NCERT solutions for class 11 Maths Chapter 12 Introduction to Three Dimensional Geometry

Question:1 A point is on the x-axis. What are its y-coordinate and z-coordinates?

Answer:

Any point on x-axis have zero y coordinate and zero z coordinate.

Question:2 A point is in the XZ-plane. What can you say about its y-coordinate?

Answer:

When a point is in XZ plane, the y coordinate of this point will always be zero.

Question:3 Name the octants in which the following points lie:
(1, 2, 3), (4, –2, 3), (4, –2, –5), (4, 2, –5), (– 4, 2, –5), (– 4, 2, 5), (–3, –1, 6) (– 2, – 4, – 7).

Answer:

The x-coordinate, y-coordinate, and z-coordinate of point (1, 2, 3) are all positive. Therefore, this point lies in octant I.

The x-coordinate, y-coordinate, and z-coordinate of point (4, -2, 3) are positive, negative, and positive respectively. Therefore, this point lies in octant IV.

The x-coordinate, y-coordinate, and z-coordinate of point (4, -2, -5) are positive, negative, and negative respectively. Therefore, this point lies in octant VIII.

The x-coordinate, y-coordinate, and z-coordinate of point (4, 2, -5) are positive, positive, and negative respectively. Therefore, this point lies in octant V.

The x-coordinate, y-coordinate, and z-coordinate of point (-4, 2, -5) are negative, positive, and negative respectively. Therefore, this point lies in octant VI.

The x-coordinate, y-coordinate, and z-coordinate of point (-4, 2, 5) are negative, positive, and positive respectively. Therefore, this point lies in octant II.

The x-coordinate, y-coordinate, and z-coordinate of point (-3, -1, 6) are negative, negative, and positive respectively. Therefore, this point lies in octant III.

The x-coordinate, y-coordinate, and z-coordinate of point (2, -4, -7) are positive, negative, and negative respectively. Therefore, this point lies in octant VIII.

Question:4 (i) Fill in the blanks: The x-axis and y-axis taken together determine a plane known as_______.

Answer:

The x-axis and y-axis taken together determine a plane known as **XY Plane** .

Question:4 (ii) Fill in the blanks: The coordinates of points in the XY-plane are of the form _______.

Answer:

The coordinates of points in the XY-plane are of the form **(x, y, 0)** .

Question:4 (iii) Fill in the blanks: Coordinate planes divide the space into ______ octants.

Answer:

Coordinate planes divide the space into **Eight** octants.

NCERT solutions for class 11 maths chapter 12 introduction to three dimensional geometry-Exercise: 12.2

Question:1 (i) Find the distance between the following pairs of points: (2, 3, 5) and (4, 3, 1)

Answer:

The distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

So, distance between (2, 3, 5) and (4, 3, 1) is given by

$$d = \sqrt{(4-2)^2 + (3-3)^2 + (1-5)^2}$$

$$d = \sqrt{4 + 0 + 16}$$

$$d = \sqrt{20}$$

$$d = 2\sqrt{5}$$

Question:1 (ii) Find the distance between the following pairs of points: (–3, 7, 2) and (2, 4, –1)

Answer:

The distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

So, distance between (–3, 7, 2) and (2, 4, –1) is given by

$$d = \sqrt{(2-(-3))^2 + (4-7)^2 + (-1-2)^2}$$

$$d = \sqrt{25+9+9}$$

$$d = \sqrt{43}$$

Question:1 (iii) Find the distance between the following pairs of points:(–1, 3, – 4) and (1, –3, 4)

Answer:

The distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by

$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

So, distance between (–1, 3, – 4) and (1, –3, 4) is given by

$$d = \sqrt{(1-(-1))^2 + (-3-3)^2 + (4-(-4))^2}$$

$$d = \sqrt{4+36+64}$$

$$d = \sqrt{104}$$

$$d = 2\sqrt{26}$$

Question:1 (iv) Find the distance between the following pairs of points: (2, –1, 3) and (–2, 1, 3).

Answer:

The distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

So, distance between (2, –1, 3) and (–2, 1, 3).) is given by

$$d = \sqrt{(-2 - (-2))^2 + (1 - (-1))^2 + (3 - 3)^2}$$

$$d = \sqrt{16 + 4 + 0}$$

$$d = \sqrt{20}$$

$$d = 2\sqrt{5}$$

Question:2 Show that the points (–2, 3, 5), (1, 2, 3) and (7, 0, –1) are collinear.

Answer:

Given,theree points A=(–2, 3, 5), B=(1, 2, 3) and C=(7, 0, –1)

The distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

The distance AB :

$$AB = \sqrt{(1 - (-2))^2 + (2 - 3)^2 + (3 - 5)^2}$$

$$AB = \sqrt{(3)^2 + (-1)^2 + (-2)^2}$$

$$AB = \sqrt{9 + 1 + 4}$$

$$AB = \sqrt{14}$$

The distance BC:

$$BC = \sqrt{(7-1)^2 + (0-2)^2 + (-1-3)^2}$$

$$BC = \sqrt{36+4+16}$$

$$BC = \sqrt{56}$$

$$BC = 2\sqrt{14}$$

The distance CA

$$CA = \sqrt{(7-(-2))^2 + (0-3)^2 + (-1-5)^2}$$

$$CA = \sqrt{81+9+36}$$

$$CA = \sqrt{126}$$

$$CA = 3\sqrt{14}$$

As we can see here,

$$AB + BC = \sqrt{14} + 2\sqrt{14} = 3\sqrt{14} = AC$$

Hence we can say that point A,B and C are colinear.

Question:3 (i) Verify the following: (0, 7, –10), (1, 6, – 6) and (4, 9, – 6) are the vertices of an isosceles triangle.

Answer:

Given Three points A=(0, 7, –10), B=(1, 6, – 6) and C=(4, 9, – 6)

The distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

The distance AB

$$AB = \sqrt{(1-0)^2 + (6-7)^2 + (-6-(-10))^2}$$

$$AB = \sqrt{1+1+16}$$

$$AB = \sqrt{18}$$

The distance BC

$$BC = \sqrt{(4-1)^2 + (9-6)^2 + (-6-(-6))^2}$$

$$BC = \sqrt{9+9+0}$$

$$BC = \sqrt{18}$$

The distance CA

$$CA = \sqrt{(4-0)^2 + (9-7)^2 + (-6-(-10))^2}$$

$$CA = \sqrt{16+4+16}$$

$$CA = \sqrt{36}$$

$$CA = 6$$

As we can see $AB = BC \neq CA$

Hence we can say that ABC is an isosceles triangle.

Question:3(ii) Verify the following

(0, 7, 10), (- 1, 6, 6) and (- 4, 9, 6) are the vertices of right angled triangle.

Answer:

Given Three points A=(0, 7, 10), B=(-1, 6, 6) and C=(-4, 9, 6)

The distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

The distance AB

$$AB = \sqrt{(-1-0)^2 + (6-7)^2 + (6-10)^2}$$

$$AB = \sqrt{1+1+16}$$

$$AB = \sqrt{18}$$

The distance BC

$$BC = \sqrt{(-4-(-1))^2 + (9-6)^2 + (6-6)^2}$$

$$BC = \sqrt{9+9+0}$$

$$BC = \sqrt{18}$$

The distance CA

$$CA = \sqrt{(-4-0)^2 + (9-7)^2 + (6-10)^2}$$

$$CA = \sqrt{16+4+16}$$

$$CA = \sqrt{36}$$

$CA = 6$

As we can see

$(AB)^2 + (BC)^2 = 18 + 18 = 36 = (CA)^2$

Since this follows pythagorus theorem, we can say that ABC is a right angle triangle.

Question:3(iii) Verify the following: (–1, 2, 1), (1, –2, 5), (4, –7, 8) and (2, –3, 4) are the vertices of a parallelogram.

Answer:

Given A=(–1, 2, 1), B=(1, –2, 5), C=(4, –7, 8) and D=(2, –3, 4)

Given Three points A=(0, 7, –10), B=(1, 6, – 6) and C=(4, 9, – 6)

The distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

The distance AB

$AB = \sqrt{(1 - (-1))^2 + (-2 - 2)^2 + (5 - 1)^2}$

$AB = \sqrt{4 + 16 + 16}$

$AB = \sqrt{36}$

$AB = 6$

The distance BC

$BC = \sqrt{(4-1)^2 + (-7-(-2))^2 + (8-5)^2}$

$BC = \sqrt{9+25+9}$

$BC = \sqrt{43}$

The distance CD

$CD = \sqrt{(2-4)^2 + (-3-(-7))^2 + (4-8)^2}$

$CA = \sqrt{4+16+16}$

$CA = \sqrt{36}$

$CA = 6$

The distance DA

$DA = \sqrt{(-1-2)^2 + (2-(-3))^2 + (1-4)^2}$

$DA = \sqrt{9+25+9}$

$DA = \sqrt{43}$

Here As we can see

$AB = 6 = CA$ And $BC = \sqrt{43} = DA$

As the opposite sides of quadrilateral are equal, we can say that ABCD is a parallelogram.

Question:4 Find the equation of the set of points which are equidistant from the points (1, 2, 3) and (3, 2, –1).

Answer:

Given, two points A=(1, 2, 3) and B=(3, 2, –1).

Let the point P= (x,y,z) be a point which is equidistance from the points A and B.

so,

The distance PA= The distance PB

$$\sqrt{(x-1)^2+(y-2)^2+(z-3)^2}=\sqrt{(x-3)^2+(y-2)^2+(z-(-1))^2}$$

$$(x-1)^2+(y-2)^2+(z-3)^2=(x-3)^2+(y-2)^2+(z-(-1))^2$$

$$\left[(x-1)^2-(x-3)^2\right]+\left[(y-2)^2-(y-2)^2\right]+\left[(z-3)^2-(z+1)^2\right]=0$$

Now lets apply the simplification property,

$$a^2-b^2=(a+b)(a-b)$$

$$[(2)(2x-4)]+0+[(-4)(2z-2)]=0$$

$$4x-8-8z+8=0$$

$$4x-8z=0$$

$$x-2z=0$$

Hence locus of the point which is equidistant from A and B is $x-2z=0$.

Question:5 Find the equation of the set of points P, the sum of whose distances from A (4, 0, 0) and B (– 4, 0, 0) is equal to 10

Answer:

Given,

Two points A (4, 0, 0) and B (– 4, 0, 0)

let the point P(x,y,z) be a point sum of whose distance from A and B is 10.

So,

The distance PA+The distance PB=10

$$\sqrt{(x-4)^2+(y-0)^2+(z-0)^2}+\sqrt{(x-(-4))^2+(y)^2+(z)^2}=10$$

$$\sqrt{(x-4)^2+(y)^2+(z)^2}+\sqrt{(x+4)^2+(y)^2+(z)^2}=10$$

$$\sqrt{(x-4)^2+(y)^2+(z)^2}=10-\sqrt{(x+4)^2+(y)^2+(z)^2}$$

Squaring on both side :

$$(x-4)^2+(y)^2+(z)^2=100-20\sqrt{(x+4)^2+(y)^2+(z)^2}+(x+4)^2+(y)^2+(z)^2$$

$$(x-4)^2-(x+4)^2=100-20\sqrt{(x+4)^2+(y)^2+(z)^2}$$

$$-16x=100-20\sqrt{(x+4)^2+(y)^2+(z)^2}$$

$$20\sqrt{(x+4)^2+(y)^2+(z)^2}=100+16x$$

$$5\sqrt{(x+4)^2+(y)^2+(z)^2}=25+4x$$

Now again squaring both sides,

$$25\left((x+4)^2+(y)^2+(z)^2\right)=625+200x+16x^2$$

$$25x^2 + 200x + 400 + 25y^2 + 25z^2 = 625 + 200x + 16x^2$$

$$9x^2 + 25y^2 + 25z^2 - 225 = 0$$

Hence the equation of the set of points P, the sum of whose distances from A and B is equal to 10 is $9x^2 + 25y^2 + 25z^2 - 225 = 0$.

NCERT solutions for class 11 maths chapter 12 introduction to three dimensional geometry-Exercise: 12.3

Question:1(i) Find the coordinates of the point which divides the line segment joining the points (– 2, 3, 5) and (1, – 4, 6) in the ratio 2 : 3 internally

Answer:

The line segment joining the points A (– 2, 3, 5) and B(1, – 4, 6)

Let point P(x,y,z) be the point that divides the line segment AB internally in the ratio 2:3.

Now, As we know by section formula, The coordinate of the point P which divides line segment $A(x_1, y_1, z_1)$ And $B(x_2, y_2, z_2)$ in ratio m:n is

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n}\right)$$

Now the point that divides A (– 2, 3, 5) and B(1, – 4, 6) in ratio 2:3 is

Hence required point is

$\left(\frac{-4}{5}, \frac{1}{5}, \frac{27}{5}\right)$.

Question:1 (ii) Find the coordinates of the point which divides the line segment joining the points (– 2, 3, 5) and (1, – 4, 6) in the ratio 2 : 3 externally.

Answer:

he line segment joining the points A (– 2, 3, 5) and B(1, – 4, 6)

Let point P(x,y,z) be the point that divides the line segment AB externally in the ratio 2:3.

Now, As we know by section formula , The coordinate of the point P which divides line segment $A(x_1, y_1, z_1)$ And $B(x_2, y_2, z_2)$ externally in ratio m:n is

$$\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}, \frac{mz_2 - nz_1}{m-n}\right)$$

Now the point that divides A (– 2, 3, 5) and B(1, – 4, 6) externally in ratio 2:3 is

$$\left(\frac{2(1) - 3(-2)}{2-3}, \frac{2(-4) - 3(3)}{2-3}, \frac{2(6) - 3(5)}{2-3}\right) = (-8, 17, 3)$$

Hence required point is

. $(-8, 17, 3)$

Question:2 Given that P (3, 2, – 4), Q (5, 4, – 6) and R (9, 8, –10) are collinear. Find the ratio in which Q divides PR.

Answer:

Given Three points,

P (3, 2, – 4), Q (5, 4, – 6) and R (9, 8, –10)

Let point Q divides PR internally in the ratio $\lambda : 1$

Now,

According to the section formula , The point Q in terms of P,Q and λ is:

$$\left(\frac{9\lambda+3}{\lambda+1}, \frac{8\lambda+2}{\lambda+1}, \frac{-10\lambda-4}{\lambda+1}\right) = (5,4,-6)$$

$$\frac{9\lambda+3}{\lambda+1} = 5$$

$$9\lambda + 3 = 5(\lambda+1)$$

$$9\lambda + 3 = 5\lambda + 5$$

$$4\lambda = 2$$

$$\lambda = \frac{1}{2}$$

Hence, point Q divides PR in ratio 1:2.

Question:3 Find the ratio in which the YZ-plane divides the line segment formed by joining the points (–2, 4, 7) and (3, –5, 8).

Answer:

Given,

two points A(–2, 4, 7) and B(3, –5, 8)

Let Y-Z plane divides AB in $\lambda : 1$

So, According to the section formula, the point which divides AB in $\lambda : 1$ is

$$\left(\frac{3\lambda - 2}{\lambda + 1}, \frac{-5\lambda + 4}{\lambda + 1}, \frac{8\lambda + 7}{\lambda + 1}\right)$$

Since this point is in YZ plane, x coordinate of this point will be zero.

So,

$$\frac{3\lambda - 2}{\lambda + 1} = 0$$

$$3\lambda - 2 = 0$$

$$\lambda = \frac{2}{3}$$

Hence YZ plane divides Line segment AB in a ratio 2:3.

Question:4 Using section formula, show that the points A (2, –3, 4), B (–1, 2, 1) and $C\left(0, 1/3, 2\right)$ are collinear.

Answer:

Given,

three points, A (2, –3, 4), B (–1, 2, 1) and C (0, 1/3, 2)

Let a point P divides Line segment AB in the ratio $\lambda : 1$

SO, according to the section formula, the point P will be

$$\left(\frac{-\lambda+2}{\lambda+1}, \frac{2\lambda-3}{\lambda+1}, \frac{\lambda+4}{\lambda+1}\right)$$

Now, let's compare this point P with point C.

$$\frac{-\lambda+2}{\lambda+1}, = 0$$

$$\lambda = 2$$

From here, we see that for $\lambda = 2$, point C divides the line segment AB in ratio 2:1. Since point C divides the line segment AB, it lies in the line joining A and B and Hence they are colinear.

Question:5 Find the coordinates of the points which trisect the line segment joining the points P (4, 2, – 6) and Q (10, –16, 6).

Answer:

Given,

two points P (4, 2, – 6) and Q (10, –16, 6).

The point which trisects the line segment are the points which divide PQ in either 1:2 or 2:1

Let R (x,y,z) be the point which divides Line segment PR in ratio 1:2

Now, according to the section formula

$$(x, y, z) = \left(\frac{10+2(4)}{1+2}, \frac{-16+2(2)}{1+2}, \frac{6-2(6)}{1+2}\right) = (6, -4, -2)$$

Let S be the point which divides the Line segment PQ in ratio 2:1

So, The point S according to section formula is

$$(x, y, z) = \left(\frac{(2)10 + (4)}{1+2}, \frac{2(-16) + (2)}{1+2}, \frac{(2)6 - (6)}{1+2} \right) = (8, -10, 2)$$

Hence the points which trisect the line segment AB are (6,-4,-2) and (8,-10,2).