $A \text{ but not } C = A - C = \{(2,4), (2,5), (2,6), (4,2), (4,3), (4,4), (4,5), (4,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$

Question: (vi) Two dice are thrown. The events A, B and C are as follows:

A: getting an even number on the first die.

B: getting an odd number on the first die.

C: getting the sum of the numbers on the dice ≤ 5

Describe the events

B or C

Answer:

Sample space when two dice are thrown:

$$S = \{(x,y) : 1 \leq x,y \leq 6\}$$

B: getting an odd number on the first die = $\{(a,b) : a \in \{1,3,5\} \text{ and } 1 \leq b \leq 6\}$

= $\{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6)\}$

C: getting the sum of the numbers on the dice ≤ 5

The possible sum are 2,3,4,5

$$C = \{(a,b): 2 \leq a + b \leq 5\} = \{(1, 1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (3,1), (3,2), (4,1)\}$$

$$(vi) B \text{ or } C = B \cup C = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6)\}$$

Question: 6.(vii) Two dice are thrown. The events A, B and C are as follows:

A: getting an even number on the first die.

B: getting an odd number on the first die.

C: getting the sum of the numbers on the dice ≤ 5

Describe the events

B and C

Answer:

Sample space when two dice are thrown:

$$S = \{(x,y): 1 \leq x,y \leq 6\}$$

B: getting an odd number on the first die = $\{(a,b): a \in \{1,3,5\} \text{ and } 1 \leq b \leq 6\}$

$$= \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6)\}$$

C: getting the sum of the numbers on the dice ≤ 5

The possible sum are 2,3,4,5

$$C = \{(a,b): 2 \leq a + b \leq 5\} = \{(1, 1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (3,1), (3,2), (4,1)\}$$

$$(vi) B \text{ and } C = B \cap C = \{(1, 1), (1,2), (1,3), (1,4), (3,1), (3,2)\}$$

Question: (viii) Two dice are thrown. The events A, B and C are as follows:

A: getting an even number on the first die.

B: getting an odd number on the first die.

C: getting the sum of the numbers on the dice ≤ 5

Describe the events

$$A \cap B' \cap C'$$

Answer:

Sample space when two dice are thrown:

$$S = \{(x,y): 1 \leq x,y \leq 6\}$$

A: getting an even number on the first die = $\{(a,b): a \in \{2,4,6\} \text{ and } 1 \leq b \leq 6\}$

$$= \{(2, 1), (2,2), (2,3), (2,4), (2,5), (2,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$$

B: getting an odd number on the first die = $\{(a,b): a \in \{1,3,5\} \text{ and } 1 \leq b \leq 6\}$

$= \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6)\}$

C: getting the sum of the numbers on the dice ≤ 5

The possible sum are 2,3,4,5

$C = \{(a,b): 2 \leq a + b \leq 5\} = \{(1, 1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (3,1), (3,2), (4,1)\}$

(viii) $A \cap B' \cap C' = A \cap A \cap C'$ (from (ii))

$= A \cap C' = A - C = \{(2,4), (2,5), (2,6), (4,2), (4,3), (4,4), (4,5), (4,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$

Question: Refer to question 6 above, state true or false: (give reason for your answer)

A and B are mutually exclusive

Answer:

Here,

$A = \{(2, 1), (2,2), (2,3), (2,4), (2,5), (2,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$

$B = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6)\}$

(i) X and Y are mutually exclusive if and only if $X \cap Y = \phi$

$A \cap B = \phi$, since A and B have no common element amongst them.

Hence, A and B are mutually exclusive. TRUE

Question:7.(ii) Refer to question 6 above, state true or false: (give reason for your answer)

A and B are mutually exclusive and exhaustive

Answer:

Here,

$A = \{(2, 1), (2,2), (2,3), (2,4), (2,5), (2,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$

$B = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6)\}$

(ii) X and Y are mutually exclusive if and only if $X \cap Y = \phi$

$A \cap B = \phi$, since A and B have no common element amongst them.

Hence, A and B are mutually exclusive.

Also,

$A \cup B = \{(2,1), (2,2)..... (2,6), (4,1), (4,2).....(4,6), (6,1), (6,2)..... (6,6), (1,1), (1,2).... (1,6), (3,1), (3,2)..... (3,6), (5,1), (5,2).... (5,6)\} = S$

Hence, A and B are exhaustive.

TRUE

Question: Refer to question 6 above, state true or false: (give reason for your answer)

$$A = B'$$

Answer:

Here,

$$S = \{(x,y): 1 \leq x,y \leq 6\}$$

$$A = \{(2, 1), (2,2), (2,3), (2,4), (2,5), (2,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$$

$$B = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6)\}$$

$$(iii) \text{ Therefore, } B' = S - B = \{(2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\} = A$$

TRUE

Question: Refer to question 6 above, state true or false: (give reason for your answer)

A and C are mutually exclusive

Answer:

Here,

$$S = \{(x,y): 1 \leq x,y \leq 6\}$$

$$A = \{(2, 1), (2,2), (2,3), (2,4), (2,5), (2,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$$

$$C = \{(1, 1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (3,1), (3,2), (4,1)\}$$

(iv) X and Y are mutually exclusive if and only if $X \cap Y = \phi$

$$A \cap C = \{(2,1), (2,2), (2,3), (4,1)\},$$

Hence, A and B are not mutually exclusive. FALSE

Question: 7.(v) Refer to question 6 above, state true or false: (give reason for your answer)

A and B' are mutually exclusive.

Answer:

X and Y are mutually exclusive if and only if $X \cap Y = \phi$

$$A \cap B' = A \cap A = A \text{ (From (iii))}$$

$$\therefore A \cap B' \neq \phi$$

Hence A and B' not mutually exclusive. FALSE

Question: 7.(vi) Refer to question 6 above, state true or false: (give reason for your answer)

A', B', C are mutually exclusive and exhaustive.

Answer:

Here,

$$S = \{(x,y): 1 \leq x,y \leq 6\}$$

$$A = \{(2, 1), (2,2), (2,3), (2,4), (2,5), (2,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$$

$$B = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6)\}$$

$$C = \{(1, 1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (3,1), (3,2), (4,1)\}$$

(vi) X and Y are mutually exclusive if and only if $X \cap Y = \phi$

$$\therefore A' \cap B' = B \cap A = \phi \text{ (from (iii) and (i))}$$

Hence A' and B' are mutually exclusive.

Again,

$$\therefore B' \cap C = A \cap C \neq \phi \text{ (from (iv))}$$

Hence B' and C are not mutually exclusive.

Hence, A' , B' and C are not mutually exclusive and exhaustive. FALSE

**NCERT solutions for class 11 maths chapter 16 probability-Exercise:
16.3**

Question:1(a) Which of the following can not be valid assignment of probabilities for outcomes of sample Space $S = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7\}$

Assignment	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7
(a)	0.1	0.01	0.05	0.03	0.01	0.2	0.6

Answer:

(a) Condition (i): Each of the number $p(\omega_i)$ is positive and less than one.

Condition (ii): Sum of probabilities = $0.1 + 0.01 + 0.05 + 0.03 + 0.01 + 0.2 + 0.6 = 1$

Therefore, the assignment is valid

Question:1(b) Which of the following cannot be the valid assignment of probabilities for outcomes of sample Space $S = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7\}$

Assignment	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7
(b)	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$

Answer:

(b) Condition (i): Each of the number $p(\omega_i)$ is positive and less than one.

Condition (ii): Sum of probabilities = $\frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} = 1$

Therefore, the assignment is valid

Question: 1.(c) Which of the following can not be valid assignment of probabilities for outcomes of sample

Space $S = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7\}$

Assignment	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7
(c)	0.1	0.2	0.3	0.4	0.5	0.6	0.7

Answer:

(c) Since sum of probabilities = $0.1 + 0.2 + 0.3 + 0.4 + 0.5 + 0.6 + 0.7 = 2.8 > 1$

Hence, Condition (ii) is not satisfied.

Therefore, the assignment is not valid

Question: 1.(d) Which of the following can not be valid assignment of probabilities for outcomes of sample

Space $S = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7\}$

Assignment	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7
(d)	-0.1	0.2	0.3	0.4	-0.2	0.1	0.3

Answer:

(d) Two of the probabilities $p(\omega_1)$ and $p(\omega_5)$ are negative, hence condition(i) is not satisfied.

Therefore, the assignment is not valid.

Question:1 Which of the following can not be valid assignment of probabilities for outcomes of sample

Space $S = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7\}$

Assignment	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7
(e)	$\frac{1}{14}$	$\frac{2}{14}$	$\frac{3}{14}$	$\frac{4}{14}$	$\frac{5}{14}$	$\frac{6}{14}$	$\frac{15}{14}$

Answer:

(e) Each of the number $p(\omega_i)$ is positive but $p(\omega_7)$ is not less than one. Hence the condition is not satisfied.

Therefore, the assignment is not valid.

Question:2 A coin is tossed twice, what is the probability that atleast one tail occurs?

Answer:

Sample space when a coin is tossed twice, $S = \{HH, HT, TH, TT\}$

[Note: A coin tossed twice is same as two coins tossed at once]

∴ Number of possible outcomes $n(S) = 4$

Let E be the event of getting at least one tail = {HT, TH, TT}

$$\therefore n(E) = 3$$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{3}{4}$$

$$= 0.75$$

Question:3.(i) A die is thrown, find the probability of following events:

A prime number will appear

Answer:

Sample space when a die is thrown, $S = \{1,2,3,4,5,6\}$

\therefore Number of possible outcomes $n(S) = 6$

Let E be the event of getting a prime number = {2,3,5}

$$\therefore n(E) = 3$$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{3}{6}$$

$$= 0.5$$

Question:3.(ii) A die is thrown, find the probability of following events:

A number greater than or equal to 3 will appear

Answer:

Sample space when a die is thrown, $S = \{1,2,3,4,5,6\}$

∴ Number of possible outcomes $n(S) = 6$

Let E be the event of getting a number greater than or equal to 3 = $\{3,4,5,6\}$

∴ $n(E) = 4$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{4}{6} = \frac{2}{3}$$

= 0.67

Question: A die is thrown, find the probability of following events:

A number less than or equal to one will appear

Answer:

Sample space when a die is thrown, $S = \{1,2,3,4,5,6\}$

∴ Number of possible outcomes $n(S) = 6$

Let E be the event of getting a number less than or equal to one = $\{1\}$

∴ $n(E) = 1$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{1}{6}$$

= 0.167

Question:3.(iv) A die is thrown, find the probability of following events:

A number more than 6 will appear

Answer:

Sample space when a die is thrown, $S = \{1,2,3,4,5,6\}$

∴ Number of possible outcomes $n(S) = 6$

Let E be the event of getting a number more than 6 will appear = ϕ

∴ $n(E) = 0$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{0}{6}$$

= 0

Question:3.(v) A die is thrown, find the probability of following events:

A number less than 6 will appear.

Answer:

Sample space when a die is thrown, $S = \{1,2,3,4,5,6\}$

∴ Number of possible outcomes $n(S) = 6$

Let E be the event of getting a number less than 6 will appear = $\{1,2,3,4,5\}$

∴ $n(E) = 5$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{5}{6}$$

= 0.83

Question:4(a). A card is selected from a pack of 52 cards.

How many points are there in the sample space?

Answer:

(a) Number of points(events) in the sample space = Number of cards in the pack = 52

Question:4(b). A card is selected from a pack of 52 cards.

Calculate the probability that the card is an ace of spades.

Answer:

Number of possible outcomes, $n(S) = 52$

Let E be the event that the card is an ace of spades

$\therefore n(E) = 1$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{1}{52}$$

The required probability that the card is an ace of spades is $\frac{1}{52}$.

Question:4(c)(i) A card is selected from a pack of 52 cards.

Calculate the probability that the card is an ace

Answer:

Number of possible outcomes, $n(S) = 52$

Let E be the event that the card is an ace. There are 4 aces.

$$\therefore n(E) = 4$$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

The required probability that the card is an ace is $\frac{1}{13}$.

Question:4(c)(ii) A card is selected from a pack of 52 cards.

Calculate the probability that the card is black card.

Answer:

Number of possible outcomes, $n(S) = 52$

Let E be the event that the card is a black card. There are 26 black cards. (Diamonds and Clubs)

$$\therefore n(E) = 26$$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{26}{52} = \frac{1}{2}$$

The required probability that the card is an ace is $\frac{1}{2}$.

Question:5.(i) A fair coin with 1 marked on one face and 6 on the other and a fair die are both tossed. find the probability that the sum of numbers that turn up is 3

Answer:

The coin and die are tossed together.

The coin can have only 1 or 6 as possible outcomes and the die can have {1,2,3,4,5,6} as possible outcomes

Sample space, $S = \{(x,y): x \in \{1,6\} \text{ and } y \in \{1,2,3,4,5,6\}\}$

$= \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$

Number of possible outcomes, $n(S) = 12$

(i) Let E be the event having sum of numbers as 3 = $\{(1, 2)\}$

$\therefore n(E) = 1$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{1}{12}$$

The required probability of having 3 as sum of numbers is $\frac{1}{12}$.

Question:5.(ii) A fair coin with 1 marked on one face and 6 on the other and a fair die are both tossed. find the probability that the sum of numbers that turn up is 12

Answer:

The coin and die are tossed together.

The coin can have only 1 or 6 as possible outcomes and the die can have {1,2,3,4,5,6} as possible outcomes

Sample space, $S = \{(x,y): x \in \{1,6\} \text{ and } y \in \{1,2,3,4,5,6\}\}$

$= \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$

Number of possible outcomes, $n(S) = 12$

(ii) Let E be the event having sum of numbers as 12 = {(6, 6)}

$$\therefore n(E) = 1$$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{1}{12}$$

The required probability of having 12 as sum of numbers is $\frac{1}{12}$.

Question:6 There are four men and six women on the city council. If one council member is selected for a committee at random, how likely is it that it is a woman?

Answer:

There are four men and six women on the city council

$$\therefore n(S) = n(\text{men}) + n(\text{women}) = 4 + 6 = 10$$

Let E be the event of selecting a woman

$$\therefore n(E) = 6$$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{6}{10} = \frac{3}{5}$$

Therefore, the required probability of selecting a woman is 0.6

Question:7. A fair coin is tossed four times, and a person win Re 1 for each head and lose Rs 1.50 for each tail that turns up. From the sample, space calculate how many different amounts of money you can have after four tosses and the probability of having each of these amounts.

Answer:

Here the sample space is,

$S = \{HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTHT, THHT, HTTH, THTH, TTHH, TTTH, TTHT, THTT, HTTT, TTTT\}$

According to question,

1.) 4 heads = $1 + 1 + 1 + 1 = \text{Rs. } 4$

2.) 3 heads and 1 tail = $1 + 1 + 1 - 1.50 = \text{Rs. } 1.50$

3.) 2 heads and 2 tails = $1 + 1 - 1.50 - 1.50 = - \text{Rs. } 1$: he will lose Re. 1

4.) 1 head and 3 tails = $1 - 1.50 - 1.50 - 1.50 = - \text{Rs. } 3.50$: he will lose Rs. 3.50

5.) 4 tails = $- 1.50 - 1.50 - 1.50 - 1.50 = - \text{Rs. } 6$ = he will lose Rs. 6

Now, sample space of amounts corresponding to S:

$S' = \{4, 1.50, 1.50, 1.50, 1.50, - 1, - 1, - 1, - 1, - 1, - 1, - 3.50, - 3.50, - 3.50, - 3.50, - 6\}$

$\therefore n(S') = 12$

\therefore Required Probabilities are:

$$P(\text{Winning Rs. } 4) = \frac{n(\text{Winning Rs. } 4)}{n(S')} = \frac{1}{16}$$

$$P(\text{Winning Rs. } 1.50) = \frac{n(\text{Winning Rs. } 1.50)}{n(S')} = \frac{4}{16} = \frac{1}{4}$$

$$P(\text{Losing Re. } 1) = \frac{n(\text{Losing Re. } 1)}{n(S')} = \frac{6}{16} = \frac{3}{8}$$

$$P(\text{Losing Rs. 3.50}) = \frac{n(\text{Losing Rs. 3.50})}{n(S')} = \frac{4}{16} = \frac{1}{4}$$

$$P(\text{Losing Rs. 6}) = \frac{n(\text{Losing Rs. 6})}{n(S')} = \frac{1}{16}$$

Question:8.(i) Three coins are tossed once. Find the probability of getting

3 heads

Answer:

Sample space when three coins are tossed: [Same as a coin tossed thrice!]

$S = \{HHH, HHT, HTH, HTT, THH, TTH, THT, TTT\}$

Number of possible outcomes, $n(S) = 8$ [Note: $2 \times 2 \times 2 = 8$]

Let E be the event of getting 3 heads = {HHH}

$\therefore n(E) = 1$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{1}{8}$$

The required probability of getting 3 heads is $\frac{1}{8}$.

Question:8.(ii) Three coins are tossed once. Find the probability of getting

2 heads

Answer:

Sample space when three coins are tossed: [Same as a coin tossed thrice!]

$$S = \{HHH, HHT, HTH, HTT, THH, TTH, THT, TTT\}$$

Number of possible outcomes, $n(S) = 8$ [Note: $2 \times 2 \times 2 = 8$]

Let E be the event of getting 2 heads = {HHT, HTH, THH}

$$\therefore n(E) = 3$$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{3}{8}$$

The required probability of getting 2 heads is $\frac{3}{8}$.

Question: 8.(iii) Three coins are tossed once. Find the probability of getting

atleast 2 heads

Answer:

Sample space when three coins are tossed: [Same as a coin tossed thrice!]

$$S = \{HHH, HHT, HTH, HTT, THH, TTH, THT, TTT\}$$

Number of possible outcomes, $n(S) = 8$ [Note: $2 \times 2 \times 2 = 8$]

Let E be the event of getting atleast 2 heads = Event of getting 2 or more heads = {HHH, HHT, HTH, THH}

$$\therefore n(E) = 4$$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{4}{8} = \frac{1}{2}$$

The required probability of getting atleast 2 heads is $\frac{1}{2}$.

Question:8.(iv) Three coins are tossed once. Find the probability of getting atleast 2 heads

Answer:

Sample space when three coins are tossed: [Same as a coin tossed thrice!]

$$S = \{HHH, HHT, HTH, HTT, THH, TTH, THT, TTT\}$$

Number of possible outcomes, $n(S) = 8$ [Note: $2 \times 2 \times 2 = 8$]

Let E be the event of getting atleast 2 heads = Event of getting 2 or less heads = {HHT, HTH, THH, TTH, HTT, THT}

$$\therefore n(E) = 6$$

$$= \frac{6}{8} = \frac{3}{4} \therefore P(E) = \frac{n(E)}{n(S)}$$

The required probability of getting atleast 2 heads is $\frac{3}{4}$.

Question:8.(v) Three coins are tossed once. Find the probability of getting no head

Answer:

Sample space when three coins are tossed: [Same as a coin tossed thrice!]

$S = \{HHH, HHT, HTH, HTT, THH, TTH, THT, TTT\}$

Number of possible outcomes, $n(S) = 8$ [Note: $2 \times 2 \times 2 = 8$]

Let E be the event of getting no head = Event of getting only tails = $\{TTT\}$

$\therefore n(E) = 1$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{1}{8}$$

The required probability of getting no head is $\frac{1}{8}$.

Question:8.(vi) Three coins are tossed once. Find the probability of getting 3 tails

Answer:

Sample space when three coins are tossed: [Same as a coin tossed thrice!]

$S = \{HHH, HHT, HTH, HTT, THH, TTH, THT, TTT\}$

Number of possible outcomes, $n(S) = 8$ [Note: $2 \times 2 \times 2 = 8$]

Let E be the event of getting 3 tails = $\{TTT\}$

$\therefore n(E) = 1$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{1}{8}$$

The required probability of getting 3 tails is $\frac{1}{8}$.

Question:8(vii) Three coins are tossed once. Find the probability of getting exactly two tails

Answer:

Sample space when three coins are tossed: [Same as a coin tossed thrice!]

$S = \{HHH, HHT, HTH, HTT, THH, TTH, THT, TTT\}$

Number of possible outcomes, $n(S) = 8$ [Note: $2 \times 2 \times 2 = 8$]

Let E be the event of getting exactly 2 tails = $\{TTH, HTT, THT\}$

$\therefore n(E) = 3$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{3}{8}$$

The required probability of getting exactly 2 tails is $\frac{3}{8}$.

Question:8.(viii) Three coins are tossed once. Find the probability of getting no tail

Answer:

Sample space when three coins are tossed: [Same as a coin tossed thrice!]

$S = \{HHH, HHT, HTH, HTT, THH, TTH, THT, TTT\}$

Number of possible outcomes, $n(S) = 8$ [Note: $2 \times 2 \times 2 = 8$]

Let E be the event of getting no tail = Event of getting only heads = $\{HHH\}$

$$\therefore n(E) = 1$$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{1}{8}$$

The required probability of getting no tail is $\frac{1}{8}$.

Question:8.(ix) Three coins are tossed once. Find the probability of getting

atmost two tails

Answer:

Sample space when three coins are tossed: [Same as a coin tossed thrice!]

$$S = \{HHH, HHT, HTH, HTT, THH, TTH, THT, TTT\}$$

Number of possible outcomes, $n(S) = 8$ [Note: $2 \times 2 \times 2 = 8$]

Let E be the event of getting atmost 2 tails = Event of getting 2 or less tails = {HHT, HTH, THH, TTH, HTT, THT}

$$\therefore n(E) = 6$$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{6}{8} = \frac{3}{4}$$

The required probability of getting atmost 2 tails is $\frac{3}{4}$.

Question:9 If $\frac{2}{11}$ is the probability of an event, what is the probability of the event 'not A'.

Answer:

Given,

$$P(E) = \frac{2}{11}$$

We know,

$$P(\text{not } E) = P(E') = 1 - P(E)$$

$$= 1 - \frac{2}{11}$$

$$= \frac{9}{11}$$

Question: 10.(i) A letter is chosen at random from the word 'ASSASSINATION'. Find the probability that letter is a vowel

Answer:

Given, 'ASSASSINATION'

No. of A's = 3; No. of S's = 4; No. of I's = 2; No. of N's = 2; No. of T = 1; No. of O = 1

No. of letters = 13

No. of vowels = {3 A's, 2 I's, O} = 6

One letter is selected:

$$n(S) = {}^{13}C_1 = 13$$

Let E be the event of getting a vowel.

$$n(E) = {}^6C_1 = 6$$

$$\therefore P(E) = \frac{6}{13}$$

Question: 10.(ii) A letter is chosen at random from the word 'ASSASSINATION'. Find the probability that letter is a consonant

Answer:

Given, 'ASSASSINATION'

No. of A's = 3; No. of S's = 4; No. of I's = 2; No. of N's = 2; No. of T = 1; No. of O = 1

No. of letters = 13

No. of consonants = {4 S's, 2 N's, T} = 7

One letter is selected:

$$n(S) = {}^{13}C_1 = 13$$

Let E be the event of getting a consonant.

$$n(E) = {}^7C_1 = 7$$

$$\therefore P(E) = \frac{7}{13}$$

Question:11 In a lottery, a person chooses six different natural numbers at random from 1 to 20, and if these six numbers match with the six numbers already fixed by the lottery committee, he wins the prize. What is the probability of winning the prize in the game? [**Hint** order of the numbers is not important.]

Answer:

Total numbers of numbers in the draw = 20

Numbers to be selected = 6

$$\therefore n(S) = {}^{20}C_6$$

Let E be the event that six numbers match with the six numbers fixed by the lottery committee.

$n(E) = 1$ (Since only one prize to be won.)

\therefore Probability of winning =

$$\begin{aligned} P(E) &= \frac{n(E)}{n(S)} = \frac{1}{{}^{20}C_6} = \frac{6!14!}{20!} \\ &= \frac{6.5.4.3.2.1.14!}{20.19.18.17.16.15.14!} \\ &= \frac{1}{38760} \end{aligned}$$

Question:12.(i) Check whether the following probabilities $P(A)$ and $P(B)$ are consistently defined:

$$P(A) = 0.5, P(B) = 0.7, P(A \cap B) = 0.6$$

Answer:

(i) Given, $P(A) = 0.5, P(B) = 0.7, P(A \cap B) = 0.6$

Now $P(A \cap B) > P(A)$

(Since $A \cap B$ is a subset of A, $P(A \cap B)$ cannot be more than $P(A)$)

Therefore, the given probabilities are not consistently defined.

Question:12.(ii) Check whether the following probabilities $P(A)$ and $P(B)$ are consistently defined

$$P(A) = 0.5, P(B) = 0.4, P(A \cup B) = 0.8$$

Answer:

(ii) Given, $P(A) = 0.5, P(B) = 0.4, P(A \cup B) = 0.8$

We know,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow 0.8 = 0.5 + 0.4 - P(A \cap B)$$

$$\Rightarrow P(A \cap B) = 0.9 - 0.8 = 0.1$$

Therefore, $P(A \cap B) < P(A)$ and $P(A \cap B) < P(B)$, which satisfies the condition.

Hence, the probabilities are consistently defined

Question:13 Fill in the blanks in following table:

	$P(A)$	$P(B)$	$P(A \cap B)$	$P(A \cup B)$
(i)	$\frac{1}{3}$	$\frac{1}{5}$	$\frac{1}{15}$...
(ii)	0.35	...	0.25	0.6

(iii)	0.5	0.35	...	0.7
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Answer:

We know,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$(i) P(A \cup B) = \frac{1}{3} + \frac{1}{5} - \frac{1}{15} = \frac{5+3-1}{15} = \frac{7}{15}$$

$$(ii) 0.6 = 0.35 + P(B) - 0.25$$

$$\Rightarrow P(B) = 0.6 - 0.1 = 0.5$$

$$(iii) 0.7 = 0.5 + 0.35 - P(A \cap B)$$

$$\Rightarrow P(A \cap B) = 0.85 - 0.7 = 0.15$$

	$P(A)$	$P(B)$	$P(A \cap B)$	$P(A \cup B)$
(i)	$\frac{1}{3}$	$\frac{1}{5}$	$\frac{1}{15}$	$\frac{7}{15}$
(ii)	0.35	0.5	0.25	0.6
(iii)	0.5	0.35	0.15	0.7

Question:14 Given $P(A) = \frac{3}{5}$ and $P(B) = \frac{1}{5}$. Find $P(A \text{ or } B)$, if A and B are mutually exclusive events.

Answer:

Given, $P(A) = \frac{3}{5}$ and $P(B) = \frac{1}{5}$

To find : $P(A \text{ or } B) = P(A \cup B)$

We know,

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = P(A) + P(B)$ [Since A and B are mutually exclusive events.]

$$\Rightarrow P(A \cup B) = \frac{3}{5} + \frac{1}{5} = \frac{4}{5}$$

Therefore, $P(A \cup B) = \frac{4}{5}$

Question:15(i) If E and F are events such

that $P(E) = \frac{1}{4}$, $P(F) = \frac{1}{2}$ and $P(E \text{ and } F) = \frac{1}{8}$, find (i) $P(E \text{ or } F)$

Answer:

Given, $P(E) = \frac{1}{4}$, $P(F) = \frac{1}{2}$ and $P(E \text{ and } F) = \frac{1}{8}$

To find : $P(E \text{ or } F) = P(E \cup F)$

We know,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\begin{aligned} \Rightarrow P(E \cup F) &= \frac{1}{4} + \frac{1}{2} - \frac{1}{8} = \frac{2+4-1}{8} \\ &= \frac{5}{8} \end{aligned}$$

Therefore, $P(E \cup F) = \frac{5}{8}$

Question:15.(ii) If E and F are events such

that $P(E) = \frac{1}{4}$, $P(F) = \frac{1}{2}$ and $P(E \text{ and } F) = \frac{1}{8}$ find $P(\text{not } E \text{ and not } F)$.

Answer:

Given, $P(E) = \frac{1}{4}$, $P(F) = \frac{1}{2}$ and $P(E \text{ and } F) = \frac{1}{8}$

To find : $P(\text{not } E \text{ and not } F) = P(E' \cap F')$

We know,

$$P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B)$$

And $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\begin{aligned} \Rightarrow P(E \cup F) &= \frac{1}{4} + \frac{1}{2} - \frac{1}{8} = \frac{2+4-1}{8} \\ &= \frac{5}{8} \end{aligned}$$

$$\Rightarrow P(E' \cap F') = 1 - P(E \cup F)$$

$$= 1 - \frac{5}{8} = \frac{3}{8}$$

Therefore, $P(E' \cap F') = \frac{3}{8}$

Question:16 Events E and F are such that $P(\text{not } E \text{ or not } F) = 0.25$, State whether E and F are mutually exclusive.

Answer:

Given, $P(\text{not } E \text{ or not } F) = 0.25$

For A and B to be mutually exclusive, $P(A \cap B) = 0$

Now, $P(\text{not } E \text{ or not } F) = P(E' \cup F') = 0.25$

We know,

$$P(A' \cup B') = P(A \cap B)' = 1 - P(A \cap B)$$

$$\implies 0.25 = 1 - P(E \cap F)$$

$$\implies P(E \cap F) = 1 - 0.25 = 0.75 \neq 0$$

Hence, E and F are not mutually exclusive.

Question:17(i) A and B are events such that $P(A) = 0.42$, $P(B) = 0.48$ and $P(A \text{ and } B) = 0.16$ Determine (i) $P(\text{not } A)$

Answer:

Given, $P(A) = 0.42$, $P(B) = 0.48$ and $P(A \text{ and } B) = 0.16$

$$(i) P(\text{not } A) = P(A') = 1 - P(A)$$

$$\implies P(\text{not } A) = 1 - 0.42 = 0.58$$

Therefore, $P(\text{not } A) = 0.58$

Question:17.(ii) A and B are events such that $P(A) = 0.42$, $P(B) = 0.48$ and $P(A \text{ and } B) = 0.16$. Determine $P(\text{not } B)$

Answer:

Given, $P(A) = 0.42$, $P(B) = 0.48$ and $P(A \text{ and } B) = 0.16$

$$(ii) P(\text{not } B) = P(B') = 1 - P(B)$$

$$\implies P(\text{not } B) = 1 - 0.48 = 0.52$$

Therefore, $P(\text{not } B) = 0.52$

Question:17(iii) A and B are events such that $P(A) = 0.42$, $P(B) = 0.48$ and $P(A \text{ and } B) = 0.16$. Determine $P(A \text{ or } B)$

Answer:

Given, $P(A) = 0.42$, $P(B) = 0.48$ and $P(A \text{ and } B) = 0.16$

(iii) We know,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\implies P(A \cup B) = 0.42 + 0.48 - 0.16 = 0.9 - 0.16$$

$$= 0.74$$

Question:18 In Class XI of a school 40% of the students study Mathematics and 30% study Biology. 10% of the class study both Mathematics and Biology. If a student is selected at random from the class, find the probability that he will be studying Mathematics or Biology.

Answer:

Let M denote the event that the student is studying Mathematics and B denote the event that the student is studying Biology

And total students in the class be 100.

$$\text{Given, } n(M) = 40 \implies P(M) = \frac{40}{100} = \frac{2}{5}$$

$$n(B) = 30 \implies P(B) = \frac{30}{100} = \frac{3}{10}$$

$$n(M \cap B) = 10 \implies P(M \cap B) = \frac{10}{100} = \frac{1}{10}$$

We know,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\implies P(M \cup B) = 0.4 + 0.3 - 0.1 = 0.6$$

Hence, the probability that he will be studying Mathematics or Biology is 0.6

Question:19 In an entrance test that is graded on the basis of two examinations, the probability of a randomly chosen student passing the first examination is 0.8 and the probability of passing the second examination is 0.7 . The probability of passing atleast one of them is 0.95 . What is the probability of passing both?

Answer:

Let A be the event that the student passes the first examination and B be the event that the students passes the second examination.

$P(A \cup B)$ is probability of passing at least one of the examination.

Therefore,

$$P(A \cup B) = 0.95, P(A) = 0.8, P(B) = 0.7$$

We know,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow P(A \cap B) = 0.8 + 0.7 - 0.95 = 1.5 - 0.95 = 0.55$$

Hence, the probability that the student will pass both the examinations is 0.55

Question:20. The probability that a student will pass the final examination in both English and Hindi is 0.5 and the probability of passing neither is 0.1. If the probability of passing the English examination is 0.75, what is the probability of passing the Hindi examination?

Answer:

Let A be the event that the student passes English examination and B be the event that the students pass Hindi examination.

Given,

$$P(A) = 0.75, P(A \cap B) = 0.5, P(A' \cap B') = 0.1$$

We know,

$$P(A' \cap B') = 1 - P(A \cup B)$$

$$\Rightarrow P(A \cup B) = 1 - 0.1 = 0.9$$

Also,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow P(B) = 0.9 - 0.75 + 0.5 = 0.65$$

Hence, the probability of passing the Hindi examination is 0.65

Question: 21.(i) In a class of 60 students, 30 opted for NCC, 32 opted for NSS and 24 opted for both NCC and NSS. If one of these students is selected at random, find the probability that

The student opted for NCC or NSS.

Answer:

Let A be the event that student opted for NCC and B be the event that the student opted for NSS.

Given,

$$n(S) = 60, n(A) = 30, n(B) = 32, n(A \cap B) = 24$$

$$\text{Therefore, } P(A) = \frac{30}{60} = \frac{1}{2}$$

$$P(B) = \frac{32}{60} = \frac{8}{15}$$

$$P(A \cap B) = \frac{24}{60} = \frac{2}{5}$$

(i) We know,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\frac{1}{2} + \frac{8}{15} - \frac{2}{5} = \frac{15 + 16 - 12}{30}$$

$$= \frac{19}{30}$$

Hence, the probability that the student opted for NCC or NSS is $\frac{19}{30}$

Question:21.(ii) In a class of 60 students, 30 opted for NCC, 32 opted for NSS and 24 opted for both NCC and NSS. If one of these students is selected at random, find the probability that

The student has opted neither NCC nor NSS.

Answer:

Let A be the event that student opted for NCC and B be the event that the student opted for NSS.

Given,

$$n(S) = 60, n(A) = 30, n(B) = 32, n(A \cap B) = 24$$

$$\text{Therefore, } P(A) = \frac{30}{60} = \frac{1}{2}$$

$$P(B) = \frac{32}{60} = \frac{8}{15}$$

$$P(A \cap B) = \frac{24}{60} = \frac{2}{5}$$

(ii) Now,

Probability that the student has opted neither NCC nor NSS = $P(A' \cap B')$

We know,

$$P(A' \cap B') = 1 - P(A \cup B) \text{ [De Morgan's law]}$$

$$\text{And, } P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{19}{30}$$

$$\therefore P(A' \cap B')$$

$$= 1 - \frac{19}{30} = \frac{11}{30}$$

Hence, the probability that the student has opted neither NCC nor NSS. is $\frac{11}{30}$

Question:21(iii) In a class of 60 students, 30 opted for NCC, 32 opted for NSS and 24 opted for both NCC and NSS. If one of these students is selected at random, find the probability that The student has opted NSS but not NCC.

Answer:

Let A be the event that student opted for NCC and B be the event that the student opted for NSS.

Given,

$$n(S) = 60, n(A) = 30, n(B) = 32, n(A \cap B) = 24$$

$$\text{Therefore, } P(A) = \frac{30}{60} = \frac{1}{2}$$

$$P(B) = \frac{32}{60} = \frac{8}{15}$$

$$P(A \cap B) = \frac{24}{60} = \frac{2}{5}$$

(iii) Now,

Probability that the student has opted NSS but not NCC = $P(B \cap A') = P(B-A)$

We know,

$$P(B-A) = P(B) - P(A \cap B)$$

$$= \frac{8}{15} - \frac{2}{5} = \frac{8-6}{15}$$

$$= \frac{2}{15}$$

Hence, the probability that the student has opted NSS but not NCC is $\frac{2}{15}$

NCERT solutions for class 11 maths chapter 16 probability- Miscellaneous Exercise

Question:1(i) A box contains 10 red marbles, 20 blue marbles and 30 green marbles. 5 marbles are drawn from the box, what is the probability that all will be blue?

Answer:

Given,

No. of red marbles = 10

No. of blue marbles = 20

No. of green marbles = 30

Total number of marbles = $10 + 20 + 30 = 60$

Number of ways of drawing 5 marbles from 60 marbles = ${}^{60}C_5$

(i) .

Number of ways of drawing 5 blue marbles from 20 blue marbles = ${}^{20}C_5$

∴ Probability of drawing all blue marbles = $\frac{{}^{20}C_5}{{}^{60}C_5}$

Question: 1.(ii) A box contains 10 red marbles, 20 blue marbles and 30 green marbles. 5 marbles are drawn from the box, what is the probability that atleast one will be green?

Answer:

Given,

No. of red marbles = 10

No. of blue marbles = 20

No. of green marbles = 30

Total number of marbles = $10 + 20 + 30 = 60$

Number of ways of drawing 5 marbles from 60 marbles = ${}^{60}C_5$

(ii). We know,

The probability that at least one marble is green = $1 - \text{Probability that no marble is green}$

Now, Number of ways of drawing no green marbles = Number of ways of drawing only red and blue marbles

$$= {}^{(20+10)}C_5 = {}^{30}C_5$$

$$\therefore \text{The probability that no marble is green} = \frac{{}^{30}C_5}{{}^{60}C_5}$$

$$\therefore \text{The probability that at least one marble is green} = 1 - \frac{{}^{30}C_5}{{}^{60}C_5}$$

Question:2 4 cards are drawn from a well – shuffled deck of 52 cards. What is the probability of obtaining 3 diamonds and one spade?

Answer:

Total number of ways of drawing 4 cards from a deck of 52 cards = ${}^{52}C_4$

We know that there are 13 diamonds and 13 spades in a deck.

Now, Number of ways of drawing 3 diamonds and 1 spade = ${}^{13}C_3 \cdot {}^{13}C_1$

\therefore Probability of obtaining 3 diamonds and 1 spade

$$= \frac{{}^{13}C_3 \cdot {}^{13}C_1}{{}^{52}C_4}$$

Question:3.(i) A die has two faces each with number '1', three faces each with number '2' and one face with number '3'. If die is rolled once, determine

$$P(2)$$

Answer:

Total number of faces of a die = 6

(i) Number of faces with number '2' = 3

$$P(2) = \frac{3}{6} = \frac{1}{2}$$

Therefore, required probability P(2) is 0.5

Question:3.(ii) A die has two faces each with number '1', three faces each with number '2' and one face with number '3'. If die is rolled once, determine

P(1 or 3)

Answer:

Total number of faces of a die = 6

(ii) $P(1 \text{ or } 3) = P(\text{not } 2) = 1 - P(2)$

$$= 1 - P(2) = 1 - \frac{3}{6} = \frac{1}{2}$$

Therefore, required probability P(1 or 3) is 0.5

Question:3.(iii) A die has two faces each with number '1', three faces each with number '2' and one face with number '3'. If die is rolled once, determine

P(not 3)

Answer:

Total number of faces of a die = 6

(iii) Number of faces with number '3' = 1

$$\therefore P(3) = \frac{1}{6}$$

$$\therefore P(\text{not } 3) = 1 - P(3)$$

$$= 1 - \frac{1}{6} = \frac{5}{6}$$

Therefore, required probability P(not 3) is $\frac{5}{6}$

Question:4(a) In a certain lottery 10,000 tickets are sold and ten equal prizes are awarded. What is the probability of **not getting a prize** if you buy one ticket

Answer:

Given, Total number of tickets sold = 10,000

Number of prizes awarded = 10

(a) If one ticket is bought,

$$P(\text{getting a prize}) = \frac{10}{10000} = \frac{1}{1000}$$

$$\therefore P(\text{not getting a prize}) = 1 - P(\text{getting a prize})$$

$$1 - \frac{1}{1000} = \frac{999}{1000}$$

Question:4(b) In a certain lottery 10,000 tickets are sold and ten equal prizes are awarded. What is the probability of not getting a prize if you buy two tickets

Answer:

Given, Total number of tickets sold = 10,000

Number of prizes awarded = 10

(b) If two tickets are bought:

Number of tickets not awarded = 10000 - 10 = 9990

$$\therefore P(\text{not getting a prize}) = \frac{{}^{9990}C_2}{{}^{10000}C_2}$$

Question:4(c) In a certain lottery 10,000 tickets are sold and ten equal prizes are awarded. What is the probability of not getting a prize if you buy 10 tickets.

Answer:

Given, Total number of tickets sold = 10,000

Number of prizes awarded = 10

(c) If ten tickets are bought:

Number of tickets not awarded = 10000 - 10 = 9990

$$\therefore P(\text{not getting a prize}) = \frac{{}^{9990}C_{10}}{{}^{10000}C_{10}}$$

Question:5.(a) Out of 100 students, two sections of 40 and 60 are formed. If you and your friend are among the 100 students, what is the probability that you both enter the same section?

Answer:

Total number of students = 100

Let A and B be the two sections consisting of 40 and 60 students respectively.

Number of ways of selecting 2 students out of 100 students. = ${}^{100}C_2$

If both are in section A:

Number of ways of selecting 2 students out of 100 = ${}^{100}C_2$ (The remaining 98 will automatically be in section B!)

Remaining 98 students are to be chosen out of (100-2 =) 98 students

∴ Required probability if they both are in section A = $\frac{{}^{98}C_2}{{}^{100}C_2}$

Similarly,

If both are in section B:

Number of ways of selecting 2 students out of 100 = ${}^{100}C_2 = {}^{100}C_2$ (The remaining 98 will automatically be in section A!)

Remaining 98 students are to be chosen out of (100-2 =) 98 students

∴ Required probability if they both are in section B = $\frac{{}^{98}C_2}{{}^{100}C_2}$

Required probability that both are in same section = Probability that both are in section

A + Probability that both are in section B

$$= \frac{{}^{98}C_2}{{}^{100}C_2} + \frac{{}^{98}C_2}{{}^{100}C_2}$$

$$= \frac{85}{165} = \frac{17}{33}$$

Hence, the required probability that both are in same section is $\frac{17}{33}$

Question:5.(b) Out of 100 students, two sections of 40 and 60 are formed. If you and your friend are among the 100 students, what is the probability that you both enter the different sections?

Answer:

Total number of students = 100

Let A and B be the two sections consisting of 40 and 60 students respectively.

We found out in (a) that the probability that both students are in same section is $\frac{17}{33}$

(b) Probability that both the students are in different section = $1 - \frac{17}{33} = \frac{16}{33}$

Question:6 Three letters are dictated to three persons and an envelope is addressed to each of them, the letters are inserted into the envelopes at random so that each envelope contains exactly one letter. Find the probability that at least one letter is in its proper envelope.

Answer:

Given, 3 letters are put in 3 envelopes.

The number of ways of putting the 3 different letters randomly = 3!

Number of ways that at least one of the 3 letters is in the correct envelope

= No. of ways that exactly 1 letter is in correct envelope + No. of ways that 2 letters are in the correct envelope (The third is automatically placed correctly)

= No. of ways that exactly 1 letter is in correct envelope + No. of ways that all the 3 letters are in the correct envelope

$$= ({}^3C_1 \times 1) + 1 = 4$$

(Explanation for ${}^3C_1 \times 1$;

No. of ways of selecting 1 envelope out of 3 = 3C_1 .

If we put the correct letter in it, there is only one way the other two are put in the wrong envelope!)

Therefore, the probability that at least one letter is in its proper envelope = $\frac{4}{3!} = \frac{4}{6} = \frac{2}{3}$

Question:7(i) A and B are two events such that $P(A) = 0.54$, $P(B) = 0.69$ and $P(A \cap B) = 0.35$ Find $P(A \cup B)$

Answer:

Given, $P(A) = 0.54$, $P(B) = 0.69$, $P(A \cap B) = 0.35$

(i) We know, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\Rightarrow P(A \cup B) = 0.54 + 0.69 - 0.35 = 0.88$$

$$\Rightarrow P(A \cup B) = 0.88$$

Question:7(ii) A and B are two events such that $P(A) = 0.54$, $P(B) = 0.69$ and $P(A \cap B) = 0.35$. Find $P(A' \cap B')$

Answer:

Given, $P(A) = 0.54$, $P(B) = 0.69$, $P(A \cap B) = 0.35$

And, $P(A \cup B) = 0.88$

(ii) $A' \cap B' = (A \cup B)'$ [De Morgan's law]

So, $P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B) = 1 - 0.88 = 0.12$

$\therefore P(A' \cap B') = 0.12$

Question:7(iii) A and B are two events such that $P(A) = 0.54$, $P(B) = 0.69$ and $P(A \cap B) = 0.35$. Find $P(A \cap B')$

Answer:

Given, $P(A) = 0.54$, $P(B) = 0.69$, $P(A \cap B) = 0.35$

And, $P(A \cup B) = 0.88$

(iii) $P(A \cap B') = P(A - B) = P(A) - P(A \cap B)$

$= 0.54 - 0.35 = 0.19$

$\therefore P(A \cap B') = 0.19$

Question:7(iv) A and B are two events such that $P(A) = 0.54$, $P(B) = 0.69$ and $P(A \cap B) = 0.35$. Find $P(B \cap A')$

Answer:

Given, $P(A) = 0.54$, $P(B) = 0.69$, $P(A \cap B) = 0.35$

And, $P(A \cup B) = 0.88$

(iv) $P(B \cap A') = P(B - A) = P(B) - P(A \cap B)$

$$= 0.69 - 0.35 = 0.34$$

$$\therefore P(B \cap A') = 0.34$$

Question:8 From the employees of a company, 5 persons are selected to represent them in the managing committee of the company. Particulars of five persons are as follows:

S. No.	Name	Sex	Age in years
1.	Harish	M	30
2.	Rohan	M	33
3.	Sheetal	F	46
4.	Alis	F	28
5.	Salim	M	41

A person is selected at random from this group to act as a spokesperson. What is the probability that the spokesperson will be either male or over 35 years?

Answer:

Given,

Total number of persons = 5

No. of male spokesperson = 3

No. of spokesperson who is over 35 years of age = 2

Let E be the event that the spokesperson is a male and F be the event that the spokesperson is over 35 years of age.

$$\therefore P(E) = \frac{3}{5} \text{ and } P(F) = \frac{2}{5}$$

Since only one male is over 35 years of age,

$$\therefore P(E \cap F) = \frac{1}{5}$$

We know,

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

$$\implies P(E \cup F) = \frac{3}{5} + \frac{2}{5} - \frac{1}{5} = \frac{4}{5}$$

Therefore, the probability that the spokesperson will either be a male or over 35 years of age is $\frac{4}{5}$.

Question:9(i) If 4 -digit numbers greater than 5000 are randomly formed from the digits 0, 1, 3, 5 and 7, what is the probability of forming a number divisible by 5 when, the digits are repeated?

Answer:

Since 4-digit numbers greater than 5000 are to be formed,

The 1000's place digit can be filled up by either 7 or 5 in 2C_1 ways

Since repetition is allowed,

Each of the remaining 3 places can be filled by any of the digits 0, 1, 3, 5, or 7 in 5 ways.

$$\begin{aligned} \therefore \text{Total number of 4-digit numbers greater than 5000} &= {}^2C_1 \times 5 \times 5 \times 5 - 1 \\ &= 250 - 1 = 249 \text{ (5000 cannot be counted, hence one less)} \end{aligned}$$

We know, a number is divisible by 5 if unit's place digit is either 0 or 5.

$$\begin{aligned} \therefore \text{Total number of 4-digit numbers greater than 5000 that are divisible by 5} \\ = {}^2C_1 \times 5 \times 5 \times {}^2C_1 - 1 = 100 - 1 = 99 \end{aligned}$$

Therefore, the required probability =

$$P(\text{with repetition}) = \frac{99}{249} = \frac{33}{83}$$

Question:9(ii) If 4 -digit numbers greater than 5,000 are randomly formed from the digits 0, 1, 3, 5 and 7, what is the probability of forming a number divisible by 5 when, the repetition of digits is not allowed?

Answer:

Since 4-digit numbers greater than 5000 are to be formed,

The 1000's place digit can be filled up by either 7 or 5 in 2C_1 ways

Since repetition is not allowed,

The remaining 3 places can be filled by remaining 4 digits in ${}^4C_3 \times 3!$ ways.

$$\begin{aligned} \therefore \text{Total number of 4-digit numbers greater than 5000} \\ = {}^2C_1 \times ({}^4C_3 \times 3!) = 2 \times 4 \times 6 = 48 \end{aligned}$$

We know, a number is divisible by 5 if unit's place digit is either 0 or 5.

Case 1. When digit at 1000's place is 5, the units place can be filled only with 0.

And the 100's & 10's places can be filled with any two of the remaining digits {1,3,7} in ${}^3C_2 \times 2!$

$$\begin{aligned} \therefore \text{Number of 4-digit numbers starting with 5 and divisible by 5} \\ = 1 \times {}^3C_2 \times 2! = 1.3.2 = 6 \end{aligned}$$

Case 2. When digit at 1000's place is 7, the units place can be filled by 0 or 5 in 2 ways.

And the 100's & 10's places can be filled with any two of the remaining 3 digits in ${}^3C_2 \times 2!$

$$\begin{aligned} \therefore \text{Number of 4-digit numbers starting with 7 and divisible by 5} \\ = 1 \times 2 \times ({}^3C_2 \times 2!) = 1.2.3.2 = 12 \end{aligned}$$

∴ Total number of 4-digit numbers greater than 5000 that are divisible by 5 = 6 + 12 = 18

Therefore, the required probability =

$$P(\text{without repetition}) = \frac{18}{48} = \frac{3}{8}$$

Question:10 The number lock of a suitcase has 4 wheels, each labelled with ten digits i.e., from 0 to 9. The lock opens with a sequence of four digits with no repeats. What is the probability of a person getting the right sequence to open the suitcase?

Answer:

Given, Each wheel can be labelled with 10 digits.

Number of ways of selecting 4 different digits out of the 10 digits = ${}^{10}C_4$

These 4 digits can arranged among themselves is $4!$ ways.

∴ Number of four digit numbers without repetitions =

$${}^{10}C_4 \times 4! = \frac{10!}{4!.6!} \times 4! = 10 \times 9 \times 8 \times 7 = 5040$$

Number of combination that can open the suitcase = 1

∴ Required probability of getting the right sequence to open the suitcase = $\frac{1}{5040}$