NCERT Solutions for Class 11 Maths Chapter 13 Limits and Derivatives

Question:1 Evaluate the following limits

$\lim_{x \to 3} x + 3$

Answer:

$\lim_{x \to 3} x + 3$

$\Rightarrow \lim_{x \to 3} 3 + 3$

$\Rightarrow 6$ Answer

Question:2 Evaluate the following limits $\lim_{x \to \pi} (x - 22/7)$

Answer:

Below you can find the solution:

$\lim_{x \to \pi} (x - 22/7) = \pi - \frac{22}{7}$

Question:3 Evaluate the following limits $\lim_{r \to 1} \pi r^2$

Answer:

The limit

$\lim_{r \to 1} \pi r^2 = \pi (1)^2 = \pi$

Answer is π

Question:4 Evaluate the following limits $\lim_{x \to 4} \frac{4x + 3}{x - 2}$

Answer:

The limit

$$\lim_{x \to 4} \frac{4x+3}{x-2}$$

$$\Rightarrow \frac{4(4)+3}{(4)-2}$$

$$\Rightarrow \frac{19}{2}$$ **(Answer)**

Question:5 Evaluate the following limits $\lim_{x \to -1} \frac{x^{10}+x^5+1}{x-1}$

Answer:

The limit

$$\lim_{x \to 4} \frac{x^{10}+x^5+1}{x-1}$$

$$\Rightarrow \frac{(-1)^{10}+(-1)^5+1}{(-1)-1}$$

$$\Rightarrow \frac{1-1+1}{-2}$$

$$\Rightarrow -\frac{1}{2}$$ **(Answer)**

Question:6 Evaluate the following limits $\lim_{x \to 0} \frac{(x+1)^5-1}{x}$

Answer:

The limit

$$\lim_{x \to 0} \frac{(x+1)^5 - 1}{x}$$

Lets put

$$x + 1 = y$$

since we have changed the function, its limit will also change,

so

$$x \to 0, y \to 0 + 1 = 1$$

So our function have became

$$\lim_{y \to 1} \frac{y^5 - 1}{y - 1}$$

Now As we know the property

$$\lim_{x \to 1} \frac{x^5 - a^n}{x - a} = na^{n-1}$$

$$\lim_{y \to 1} \frac{y^5 - 1}{y - 1} = 5(1)^5 = 5$$

Hence,

$$\lim_{x \to 0} \frac{(x+1)^5 - 1}{x} = 5$$

Question:7 Evaluate the following limits $\lim_{x \to 2} \frac{3x^2 - x - 10}{x^2 - 4}$

Answer:

The limit

$$\lim_{x\to 2}\frac{3x^2-x-10}{x^2-4}$$

$$\Rightarrow \lim_{x\to 2}\frac{(x-2)(3x+5)}{(x-2)(x+2)}$$

$$\Rightarrow \lim_{x\to 2}\frac{(3x+5)}{(x+2)}$$

$$\Rightarrow \frac{(3(2)+5)}{((2)+2)}$$

$\Rightarrow \frac{11}{4}$ **(Answer)**

Question:8 Evaluate the following limits $\lim_{x\to 3}\frac{x^4-81}{2x^2-5x-3}$

Answer:

The limit

$$\lim_{x\to 3}\frac{x^4-81}{2x^2-5x-3}$$

At x = 2 both numerator and denominator becomes zero, so lets factorise the function

$$\lim_{x\to 3}\frac{(x-3)(x+3)(x^2+9)}{(x-3)(2x+1)}$$

$$\lim_{x\to 3}\frac{(x+3)(x^2+9)}{(2x+1)}$$

Now we can put the limit directly, so

$$\lim_{x\to 3}\frac{(x+3)(x^2+9)}{(2x+1)}$$

$$\Rightarrow \frac{((3)+3)((3)^2+9)}{(2(3)+1)}$$

$$\Rightarrow \frac{6 \times 18}{7}$$

$$\Rightarrow \frac{108}{7}$$

Question:9 Evaluate the following limits $\lim_{x \to 0} \frac{ax+b}{cx+1}$

Answer:

The limit,

$$\lim_{x \to 0} \frac{ax+b}{cx+1}$$

$$\Rightarrow \frac{a(0)+b}{c(0)+1}$$

$\Rightarrow b$ **(Answer)**

Question:10 Evaluate the following limits $\lim_{z \to 1} \frac{z^{1/3}-1}{z^{1/6}-1}$

Answer:

The limit

$$\lim_{z \to 1} \frac{z^{1/3}-1}{z^{1/6}-1}$$

Here on directly putting limit , both numerator and the deniminator becomes zero so we factorize the function and then put the limit.

$$\lim_{z\to 1}\frac{z^{1/3}-1}{z^{1/6}-1}=\lim_{z\to 1}\frac{z^{(1/6)^2}-1^2}{z^{1/6}-1}$$

$$=\lim_{z\to 1}\frac{(z^{(1/6)}-1)(z^{(1/6)}+1)}{z^{1/6}-1}$$

$$=\lim_{z\to 1}(z^{(1/6)}+1)$$

$$=(1^{(1/6)}+1)$$

$$=1+1$$

$=2$ **(Answer)**

Question:11 Evaluate the following limits $\lim_{x\to 1}\frac{ax^2+bx+c}{cx^2+bx+a}, a+b+c\neq 0$

Answer:

The limit:

$$\lim_{x\to 1}\frac{ax^2+bx+c}{cx^2+bx+a}, a+b+c\neq 0$$

Since Denominator is not zero on directly putting the limit, we can directly put the limits, so,

$$\lim_{x\to 1}\frac{ax^2+bx+c}{cx^2+bx+a}, a+b+c\neq 0$$

$$=\frac{a(1)^2+b(1)+c}{c(1)^2+b(1)+a},$$

$$=\frac{a+b+c}{a+b+c},$$

$=1$ **(Answer)**

Question:12 Evaluate the following limits $\lim_{x \to -2} \frac{\frac{1}{x} + \frac{x}{2}}{x + 2}$

Answer:

$$\lim_{x \to -2} \frac{\frac{1}{x} + \frac{x}{2}}{x + 2}$$

Here, since denominator becomes zero on putting the limit directly, so we first simplify the function and then put the limit,

$$\lim_{x \to -2} \frac{\frac{1}{x} + \frac{x}{2}}{x + 2}$$

$$= \lim_{x \to -2} \frac{\frac{x+2}{2x}}{x + 2}$$

$$= \lim_{x \to -2} \frac{1}{2x}$$

$$= \frac{1}{2(-2)}$$

$= -\frac{1}{4}$ **(Answer)**

Question:13 Evaluate the following limits $\lim_{x \to 0} \frac{\sin ax}{bx}$

Answer:

The limit

$$\lim_{x \to 0} \frac{\sin ax}{bx}$$

Here on directly putting the limits, the function becomes $\frac{0}{0}$ form. so we try to make the function in the form of $\frac{sinx}{x}$. so,

$$\lim_{x\to 0}\frac{\sin ax}{bx}$$

$$=\lim_{x\to 0}\frac{\sin ax(ax)}{bx(ax)}$$

$$=\lim_{x\to 0}\frac{\sin ax}{ax}\frac{a}{b}$$

As $\lim_{x\to 0}\frac{sinx}{x}=1$

$$=1\cdot\frac{a}{b}$$

$=\frac{a}{b}$ **(Answer)**

Question:14 Evaluate the following limits $\lim_{x\to 0}\frac{\sin ax}{\sin bx}, a, b\neq 0$

Answer:

The limit,

$$\lim_{x\to 0}\frac{\sin ax}{\sin bx}, a, b\neq 0$$

On putting the limit directly, the function takes the zero by zero form So,we convert it in the form of $\frac{sina}{a}$ .and then put the limit,

$$\Rightarrow\lim_{x\to 0}\frac{\frac{sinax}{ax}}{\frac{sinbx}{bx}}\cdot\frac{ax}{bx}$$

$$= \frac{\lim_{ax \to 0} \frac{sinax}{ax}}{\lim_{bx \to 0} \frac{sinbx}{bx}} \cdot \frac{a}{b}$$

$= \dfrac{a}{b}$ **(Answer)**

Question:15 Evaluate the following limits $\lim\limits_{x \to \pi} \dfrac{\sin(\pi - x)}{\pi(\pi - x)}$

Answer:

The limit

$$\lim_{x \to \pi} \frac{\sin(\pi - x)}{\pi(\pi - x)}$$

$$\Rightarrow \lim_{x \to \pi} \frac{\sin(\pi - x)}{(\pi - x)} \times \frac{1}{\pi}$$

$$= 1 \times \frac{1}{\pi}$$

$= \dfrac{1}{\pi}$ **(Answer)**

Question:16 Evaluate the following limits $\lim\limits_{x \to 0} \dfrac{\cos x}{\pi - x}$

Answer:

The limit

$$\lim_{x \to 0} \frac{\cos x}{\pi - x}$$

the function behaves well on directly putting the limit,so we put the limit directly. So.

$$\lim_{x \to 0} \frac{\cos x}{\pi - x}$$

$$= \frac{\cos(0)}{\pi - (0)}$$

$$= \frac{1}{\pi}$$ **(Answer)**

Question:17 Evaluate the following limits $\lim_{x \to 0} \frac{\cos 2x - 1}{\cos x - 1}$

Answer:

The limit:

$$\lim_{x \to 0} \frac{\cos 2x - 1}{\cos x - 1}$$

The function takes the zero by zero form when the limit is put directly, so we simplify the function and then put the limit

$$\lim_{x \to 0} \frac{\cos 2x - 1}{\cos x - 1}$$

$$\lim_{x \to 0} \frac{-2(sin^2 x)}{-2(sin^2(\frac{x}{2}))}$$

$$= \lim_{x \to 0}$$

$$= \frac{1^2}{1^2} \times 4$$

$= 4$ **(Answer)**

Question:18 Evaluate the following limits $\lim_{x \to 0} \frac{ax + x\cos x}{b \sin x}$

Answer:

$$\lim_{x\to 0}\frac{ax + x\cos x}{b\sin x}$$

The function takes the form zero by zero when we put the limit directly in the function,.

since function consist of sin function and cos function, we try to make the function in the form of $\frac{sin x}{x}$ as we know that it tends to 1 when x tends to 0.

So,

$$\lim_{x\to 0}\frac{ax + x\cos x}{b\sin x}$$

$$= \frac{1}{b}\lim_{x\to 0}\frac{x(a+\cos x)}{\sin x}$$

$$= \frac{1}{b}\lim_{x\to 0}\frac{x}{\sin x}\times (a+\cos x)$$

$$= \frac{1}{b}\times 1\times (a+\cos(0))$$

$= \frac{a+1}{b}$ **(Answer)**

Question:19 Evaluate the following limits $\lim_{x\to 0} x\sec x$

Answer:

$$\lim_{x\to 0} x\sec x$$

As function doesn't create any abnormality on putting the limit directly,we can put limit directly. So,

$$\lim_{x\to 0} x\sec x$$

$$= (0)\sec(0)$$

$= (0) \times 1$

$= 0$. **(Answer)**

Question:20 Evaluate the following limits $\lim_{x\to 0} \frac{\sin ax + bx}{ax + \sin bx} a, b, a+b \neq 0$

Answer:

$$\lim_{x\to 0} \frac{\sin ax + bx}{ax + \sin bx} a, b, a+b \neq 0$$

The function takes the zero by zero form when we put the limit into the function directly, so we try to eliminate this case by simplifying the function. So

$$\lim_{x\to 0} \frac{\sin ax + bx}{ax + \sin bx} a, b, a+b \neq 0$$

$$= \lim_{x\to 0} \frac{\frac{\sin ax}{ax} \cdot ax + bx}{ax + \frac{\sin bx}{bx} \cdot bx}$$

$$= \lim_{x\to 0} \frac{\frac{\sin ax}{ax} \cdot a + b}{a + \frac{\sin bx}{bx} \cdot b}$$

$$= \frac{1 \cdot a + b}{a + 1 \cdot b}$$

$$= \frac{a+b}{a+b}$$

$= 1$ **(Answer)**

Question:21 Evaluate the following limits $\lim_{x\to 0} (\csc x - \cot x)$

Answer:

$$\lim_{x\to 0} (\csc x - \cot x)$$

On putting the limit directly the function takes infinity by infinity form, So we simplify the function and then put the limit

$$\lim_{x\to 0}(\csc x - \cot x)$$

$$= \lim_{x\to 0}\left(\frac{1}{sinx} - \frac{cosx}{sinx}\right)$$

$$= \lim_{x\to 0}\left(\frac{1 - cosx}{sinx}\right)$$

$$= \lim_{x\to 0}\left(\frac{2sin^2(\frac{x}{2})}{sinx}\right)$$

$$= \frac{2}{4} \times (1)^2 \times 0$$

$= 0$ **(Answer)**

Question:22 Evaluate the following limits $\lim_{x\to\pi/2} \frac{\tan 2x}{x - \pi/2}$

Answer:

$$\lim_{x\to\pi/2} \frac{\tan 2x}{x - \pi/2}$$

The function takes zero by zero form when the limit is put directly, so we simplify the function and then put the limits,

So

Let's put

$$y = x - \frac{\pi}{2}$$

Since we are changing the variable, limit will also change.

as

$$x \to \frac{\pi}{2}, y = x - \frac{\pi}{2} \to \frac{\pi}{2} - \frac{\pi}{2} = 0$$

So function in new variable becomes,

$$\lim_{y \to 0} \frac{\tan 2(y + \frac{\pi}{2})}{y + \pi/2 - \pi/2}$$

$$= \lim_{y \to 0} \frac{\tan(2y + \pi)}{y}$$

As we know tha property $tan(\pi + x) = tanx$

$$= \lim_{y \to 0} \frac{\tan(2y)}{y}$$

$$= \lim_{y \to 0} \frac{sin2y}{2y} \cdot \frac{2}{cos2y}$$

$$= 1 \times 2$$

$= 2$ **(Answer)**

Question:23 Find

Answer:

Given Function

$$f(x) = \begin{cases} 2x+3 & x \le 0 \\ 3(x+1) & x > 0 \end{cases}$$

Now,

Limit at x = 0 :

$at\ x = 0^-$

$: \lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} (2x+3) = 2(0)+3 = 3$

$at\ x = 0^+$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} 3(x+1) = 3(0+1) = 3$

Hence limit at x = 0 is 3.

Limit at x = 1

$at\ x = 1^+$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} 3(x+1) = 3(1+1) = 6$

$at\ x = 1^-$

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} 3(x+1) = 3(1+1) = 6$

Hence limit at x = 1 is 6.

Question:24 Find

Answer:

Limit at $x = 1^+$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1}(-x^2 - 1) = -(1)^2 - 1 = -2$$

Limit at $x = 1^-$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1}(x^2 - 1) = (1)^2 - 1 = 0$$

As we can see that Limit at $x = 1^+$ is not equal to Limit at $x = 1^-$,The limit of this function at x = 1 does not exists.

Question:25 Evaluate

Answer:

The right-hand Limit or Limit at $x = 0^+$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \frac{|x|}{x} = \frac{x}{x} = 1$$

The left-hand limit or Limit at $x = 0^-$

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \frac{|x|}{x} = \frac{-x}{x} = -1$$

Since Left-hand limit and right-hand limit are not equal, The limit of this function at x = 0 does not exists.

Question:26 Evaluate

Answer:

The right-hand Limit or Limit at $x = 0^+$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \frac{x}{|x|} = \frac{x}{x} = 1$$

The left-hand limit or Limit at $x = 0^-$

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \frac{x}{|x|} = \frac{x}{-x} = -1$$

Since Left-hand limit and right-hand limit are not equal, The limit of this function at x = 0 does not exists.

Question:27 Find $\lim_{x \to 5} f(x), where f(x) = |x| - 5$

Answer:

$$\lim_{x \to 5} f(x), where f(x) = |x| - 5$$

The right-hand Limit or Limit at $x = 5^+$

$$\lim_{x \to 5^+} f(x) = \lim_{x \to 5^+} |x| - 5 = 5 - 5 = 0$$

The left-hand limit or Limit at $x = 5^-$

$$\lim_{x \to 5^-} f(x) = \lim_{x \to 5^-} |x| - 5 = 5 - 5 = 0$$

Since Left-hand limit and right-hand limit are equal, The limit of this function at x = 5 is 0.

Question:28 Suppose

$$f(x)=\begin{cases}a+bx & x<1\\ 4 & x=1\\ b-ax & x>1\end{cases}$$ f (x) = f (1) what are possible values of a and b?

Answer:

Given,

$$f(x)=\begin{cases}a+bx & x<1\\ 4 & x=1\\ b-ax & x>1\end{cases}$$

And

$$\lim_{x\to 1} f(x)=f(1)$$

Since the limit exists,

left-hand limit = Right-hand limit = f(1).

Left-hand limit = f(1)

$$\lim_{x\to 1^-} f(x)=\lim_{x\to 1}(a+bx)=a+b(1)=a+b=4$$

Right-hand limit

$$\lim_{x\to 1^+} f(x)=\lim_{x\to 1}(b-ax)=b-a(1)=b-a=4$$

From both equations, we get that,

$a=0$ and $b=4$

Hence the possible value of a and b are 0 and 4 respectively.

Question:29 Let a1, a2, . . ., an be fixed real numbers and define a function $f(x) = (x - a_1)(x - a_2)...(x - a_n)$. What is $\lim_{x \to a_1}$ f (x) ? For some $a \neq a_1, a_2....a_n$, compute l $\lim_{x \to a} f(x)$

Answer:

Given,

$f(x) = (x - a_1)(x - a_2)...(x - a_n).$

Now,

Hence

, $\lim_{x \to a_1} f(x) = 0$

Now,

$$\lim_{x \to a} f(x) = \lim_{x \to a}(x - a_1)(x - a_2)...(x - a_n)$$

$$\lim_{x \to a} f(x) = (a - a_1)(a - a_2)(a - a_3)$$

Hence

$$\lim_{x \to a} f(x) = (a - a_1)(a - a_2)(a - a_3)$$.

Question:30 If $f(x) = \begin{cases} |x| + 1 & x < 0 \\ 0 & x = 0 \\ |x| - 1 & x > 0 \end{cases}$ For what value (s) of a does $\lim_{x \to a} f(x)$ exists ?

Answer:

$$f(x) = \begin{cases} |x| + 1 & x < 0 \\ 0 & x = 0 \\ |x| - 1 & x > 0 \end{cases}$$

Limit at x = a exists when the right-hand limit is equal to the left-hand limit. So,

Case 1: when a = 0

The right-hand Limit or Limit at $x = 0^+$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} |x| - 1 = 1 - 1 = 0$$

The left-hand limit or Limit at $x = 0^-$

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} |x| + 1 = 0 + 1 = 1$$

Since Left-hand limit and right-hand limit are not equal, The limit of this function at x = 0 does not exists.

Case 2: When a < 0

The right-hand Limit or Limit at $x = a^+$

$$\lim_{x \to a^+} f(x) = \lim_{x \to a^+} |x| - 1 = a - 1$$

The left-hand limit or Limit at $x = a^-$

$$\lim_{x \to a^-} f(x) = \lim_{x \to a^-} |x| - 1 = a - 1$$

Since LHL = RHL, Limit exists at x = a and is equal to a-1.

Case 3: When a > 0

The right-hand Limit or Limit at $x = a^+$

$$\lim_{x\to a^+} f(x) = \lim_{x\to a^+} |x| + 1 = a + 1$$

The left-hand limit or Limit at $x = a^-$

$$\lim_{x\to a^-} f(x) = \lim_{x\to a^-} |x| + 1 = a + 1$$

Since LHL = RHL, Limit exists at x = a and is equal to a+1

Hence,

The limit exists at all points except at x=0.

Question:31 If the function f(x) satisfies $\lim_{x\to 1} \frac{f(x)-2}{x^2-1} = \pi$, evaluate $\lim_{x\to 1} f(x)$

Answer:

Given

$$\lim_{x\to 1} \frac{f(x)-2}{x^2-1} = \pi$$

Now,

$$\lim_{x\to 1}(f(x)-2) = \pi \lim_{x\to 1}(x^2-1)$$

$$\lim_{x\to 1}(f(x)-2) = \pi(1-1)$$

$$\lim_{x\to 1}(f(x)-2) = 0$$

$$\lim_{x\to 1} f(x) = 2$$

Question:32 If

. For what integers m and n does both $\lim_{x\to 0} f(x)\ and\ \lim_{x\to 1} f(x)$ exist ?

Answer:

Given,

Case 1: Limit at x = 0

The right-hand Limit or Limit at $x = 0^+$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} nx + m = n(0) + m = m$$

The left-hand limit or Limit at $x = 0^-$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} mx^2 + n = m(0)^2 + n = n$$

Hence Limit will exist at x = 0 when m = n .

Case 2: Limit at x = 1

The right-hand Limit or Limit at $x = 1^+$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} nx^3 + m = n(1)^3 + m = n + m$$

The left-hand limit or Limit at $x = 1^-$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} nx + m = n(1) + m = n + m$$

Hence Limit at 1 exists at all integers.

NCERT solutions for class 11 maths chapter 13 limits and derivatives- Exercise: 13.2

Question:1 Find the derivative of $x^2 - 2 \ at \ x = 10$

Answer:

F(x)= $x^2 - 2$

Now, As we know, The derivative of any function at x is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

The derivative of f(x) at x = 10:

$$f'(10) = \lim_{h \to 0} \frac{f(10+h) - f(10)}{h}$$

$$f'(10) = \lim_{h \to 0} \frac{(10+h)^2 - 2 - ((10)^2 - 2)}{h}$$

$$f'(10) = \lim_{h \to 0} \frac{100 + 20h + h^2 - 2 - 100 + 2}{h}$$

$$f'(10) = \lim_{h \to 0} \frac{20h + h^2}{h}$$

$$f'(10) = \lim_{h \to 0} 20 + h$$

$$f'(10) = 20 + 0$$

$f'(10) = 20$

Question:2 Find the derivative of x at x = 1.

Answer:

Given

f(x)= x

Now, As we know, The derivative of any function at x is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

The derivative of f(x) at x = 1:

$$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$$f'(1) = \lim_{h \to 0} \frac{(1+h) - (1)}{h}$$

$$f'(1) = \lim_{h \to 0} \frac{(h)}{h}$$

$f'(1) = 1$ **(Answer)**

Question:3 Find the derivative of 99x at x = l00.

Answer:

f(x)= 99x

Now, As we know, The derivative of any function at x is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

The derivative of f(x) at x = 100:

$$f'(100) = \lim_{h \to 0} \frac{f(100+h) - f(100)}{h}$$

$$f'(100) = \lim_{h \to 0} \frac{99(100+h) - 99(100)}{h}$$

$$f'(100) = \lim_{h \to 0} \frac{99h}{h}$$

$$f'(100) = 99$$

Question:4 (i) Find the derivative of the following functions from first principle. $x^3 - 27$

Answer:

Given

f(x)= $x^3 - 27$

Now, As we know, The derivative of any function at x is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^3 - 27 - ((x)^3 - 27)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{x^3 + h^3 + 3x^2h + 3h^2x - 27 + x^3 + 27}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{h^3 + 3x^2h + 3h^2x}{h}$$

$f'(x) = \lim_{h \to 0} h^2 + 3x^2 + 3hx$

$f'(x) = 3x^2$

Question:4.(ii) Find the derivative of the following function from first principle. $(x-1)(x-2)$

Answer:

f(x)= $(x-1)(x-2)$

Now, As we know, The derivative of any function at x is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{(x+h-1)(x+h-2) - (x-1)(x-2)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{x^2 + xh - 2x + hx + h^2 - 2h - x - h + 2 - x^2 + 2x + x - 2}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{2hx + h^2 - 3h}{h}$$

$$f'(x) = \lim_{h \to 0} 2x + h - 3$$

$f'(x) = 2x - 3$ **(Answer)**

Question:4(iii) Find the derivative of the following functions from first principle. $1/x^2$

Answer:

f(x)= $1/x^2$

Now, As we know, The derivative of any function at x is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{1/(x+h)^2 - 1/(x^2)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\frac{x^2-(x+h)^2}{(x+h)^2 x^2}}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{x^2 - x^2 - 2xh - h^2}{h(x+h)^2 x^2}$$

$$f'(x) = \lim_{h \to 0} \frac{-2xh - h^2}{h(x+h)^2 x^2}$$

$$f'(x) = \lim_{h \to 0} \frac{-2x - h}{(x+h)^2 x^2}$$

$$f'(x) = \frac{-2x - 0}{(x+0)^2 x^2}$$

$f'(x) = \dfrac{-2}{x^3}$ **(Answer)**

Question:4(iv) Find the derivative of the following functions from first principle. $\dfrac{x+1}{x-1}$

Answer:

Given:

$$f(x) = \frac{x+1}{x-1}$$

Now, As we know, The derivative of any function at x is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x)=\lim_{h\to 0}\frac{\frac{x+h+1}{x+h-1}-\frac{x+1}{x-1}}{h}$$

$$f'(x)=\lim_{h\to 0}\frac{\frac{(x+h+1)(x-1)-(x+1)(x+h-1)}{(x-1)(x+h-1)}}{h}$$

$$f'(x)=\lim_{h\to 0}\frac{x^2-x+hx-h+x-1-x^2-xh+x-x-h+1}{(x-1)(x+h-1)h}$$

$$f'(x)=\lim_{h\to 0}\frac{-2h}{(x-1)(x+h-1)h}$$

$$f'(x)=\lim_{h\to 0}\frac{-2}{(x-1)(x+h-1)}$$

$$f'(x)=\frac{-2}{(x-1)(x+0-1)}$$

$$f'(x)=\frac{-2}{(x-1)^2}$$

Question:5 For the function $f(x)=\frac{x^{100}}{100}+\frac{x^{99}}{99}+....+\frac{x^2}{2}+x+1$ Prove that f '(1) =100 f '(0).

Answer:

$$f(x)=\frac{x^{100}}{100}+\frac{x^{99}}{99}+....+\frac{x^2}{2}+x+1$$

As we know, the property,

$$f'(x^n)=nx^{n-1}$$

applying that property we get

$$f'(x)=100\frac{x^{99}}{100}+99\frac{x^{98}}{99}+....+2\frac{x}{2}+1+0$$

$f'(x) = x^{99} + x^{98} + ...x + 1$

Now.

$f'(0) = 0^{99} + 0^{98} + ...0 + 1 = 1$

$f'(1) = 1^{99} + 1^{98} + ...1 + 1 = 100$

So,

Here

$1 \times 100 = 100$

$f'(0) \times 100 = f'(1)$

Hence Proved.

Question:6 Find the derivative of $x^n + ax^{n-1} + a^2x^{n-2} + ... + a^{n-1}x + a^n$ for some fixed real number a.

Answer:

Given

$f(x) = x^n + ax^{n-1} + a^2x^{n-2} + ... + a^{n-1}x + a^n$

As we know, the property,

$f'(x^n) = nx^{n-1}$

applying that property we get

$$f'(x) = nx^{n-1} + a(n-1)x^{n-2} + a^2(n-2)x^{n-3} + ... + a^{n-1}1 + 0$$

$$f'(x) = nx^{n-1} + a(n-1)x^{n-2} + a^2(n-2)x^{n-3} + ... + a^{n-1}$$

Question:7(i) For some constants a and b, find the derivative of $(x-a)(x-b)$

Answer:

Given

$$f(x) = (x-a)(x-b) = x^2 - ax - bx + ab$$

As we know, the property,

$$f'(x^n) = nx^{n-1}$$

and the property

$$\frac{d(y_1 + y_2)}{dx} = \frac{dy_1}{dx} + \frac{dy_2}{dx}$$

applying that property we get

$$f'(x) = 2x - a - b$$

Question:7(ii) For some constants a and b, find the derivative of $(ax^2+b)^2$

Answer:

Given

$$f(x) = (ax^2+b)^2 = a^2x^4 + 2abx^2 + b^2$$

As we know, the property,

$$f'(x^n) = nx^{n-1}$$

and the property

$$\frac{d(y_1+y_2)}{dx} = \frac{dy_1}{dx} + \frac{dy_2}{dx}$$

applying those properties we get

$$f'(x) = 4a^2x^3 + 2(2)abx + 0$$

$$f'(x) = 4a^2x^3 + 4abx$$

$$f'(x) = 4ax(ax^2 + b)$$

Question:7(iii) For some constants a and b, find the derivative of $\frac{x-a}{x-b}$

Answer:

Given,

$$f(x) = \frac{x-a}{x-b}$$

Now As we know the quotient rule of derivative,

$$\frac{d(\frac{y_1}{y_2})}{dx} = \frac{y_2\frac{dy_1}{dx} - y_1\frac{dy_2}{dx}}{y_2^2}$$

So applying this rule, we get

$$\frac{d(\frac{x-a}{x-b})}{dx} = \frac{(x-b)\frac{d(x-a)}{dx} - (x-a)\frac{d(x-b)}{dx}}{(x-b)^2}$$

$$\frac{d(\frac{x-a}{x-b})}{dx} = \frac{(x-b)-(x-a)}{(x-b)^2}$$

$$\frac{d(\frac{x-a}{x-b})}{dx} = \frac{a-b}{(x-b)^2}$$

Hence

$$f'(x) = \frac{a-b}{(x-b)^2}$$

Question:8 Find the derivative of $\frac{x^n - a^n}{x-a}$ for some constant a.

Answer:

Given,

$$f(x) = \frac{x^n - a^n}{x-a}$$

Now As we know the quotient rule of derivative,

$$\frac{d(\frac{y_1}{y_2})}{dx} = \frac{y_2\frac{dy_1}{dx} - y_1\frac{dy_2}{dx}}{y_2^2}$$

So applying this rule, we get

$$\frac{d(\frac{x^n-a^n}{x-a})}{dx} = \frac{(x-a)\frac{d(x^n-a^n)}{dx} - (x^n-a^n)\frac{d(x-a)}{dx}}{(x-a)^2}$$

$$\frac{d(\frac{x^n-a^n}{x-a})}{dx} = \frac{(x-a)nx^{n-1} - (x^n-a^n)}{(x-a)^2}$$

$$\frac{d(\frac{x^n-a^n}{x-a})}{dx} = \frac{nx^n - anx^{n-1} - x^n + a^n}{(x-a)^2}$$

Hence

$$f'(x) = \frac{nx^n - anx^{n-1} - x^n + a^n}{(x-a)^2}$$

Question:9(i) Find the derivative of $2x - 3/4$

Answer:

Given:

$f(x) = 2x - 3/4$

As we know, the property,

$f'(x^n) = nx^{n-1}$

and the property

$\frac{d(y_1 + y_2)}{dx} = \frac{dy_1}{dx} + \frac{dy_2}{dx}$

applying that property we get

$f'(x) = 2 - 0$

$f'(x) = 2$

Question:9(ii) Find the derivative of $(5x^3 + 3x - 1)(x - 1)$

Answer:

Given.

$f(x) = (5x^3 + 3x - 1)(x - 1) = 5x^4 + 3x^2 - x - 5x^3 - 3x + 1$

$f(x) = 5x^4 - 5x^3 + 3x^2 - 4x + 1$

As we know, the property,

$$f'(x^n) = nx^{n-1}$$

and the property

$$\frac{d(y_1 + y_2)}{dx} = \frac{dy_1}{dx} + \frac{dy_2}{dx}$$

applying that property we get

$$f'(x) = 5(4)x^3 - 5(3)x^2 + 3(2)x - 4 + 0$$

$$f'(x) = 20x^3 - 15x^2 + 6x - 4$$

Question:9(iii) Find the derivative of $x^{-3}(5 + 3x)$

Answer:

Given

$$f(x) = x^{-3}(5 + 3x) = 5x^{-3} + 3x^{-2}$$

As we know, the property,

$$f'(x^n) = nx^{n-1}$$

and the property

$$\frac{d(y_1 + y_2)}{dx} = \frac{dy_1}{dx} + \frac{dy_2}{dx}$$

applying that property we get

$$f'(x) = (-3)5x^{-4} + 3(-2)x^{-3}$$

$$f'(x) = -15x^{-4} - 6x^{-3}$$

Question:9(iv) Find the derivative of $x^5(3-6x^{-9})$

Answer:

Given

$$f(x) = x^5(3-6x^{-9}) = 3x^5 - 6x^{-4}$$

As we know, the property,

$$f'(x^n) = nx^{n-1}$$

and the property

$$\frac{d(y_1+y_2)}{dx} = \frac{dy_1}{dx} + \frac{dy_2}{dx}$$

applying that property we get

$$f'(x) = (5)3x^4 - 6(-4)x^{-5}$$

$$f'(x) = 15x^4 + 24x^{-5}$$

Question:9(v) Find the derivative of $x^{-4}(3-4x^{-5})$

Answer:

Given

$$f(x) = x^{-4}(3-4x^{-5}) = 3x^{-4} - 4x^{-9}$$

As we know, the property,

$$f'(x^n) = nx^{n-1}$$

and the property

$$\frac{d(y_1+y_2)}{dx}=\frac{dy_1}{dx}+\frac{dy_2}{dx}$$

applying that property we get

$$f'(x)=(-4)3x^{-5}-(-9)4x^{-10}$$

$$f'(x)=-12x^{-5}+36x^{-10}$$

Question:9(vi) Find the derivative of $\frac{2}{x+1}-\frac{x^2}{3x-1}$

Answer:

Given

$$f(x)=\frac{2}{x+1}-\frac{x^2}{3x-1}$$

As we know the quotient rule of derivative:

$$\frac{d(\frac{y_1}{y_2})}{dx}=\frac{y_2\frac{dy_1}{dx}-y_1\frac{dy_2}{dx}}{y_2^2}$$

and the property

$$\frac{d(y_1+y_2)}{dx}=\frac{dy_1}{dx}+\frac{dy_2}{dx}$$

So applying this rule, we get

$$\frac{d(\frac{2}{x+1}-\frac{x^2}{3x-1})}{dx}=\frac{-2}{(x+1)^2}-\frac{(3x-1)2x-x^23}{(3x-1)^2}$$

$$\frac{d(\frac{2}{x+1}-\frac{x^2}{3x-1})}{dx}=\frac{-2}{(x+1)^2}-\frac{6x^2-2x-3x^2}{(3x-1)^2}$$

$$\frac{d(\frac{2}{x+1}-\frac{x^2}{3x-1})}{dx}=\frac{-2}{(x+1)^2}-\frac{3x^2-2x}{(3x-1)^2}$$

Hence

$$f'(x)=\frac{-2}{(x+1)^2}-\frac{3x^2-2x}{(3x-1)^2}$$

Question:10 Find the derivative of $\cos x$ from first principle.

Answer:

Given,

f(x)= $\cos x$

Now, As we know, The derivative of any function at x is

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$f'(x)=\lim_{h\to 0}\frac{\cos(x+h)-\cos(x)}{h}$$

$$f'(x)=\lim_{h\to 0}\frac{\cos(x)\cos(h)-\sin(x)\sin(h)-\cos(x)}{h}$$

$$f'(x)=\lim_{h\to 0}\frac{\cos(x)\cos(h)-\cos(x)}{h}-\frac{\sin(x)\sin(h)}{h}$$

$$f'(x)=\lim_{h\to 0}\frac{\cos(x)(\cos(h)-1)}{h}-\frac{\sin(x)\sin(h)}{h}$$

$$f'(x) = \lim_{h\to 0} \cos(x)\frac{-2sin^2(h/2)}{h} \cdot - \sin(x)\frac{sinh}{h}$$

$$f'(x) = \cos(x)(0) - sinx(1)$$

$$f'(x) = -\sin(x)$$

Question:11(i) Find the derivative of the following functions: $\sin x \cos x$

Answer:

Given,

f(x)= $\sin x \cos x$

Now, As we know the product rule of derivative,

$$\frac{d(y_1y_2)}{dx} = y_1\frac{dy_2}{dx} + y_2\frac{dy_1}{dx}$$

So, applying the rule here,

$$\frac{d(\sin x \cos x)}{dx} = \sin x\frac{d \cos x}{dx} + \cos x\frac{d \sin x}{dx}$$

$$\frac{d(\sin x \cos x)}{dx} = \sin x(-\sin x) + \cos x(\cos x)$$

$$\frac{d(\sin x \cos x)}{dx} = -\sin^2 x + \cos^2 x$$

$$\frac{d(\sin x \cos x)}{dx} = \cos 2x$$

Question:11(ii) Find the derivative of the following functions: $\sec x$

Answer:

Given

$$f(x) = \sec x = \frac{1}{\cos x}$$

Now As we know the quotient rule of derivative,

$$\frac{d(\frac{y_1}{y_2})}{dx} = \frac{y_2\frac{dy_1}{dx} - y_1\frac{dy_2}{dx}}{y_2^2}$$

So applying this rule, we get

$$\frac{d(\frac{1}{\cos x})}{dx} = \frac{\cos x\frac{d(1)}{dx} - 1\frac{d(\cos x)}{dx}}{\cos^2 x}$$

$$\frac{d(\frac{1}{\cos x})}{dx} = \frac{-1(-\sin x)}{\cos^2 x}$$

$$\frac{d(\frac{1}{\cos x})}{dx} = \frac{\sin x}{\cos^2 x} = \frac{\sin x}{\cos x}\frac{1}{\cos x}$$

$$\frac{d(\sec x)}{dx} = \tan x \sec x$$

Question:11 (iii) Find the derivative of the following functions: $5\sec x + 4\cos x$

Answer:

Given

$$f(x) = 5\sec x + 4\cos x$$

As we know the property

$$\frac{d(y_1 + y_2)}{dx} = \frac{dy_1}{dx} + \frac{dy_2}{dx}$$

Applying the property, we get

$$\frac{d(5\sec x+4\cos x)}{dx}=\frac{d(5\sec x)}{dx}+\frac{d(4\cos x)}{dx}$$

$$\frac{d(5\sec x+4\cos x)}{dx}=5\tan x\sec x+4(-\sin x)$$

$$\frac{d(5\sec x+4\cos x)}{dx}=5\tan x\sec x-4\sin x$$

Question:11(iv) Find the derivative of the following functions: $\csc x$

Answer:

Given :

$$f(x)=\csc x=\frac{1}{\sin x}$$

Now As we know the quotient rule of derivative,

$$\frac{d(\frac{y_1}{y_2})}{dx}=\frac{y_2\frac{dy_1}{dx}-y_1\frac{dy_2}{dx}}{y_2^2}$$

So applying this rule, we get

$$\frac{d(\frac{1}{\sin x})}{dx}=\frac{(\sin x)\frac{d(1)}{dx}-1\frac{d(\sin x)}{dx}}{(\sin x)^2}$$

$$\frac{d(\frac{1}{\sin x})}{dx}=\frac{-1(\cos x)}{(\sin x)^2}$$

$$\frac{d(\frac{1}{\sin x})}{dx}=-\frac{(\cos x)}{(\sin x)}\frac{1}{\sin x}$$

$$\frac{d(\csc x)}{dx}=-\cot x\csc x$$

Question:11(v) Find the derivative of the following functions: $3\cot x+5\csc x$

Answer:

Given,

$$f(x) = 3\cot x + 5\csc x$$

As we know the property

$$\frac{d(y_1 + y_2)}{dx} = \frac{dy_1}{dx} + \frac{dy_2}{dx}$$

Applying the property,

$$\frac{d(3\cot x + 5\csc x)}{dx} = \frac{d(3\cot x)}{dx} + \frac{d(5\csc x)}{dx}$$

$$\frac{d(3\cot x + 5\csc x)}{dx} = 3\frac{d(\frac{\cos x}{\sin x})}{dx} + \frac{d(5\csc x)}{dx}$$

$$\frac{d(3\cot x + 5\csc x)}{dx} = -5\csc x\cot x + 3\frac{d(\frac{\cos x}{\sin x})}{dx}$$

Now As we know the quotient rule of derivative,

$$\frac{d(\frac{y_1}{y_2})}{dx} = \frac{y_2\frac{dy_1}{dx} - y_1\frac{dy_2}{dx}}{y_2^2}$$

So applying this rule, we get

$$\frac{d(3\cot x + 5\csc x)}{dx} = -5\csc x\cot x + 3\left[\frac{\sin x(-\sin x) - \cos x(\cos x)}{\sin^2 x}\right]$$

$$\frac{d(3\cot x + 5\csc x)}{dx} = -5\csc x \cot x + 3\left[\frac{-\sin^2 x - \cos^2 x}{\sin^2 x}\right]$$

$$\frac{d(3\cot x + 5\csc x)}{dx} = -5\csc x \cot x - 3\left[\frac{\sin^2 x + \cos^2 x}{\sin^2 x}\right]$$

$$\frac{d(3\cot x + 5\csc x)}{dx} = -5\csc x \cot x - 3\left[\frac{1}{\sin^2 x}\right]$$

$$\frac{d(3\cot x + 5\csc x)}{dx} = -5\csc x \cot x - 3\csc^2 x$$

Question:11(vi) Find the derivative of the following functions: $5\sin x - 6\cos x + 7$

Answer:

Given,

$$f(x) = 5\sin x - 6\cos x + 7$$

Now as we know the property

$$\frac{d(y_1 + y_2)}{dx} = \frac{dy_1}{dx} + \frac{dy_2}{dx}$$

So, applying the property,

$$f'(x) = 5\cos x - 6(-\sin x) + 0$$

$$f'(x) = 5\cos x + 6(\sin x)$$

$$f'(x) = 5\cos x + 6\sin x$$

Question:11(vii) Find the derivative of the following functions: $2\tan x - 7\sec x$

Answer:

Given

$f(x) = 2\tan x - 7\sec x$

As we know the property

$$\frac{d(y_1 + y_2)}{dx} = \frac{dy_1}{dx} + \frac{dy_2}{dx}$$

Applying this property,

$$\frac{d(2\tan x + 7\sec x)}{dx} = 2\frac{d\tan x}{dx} + 7\frac{d\sec x}{dx}$$

$$\frac{d(2\tan x + 7\sec x)}{dx} = 2\sec^2 x + 7(-\sec x\tan x)$$

$$\frac{d(2\tan x + 7\sec x)}{dx} = 2\sec^2 x - 7\sec x\tan x$$

NCERT solutions for class 11 maths chapter 13 limits and derivatives- Miscellaneous Exercise

Question:1(i) Find the derivative of the following functions from first principle: -x

Answer:

Given.

f(x)=-x

Now, As we know, The derivative of any function at x is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x)=\lim_{h\to 0}\frac{-(x+h)-(-x)}{h}$$

$$f'(x)=\lim_{h\to 0}\frac{-h}{h}$$

$$f'(x)=-1$$

Question:1(ii) Find the derivative of the following functions from first principle: $(-x)^{-1}$

Answer:

Given.

f(x)= $(-x)^{-1}$

Now, As we know, The derivative of any function at x is

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$f'(x)=\lim_{h\to 0}\frac{-(x+h)^{-1}-(-x)^{-1}}{h}$$

$$f'(x)=\lim_{h\to 0}\frac{-\frac{1}{x+h}+\frac{1}{x}}{h}$$

$$f'(x)=\lim_{h\to 0}\frac{\frac{-x+x+h}{x(x+h)}}{h}$$

$$f'(x)=\lim_{h\to 0}\frac{\frac{h}{(x+h)(x)}}{h}$$

$$f'(x)=\lim_{h\to 0}\frac{1}{x(x+h)}$$

$$f'(x)=\frac{1}{x^2}$$

Question:1(iii) Find the derivative of the following functions from first principle: $\sin(x+1)$

Answer:

Given.

$$f(x) = \sin(x+1)$$

Now, As we know, The derivative of any function at x is

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$f'(x) = \lim_{h\to 0}\frac{\sin(x+h+1)-\sin(x+1)}{h}$$

$$f'(x) = \lim_{h\to 0}\frac{2\cos(\frac{x+h+1+x+1}{2})\sin(\frac{x+h+1-x-1}{2})}{h}$$

$$f'(x) = \lim_{h\to 0}\frac{2\cos(\frac{2x+h+2}{2})\sin(\frac{h}{2})}{h}$$

$$f'(x) = \lim_{h\to 0}\frac{\cos(\frac{2x+h+2}{2})\sin(\frac{h}{2})}{\frac{h}{2}}$$

$$f'(x) = \cos(\frac{2x+0+2}{2})\times 1$$

$$f'(x) = \cos(x+1)$$

Question:1(iv) Find the derivative of the following functions from first principle: $\cos(x-\pi/8)$

Answer:

Given.

$$f(x) = \cos(x - \pi/8)$$

Now, As we know, The derivative of any function at x is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\cos(x + h - \pi/8) - cos(x - \pi/8)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{-2\sin\left(\frac{2x+h-\pi/4}{2}\right)\sin\left(\frac{h}{2}\right)}{h}$$

$$f'(x) = \sin\left(\frac{2x + 0 - \pi/4}{2}\right) \times 1$$

$$f'(x) = -\sin\left(x - \pi/8\right)$$

Question:2 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): (x+a)

Answer:

Given

f(x)= x + a

As we know, the property,

$f'(x^n) = nx^{n-1}$

applying that property we get

$f'(x) = 1 + 0$

$f'(x) = 1$

Question:3 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $(px + q)\left(\frac{r}{x} + s\right)$

Answer:

Given

$$f(x) = (px + q)\left(\frac{r}{x} + s\right)$$

$$f(x) = pr + psx + \frac{qr}{x} + qs$$

As we know, the property,

$$f'(x^n) = nx^{n-1}$$

applying that property we get

$$f'(x) = 0 + ps + \frac{-qr}{x^2} + 0$$

$$f'(x) = ps + q\left(\frac{-r}{x^2}\right)$$

$$f'(x) = ps - \left(\frac{qr}{x^2}\right)$$

Question:4 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $(ax+b)(cx+d)^2$

Answer:

Given,

$$f(x)=(ax+b)(cx+d)^2$$

$$f(x)=(ax+b)(c^2x^2+2cdx+d^2)$$

$$f(x)=ac^2x^3+2acdx^2+ad^2x+bc^2x^2+2bcdx+bd^2$$

Now,

As we know, the property,

$$f'(x^n)=nx^{n-1}$$

and the property

$$\frac{d(y_1+y_2)}{dx}=\frac{dy_1}{dx}+\frac{dy_2}{dx}$$

applying that property we get

$$f'(x)=3ac^2x^2+4acdx+ad^2+2bc^2x+2bcd+0$$

$$f'(x)=3ac^2x^2+4acdx+ad^2+2bc^2x+2bcd$$

$$f'(x)=3ac^2x^2+(4acd+2bc^2)x+ad^2+2bcd$$

Question:5 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $ax + b/cx + d$

Answer:

Given,

$$f(x) = \frac{ax+b}{cx+d}$$

Now, As we know the derivative of any function

$$\frac{d(\frac{y_1}{y_2})}{dx} = \frac{y_2 d(\frac{dy_1}{dx}) - y_1(\frac{dy_2}{dx})}{y_2^2}$$

Hence, The derivative of f(x) is

$$\frac{d(\frac{ax+b}{cx+d})}{dx} = \frac{(cx+d)d(\frac{d(ax+b)}{dx}) - (ax+b)(\frac{d(cx+d)}{dx})}{(cx+d)^2}$$

$$\frac{d(\frac{ax+b}{cx+d})}{dx} = \frac{(cx+d)a - (ax+b)c}{(cx+d)^2}$$

$$\frac{d(\frac{ax+b}{cx+d})}{dx} = \frac{acx + ad - acx - bc}{(cx+d)^2}$$

$$\frac{d(\frac{ax+b}{cx+d})}{dx} = \frac{ad - bc}{(cx+d)^2}$$

Hence Derivative of the function is

$$\frac{ad-bc}{(cx+d)^2}.$$

Question:6 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):

$$\frac{1+\frac{1}{x}}{1-\frac{1}{x}}$$

Answer:

Given,

$$f(x)=\frac{1+\frac{1}{x}}{1-\frac{1}{x}}$$

Also can be written as

$$f(x)=\frac{x+1}{x-1}$$

Now, As we know the derivative of any function

$$\frac{d(\frac{y_1}{y_2})}{dx}=\frac{y_2 d(\frac{dy_1}{dx})-y_1(\frac{dy_2}{dx})}{y_2^2}$$

Hence, The derivative of f(x) is

$$\frac{d(\frac{x+1}{x-1})}{dx}=\frac{(x-1)d(\frac{d(x+1)}{dx})-(x+1)(\frac{d(x-1)}{dx})}{(x-1)^2}$$

$$\frac{d(\frac{x+1}{x-1})}{dx}=\frac{(x-1)1-(x+1)1}{(x-1)^2}$$

$$\frac{d(\frac{x+1}{x-1})}{dx}=\frac{x-1-x-1}{(x-1)^2}$$

$$\frac{d(\frac{x+1}{x-1})}{dx}=\frac{-2}{(x-1)^2}$$

Hence Derivative of the function is

$$\frac{-2}{(x-1)^2}$$

Question:7 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):

$$\frac{1}{ax^2+bx+c}$$

Answer:

Given,

$$f(x)=\frac{1}{ax^2+bx+c}$$

Now, As we know the derivative of any such function is given by

$$\frac{d(\frac{y_1}{y_2})}{dx}=\frac{y_2 d(\frac{dy_1}{dx})-y_1(\frac{dy_2}{dx})}{y_2^2}$$

Hence, The derivative of f(x) is

$$\frac{d(\frac{1}{ax^2+bx+c})}{dx}=\frac{(ax^2+bx+c)d(\frac{d(1)}{dx})-1(\frac{d(ax^2+bx+c)}{dx})}{(ax^2+bx+c)^2}$$

$$\frac{d(\frac{1}{ax^2+bx+c})}{dx}=\frac{0-(2ax+b)}{(ax^2+bx+c)^2}$$

$$\frac{d(\frac{1}{ax^2+bx+c})}{dx}=-\frac{(2ax+b)}{(ax^2+bx+c)^2}$$

Question:8 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):

$$\frac{ax+b}{px^2+qx+r}$$

Answer:

Given,

$$f(x) = \frac{ax+b}{px^2+qx+r}$$

Now, As we know the derivative of any function

$$\frac{d(\frac{y_1}{y_2})}{dx} = \frac{y_2 d(\frac{dy_1}{dx}) - y_1(\frac{dy_2}{dx})}{y_2^2}$$

Hence, The derivative of f(x) is

$$\frac{d(\frac{ax+b}{px^2+qx+r})}{dx} = \frac{(px^2+qx+r)a - (ax+b)(2px+q)}{(px^2+qx+r)^2}$$

$$\frac{d(\frac{ax+b}{px^2+qx+r})}{dx} = \frac{apx^2+aqx+ar-2apx^2-aqx-2bpx-bq}{(px^2+qx+r)^2}$$

$$\frac{d(\frac{ax+b}{px^2+qx+r})}{dx} = \frac{-apx^2+ar-2bpx-bq}{(px^2+qx+r)^2}$$

Question:9 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):

$$\frac{px^2+qx+r}{ax+b}$$

Answer:

Given,

$$f(x) = \frac{px^2 + qx + r}{ax + b}$$

Now, As we know the derivative of any function

$$\frac{d(\frac{y_1}{y_2})}{dx} = \frac{y_2 d(\frac{dy_1}{dx}) - y_1(\frac{dy_2}{dx})}{y_2^2}$$

Hence, The derivative of f(x) is

$$\frac{d(\frac{px^2+qx+r}{ax+b})}{dx} = \frac{(ax+b)(2px+q) - (px^2+qx+r)(a)}{(ax+b)^2}$$

$$\frac{d(\frac{px^2+qx+r}{ax+b})}{dx} = \frac{2apx^2 + aqx + 2bpx + bq - apx^2 - aqx - ar}{(ax+b)^2}$$

$$\frac{d(\frac{px^2+qx+r}{ax+b})}{dx} = \frac{apx^2 + 2bpx + bq - ar}{(ax+b)^2}$$

Question:10 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):

$$\frac{a}{x^4} - \frac{b}{x^2} + \cos x$$

Answer:

Given

$$f(x) = \frac{a}{x^4} - \frac{b}{x^2} + \cos x$$

As we know, the property,

$$f'(x^n) = nx^{n-1}$$

and the property

$$\frac{d(y_1 + y_2)}{dx} = \frac{dy_1}{dx} + \frac{dy_2}{dx}$$

applying that property we get

$$f'(x) = \frac{-4a}{x^5} - (\frac{-2b}{x^3}) + (-\sin x)$$

$$f'(x) = \frac{-4a}{x^5} + \frac{2b}{x^3} - \sin x$$

Question:11 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $4\sqrt{x} - 2$

Answer:

Given

$$f(x) = 4\sqrt{x} - 2$$

It can also be written as

$$f(x) = 4x^{\frac{1}{2}} - 2$$

Now,

As we know, the property,

$$f'(x^n) = nx^{n-1}$$

and the property

$$\frac{d(y_1 + y_2)}{dx} = \frac{dy_1}{dx} + \frac{dy_2}{dx}$$

applying that property we get

$$f'(x) = 4(\frac{1}{2})x^{-\frac{1}{2}} - 0$$

$$f'(x) = 2x^{-\frac{1}{2}}$$

$$f'(x) = \frac{2}{\sqrt{x}}$$

Question:12 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):

$$(ax + b)^n$$

Answer:

Given

$$f(x) = (ax + b)^n$$

Now, As we know the chain rule of derivative,

$$[f(g(x))]' = f'(g(x)) \times g'(x)$$

And, the property,

$$f'(x^n) = nx^{n-1}$$

Also the property

$$\frac{d(y_1+y_2)}{dx}=\frac{dy_1}{dx}+\frac{dy_2}{dx}$$

applying those properties we get,

$$f'(x)=n(ax+b)^{n-1}\times a$$

$$f'(x)=an(ax+b)^{n-1}$$

Question:13 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $(ax+b)^n(cx+d)^m$

Answer:

Given

$$f(x)=(ax+b)^n(cx+d)^m$$

Now, As we know the chain rule of derivative,

$$[f(g(x))]'=f'(g(x))\times g'(x)$$

And the Multiplication property of derivative,

$$\frac{d(y_1y_2)}{dx}=y_1\frac{dy_2}{dx}+y_2\frac{dy_1}{dx}$$

And, the property,

$$f'(x^n)=nx^{n-1}$$

Also the property

$$\frac{d(y_1+y_2)}{dx}=\frac{dy_1}{dx}+\frac{dy_2}{dx}$$

Applying those properties we get,

$$f'(x)=(ax+b)^n(m(cx+d)^{m-1})+(cx+d)(n(ax+b)^{n-1})$$

$$f'(x)=m(ax+b)^n(cx+d)^{m-1}+n(cx+d)(ax+b)^{n-1}$$

Question:14 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $\sin(x+a)$

Answer:

Given,

$$f(x)=\sin(x+a)$$

Now, As we know the chain rule of derivative,

$$[f(g(x))]'=f'(g(x))\times g'(x)$$

Applying this property we get,

$$f'(x)=\cos(x+a)\times 1$$

$$f'(x)=\cos(x+a)$$

Question:15 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $\csc x \cot x$

Answer:

Given,

$f(x) = \csc x \cot x$

the Multiplication property of derivative,

$$\frac{d(y_1 y_2)}{dx} = y_1 \frac{dy_2}{dx} + y_2 \frac{dy_1}{dx}$$

Applying the property

$$\frac{d(\csc x)(\cot x))}{dx} = \csc x \frac{d \cot x}{dx} + \cot x \frac{d \csc x}{dx}$$

$$\frac{d(\csc x)(\cot x))}{dx} = \csc x(-\csc^2 x) + \cot x(-\csc x \cot x)$$

$$\frac{d(\csc x)(\cot x))}{dx} = -\csc^3 x - \cot^2 x \csc x$$

Hence derivative of the function is $-\csc^3 x - \cot^2 x \csc x$.

Question:16 Find the derivative of the following functions (it is to be understood that a, b, c, d,p, q, r and s are fixed non-zero constants and m and n are integers):

$$\frac{\cos x}{1 + \sin x}$$

Answer:

Given,

$$f(x) = \frac{\cos x}{1 + \sin x}$$

Now, As we know the derivative of any function

Hence, The derivative of f(x) is

$$\frac{d(\frac{\cos x}{1+\sin x})}{dx} = \frac{(1+\sin x)(-\sin x) - \cos x(\cos x)}{(1+\sin x)^2}$$

$$\frac{d(\frac{\cos x}{1+\sin x})}{dx} = \frac{-\sin x - \sin^2 x - \cos^2 x}{(1+\sin x)^2}$$

$$\frac{d(\frac{\cos x}{1+\sin x})}{dx} = \frac{-\sin x - 1}{(1+\sin x)^2}$$

$$\frac{d(\frac{\cos x}{1+\sin x})}{dx} = -\frac{1}{(1+\sin x)}$$

Question:17 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $\frac{\sin x + \cos x}{\sin x - \cos x}$

Answer:

Given

$$f(x) = \frac{\sin x + \cos x}{\sin x - \cos x}$$

Also can be written as

$$f(x) = \frac{\tan x + 1}{\tan x - 1}$$

which further can be written as

$$f(x) = -\frac{\tan x + tan(\pi/4)}{1 - \tan(\pi/4)\tan x}$$

$$f(x) = -\tan(x - \pi/4)$$

Now,

$$f'(x) = -\sec^2(x - \pi/4)$$

$$f'(x) = -\frac{1}{\cos^2(x - \pi/4)}$$

Question:18 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):

$$\frac{\sec x - 1}{\sec x + 1}$$

Answer:

Given,

$$f(x) = \frac{\sec x - 1}{\sec x + 1}$$

which also can be written as

$$f(x) = \frac{1 - \cos x}{1 + \cos x}$$

Now,

As we know the derivative of such function

$$\frac{d(\frac{y_1}{y_2})}{dx} = \frac{y_2 d(\frac{dy_1}{dx}) - y_1(\frac{dy_2}{dx})}{y_2^2}$$

So, The derivative of the function is,

$$\frac{d(\frac{1-\cos x}{1+\cos x})}{dx}=\frac{(1+\cos x)(-(-\sin x))-(1-\cos x)(-\sin x)}{(1+\cos x)^2}$$

$$\frac{d(\frac{1-\cos x}{1+\cos x})}{dx}=\frac{\sin x+\sin x\cos x+\sin x-\cos x\sin x}{(1+\cos x)^2}$$

$$\frac{d(\frac{1-\cos x}{1+\cos x})}{dx}=\frac{2\sin x}{(1+\cos x)^2}$$

Which can also be written as

$$\frac{d(\frac{1-\cos x}{1+\cos x})}{dx}=\frac{2\sec x\tan x}{(1+\sec x)^2}.$$

Question:19 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $\sin^n x$

Answer:

Given,

$$f(x)=\sin^n x$$

Now, As we know the chain rule of derivative,

$$[f(g(x))]'=f'(g(x))\times g'(x)$$

And, the property,

$$f'(x^n)=nx^{n-1}$$

Applying those properties, we get

$$f'(x)=n\sin^{n-1}x\cos x$$

Hence Derivative of the given function is $n\sin^{n-1}x\cos x$

Question:20 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):

$$\frac{a+b\sin x}{c+d\cos x}$$

Answer:

Given Function

$$f(x)=\frac{a+b\sin x}{c+d\cos x}$$

Now, As we know the derivative of any function of this type is:

$$\frac{d(\frac{y_1}{y_2})}{dx}=\frac{y_2 d(\frac{dy_1}{dx})-y_1(\frac{dy_2}{dx})}{y_2^2}$$

Hence derivative of the given function will be:

$$\frac{d(\frac{a+b\sin x}{c+d\cos x})}{dx}=\frac{(c+d\cos x)(b\cos x)-(a+b\sin x)(d(-\sin x))}{(c+d\cos x)^2}$$

$$\frac{d(\frac{a+b\sin x}{c+d\cos x})}{dx}=\frac{cb\cos x+bd\cos^2 x+ad\sin x+bd\sin^2 x}{(c+d\cos x)^2}$$

$$\frac{d(\frac{a+b\sin x}{c+d\cos x})}{dx}=\frac{cb\cos x+ad\sin x+bd(\sin^2 x+\cos^2 x)}{(c+d\cos x)^2}$$

$$\frac{d(\frac{a+b\sin x}{c+d\cos x})}{dx}=\frac{cb\cos x+ad\sin x+bd}{(c+d\cos x)^2}$$

Question:21 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):

$$\frac{\sin(x+a)}{\cos x}$$

Answer:

Given,

$$f(x)=\frac{\sin(x+a)}{\cos x}$$

Now, As we know the derivative of any function

$$\frac{d(\frac{y_1}{y_2})}{dx}=\frac{y_2 d(\frac{dy_1}{dx})-y_1(\frac{dy_2}{dx})}{y_2^2}$$

Hence the derivative of the given function is:

$$\frac{d(\frac{\sin(x+a)}{\cos x})}{dx}=\frac{(\cos x)(\cos(x+a))-\sin(x+a)(-\sin(x))}{(\cos x)^2}$$

$$\frac{d(\frac{\sin(x+a)}{\cos x})}{dx}=\frac{(\cos x)(\cos(x+a))+\sin(x+a)(\sin(x))}{(\cos x)^2}$$

$$\frac{d(\frac{\sin(x+a)}{\cos x})}{dx}=\frac{\cos(x+a-x)}{(\cos x)^2}$$

$$\frac{d(\frac{\sin(x+a)}{\cos x})}{dx}=\frac{\cos(a)}{(\cos x)^2}$$

Question:22 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $x^4(5\sin x - 3\cos x)$

Answer:

Given

$$f(x) = x^4(5\sin x - 3\cos x)$$

Now, As we know, the Multiplication property of derivative,

$$\frac{d(y_1y_2)}{dx} = y_1\frac{dy_2}{dx} + y_2\frac{dy_1}{dx}$$

Hence derivative of the given function is:

$$\frac{d(x^4(5\sin x - 3\cos x))}{dx} = x^4(5\cos x + 3\sin x) + (5\sin x - 3\cos x)4x^3$$

$$\frac{d(x^4(5\sin x - 3\cos x))}{dx} = 5x^4\cos x + 3x^4\sin x + 20x^3\sin x - 12x^3\cos x$$

Question:23 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $(x^2+1)\cos x$

Answer:

Given

$$f(x) = (x^2+1)\cos x$$

Now, As we know the product rule of derivative,

$$\frac{d(y_1y_2)}{dx} = y_1\frac{dy_2}{dx} + y_2\frac{dy_1}{dx}$$

The derivative of the given function is

$$\frac{d((x^2+1)\cos x)}{dx} = (x^2+1)\frac{d\cos x}{dx} + \cos x\frac{d(x^2+1)}{dx}$$

$$\frac{d((x^2+1)\cos x)}{dx} = (x^2+1)(-\sin x) + \cos x(2x)$$

$$\frac{d((x^2+1)\cos x)}{dx} = -x^2\sin x - \sin x + 2x\cos x$$

Question:24 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $(ax^2+\sin x)(p+q\cos x)$

Answer:

Given,

$$f(x) = (ax^2+\sin x)(p+q\cos x)$$

Now As we know the Multiplication property of derivative,(the product rule)

$$\frac{d(y_1y_2)}{dx} = y_1\frac{dy_2}{dx} + y_2\frac{dy_1}{dx}$$

And also the property

$$\frac{d(y_1+y_2)}{dx} = \frac{dy_1}{dx} + \frac{dy_2}{dx}$$

Applying those properties we get,

$$\frac{d((ax^2+\sin x)(p+q\cos x))}{dx}=(ax^2+\sin x)(-q\sin x)+(p+qx)(2ax+\cos x)$$

$$f'(x)=-aqx^2\sin x-q\sin^2 x+2apx+p\cos x+2aqx^2+qx\cos x$$

$$f'(x)=x^2(-aq\sin x+2aq)+x(2ap+q\cos x)+p\cos x-q\sin^2 x$$

Question:25 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $(x+\cos x)(x-\tan x)$

Answer:

Given,

$$f(x)=(x+\cos x)(x-\tan x)$$

And the Multiplication property of derivative,

$$\frac{d(y_1y_2)}{dx}=y_1\frac{dy_2}{dx}+y_2\frac{dy_1}{dx}$$

Also the property

$$\frac{d(y_1+y_2)}{dx}=\frac{dy_1}{dx}+\frac{dy_2}{dx}$$

Applying those properties we get,

$$=(x+\cos x)(1-\sec^2 x)+(x-\tan x)(1-\sin x)$$

$= (x + \cos x)(-\tan^2 x) + (x - \tan x)(1 - \sin x)$

$= (-\tan^2 x)(x + \cos x) + (x - \tan x)(1 - \sin x)$

Question:26 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):

$$\frac{4x + 5\sin x}{3x + 7\cos x}$$

Answer:

Given,

$$f(x) = \frac{4x + 5\sin x}{3x + 7\cos x}$$

Now, As we know the derivative of any function

$$\frac{d(\frac{y_1}{y_2})}{dx} = \frac{y_2 d(\frac{dy_1}{dx}) - y_1(\frac{dy_2}{dx})}{y_2^2}$$

Also the property

$$\frac{d(y_1 + y_2)}{dx} = \frac{dy_1}{dx} + \frac{dy_2}{dx}$$

Applying those properties,we get

$$\frac{d(\frac{4x+5\sin x}{3x+7\cos x})}{dx} = \frac{(3x + 7\cos x)(4 + 5\cos x) - (4x + 5\sin x)(3 - 7\sin x)}{(3x + 7\cos x)^2}$$

$$= \frac{12x + 28\cos x + 15x\cos x + 35\cos^2 x - 12x - 15\sin x + 28x\sin x + 35\sin^2 x}{(3x + 7\cos x)^2}$$

$$= \frac{12x + 28\cos x + 15x\cos x - 12x - 15\sin x + 28x\sin x + 35(\sin^2 x + \cos^2 x)}{(3x + 7\cos x)^2}$$

$$= \frac{28\cos x + 15x\cos x - 15\sin x + 28x\sin x + 35}{(3x + 7\cos x)^2}$$

Question:27 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):

$$\frac{x^2\cos(\pi/4)}{\sin x}$$

Answer:

Given,

$$f(x) = \frac{x^2\cos(\pi/4)}{\sin x}$$

Now, As we know the derivative of any function

Now, As we know the derivative of any function

$$\frac{d(\frac{y_1}{y_2})}{dx} = \frac{y_2 d(\frac{dy_1}{dx}) - y_1(\frac{dy_2}{dx})}{y_2^2}$$

Hence the derivative of the given function is

$$\frac{d(\frac{x^2\cos(\pi/4)}{\sin x})}{dx} = \frac{(\sin x)(2x\cos(\pi/4)) - (x^2\cos(\pi/4))(\cos x)}{\sin^2 x}$$

$$\frac{d(\frac{x^2\cos(\pi/4)}{\sin x})}{dx} = \frac{2x\sin x\cos(\pi/4) - x^2\cos x\cos(\pi/4)}{\sin^2 x}$$

Question:28 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):

$$\frac{x}{1+\tan x}$$

Answer:

Given

$$f(x)=\frac{x}{1+\tan x}$$

Now, As we know the derivative of any function

$$\frac{d(\frac{x}{1+\tan x})}{dx}=\frac{(1+\tan x)d(\frac{dx}{dx})-x(\frac{d(1+\tan x)}{dx})}{(1+\tan x)^2}$$

$$\frac{d(\frac{x}{1+\tan x})}{dx}=\frac{(1+\tan x)(1)-x(\sec^2 x)}{(1+\tan x)^2}$$

$$\frac{d(\frac{x}{1+\tan x})}{dx}=\frac{1+\tan x-x\sec^2 x}{(1+\tan x)^2}$$

Question:29 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $(x+\sec x)(x-\tan x)$

Answer:

Given

$$f(x)=(x+\sec x)(x-\tan x)$$

Now, As we know the Multiplication property of derivative,

$$\frac{d(y_1y_2)}{dx} = y_1\frac{dy_2}{dx} + y_2\frac{dy_1}{dx}$$

Also the property

$$\frac{d(y_1+y_2)}{dx} = \frac{dy_1}{dx} + \frac{dy_2}{dx}$$

Applying those properties we get,

the derivative of the given function is,

$$\frac{d((x+\sec x)(x-\tan x)}{dx} = (x+\sec x)(1-\sec^2 x)+(x-\tan x)(1+\sec x\tan x)$$

Question:30 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):

$$\frac{x}{\sin^n x}$$

Answer:

Given,

$$f(x) = \frac{x}{\sin^n x}$$

Now, As we know the derivative of any function

$$\frac{d(\frac{y_1}{y_2})}{dx} = \frac{y_2 d(\frac{dy_1}{dx}) - y_1(\frac{dy_2}{dx})}{y_2^2}$$

Also chain rule of derivative,

$$[f(g(x))]' = f'(g(x)) \times g'(x)$$

Hence the derivative of the given function is

$$\frac{d(\frac{x}{\sin^n x})}{dx} = \frac{\sin^n x d(\frac{dx}{dx}) - x(\frac{d(\sin^n x)}{dx})}{sin^{2n}x}$$

$$\frac{d(\frac{x}{\sin^n x})}{dx} = \frac{\sin^n x(1) - x(n\sin^{n-1} x \times \cos x)}{sin^{2n}x}$$

$$\frac{d(\frac{x}{\sin^n x})}{dx} = \frac{\sin^n x - x\cos xn\sin^{n-1} x}{sin^{2n}x}$$

$$\frac{d(\frac{x}{\sin^n x})}{dx} = \frac{\sin^n x - x\cos xn\sin^{n-1} x}{sin^{2n}x}$$

$$\frac{d(\frac{x}{\sin^n x})}{dx} = \frac{\sin^{n-1} x(\sin x - nx\cos x)}{sin^{2n}x}$$

$$\frac{d(\frac{x}{\sin^n x})}{dx} = \frac{(\sin x - nx\cos x)}{sin^{n+1}x}$$