NCERT solutions for class 11 maths chapter 11 conic sections-Exercise: 11.1

Question:1 Find the equation of the circle with

centre (0,2) and radius 2

Answer:

As we know,

The equation of the circle with centre (h, k) and radius r is given by ;

$(x-h)^2+(y-k)^2=r^2$

So Given Here

$(h,k)=(0,2)$

AND $r=2$

So the equation of the circle is:

, $(x-0)^2+(y-2)^2=2^2$

$x^2+y^2-4y+4=4$

$x^2+y^2-4y=0$

Question:2 Find the equation of the circle with

centre (–2,3) and radius 4

Answer:

As we know,

The equation of the circle with centre (h, k) and radius r is given by ;

$(x-h)^2+(y-k)^2=r^2$

So Given Here

$(h,k)=(-2,3)$

AND $r=4$

So the equation of the circle is:

, $(x-(-2))^2+(y-3)^2=4^2$

$x^2+4x+4+y^2-6y+9=16$

$x^2+y^2+4x-6y-3=0$

Question:3 Find the equation of the circle with centre $\left(\frac{1}{2},\frac{1}{4}\right)$ and radius $\frac{1}{12}$

Answer:

As we know,

The equation of the circle with center (h, k) and radius r is give by ;

$(x-h)^2+(y-k)^2=r^2$

So Given Here

$$(h, k) = \left(\frac{1}{2}, \frac{1}{4}\right)$$

AND

$$r = \frac{1}{12}$$

So the equation of circle is:

$$\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{4}\right)^2 = \left(\frac{1}{12}\right)^2$$

$$x^2 - x + \frac{1}{4} + y^2 - \frac{1}{2}y + \frac{1}{16} = \frac{1}{144}$$

$$x^2 + y^2 - x - \frac{1}{2}y - \frac{11}{36} = 0$$

$$36x^2 + 36y^2 - 36x - 18y - 11 = 0$$

Question:4 Find the equation of the circle with

centre (1,1) and radius $\sqrt{2}$

Answer:

As we know,

The equation of the circle with centre (h, k) and radius r is given by ;

$$(x - h)^2 + (y - k)^2 = r^2$$

So Given Here

$(h, k) = (1, 1)$

AND $r = \sqrt{2}$

So the equation of the circle is:

, $(x-1)^2 + (y-1)^2 = (\sqrt{2})^2$

$x^2 - 2x + 1 + y^2 - 2y + 1 = 2$

$x^2 + y^2 - 2x - 2y = 0$

Question:5 Find the equation of the circle with

centre $(-a, -b)$ and radius $\sqrt{a^2 - b^2}$

Answer:

As we know,

The equation of the circle with centre (h, k) and radius r is given by ;

$(x-h)^2 + (y-k)^2 = r^2$

So Given Here

$(h, k) = (-a, -b)$

AND $r = \sqrt{a^2 - b^2}$

So the equation of the circle is:

, $(x-(-a))^2 + (y-(-b))^2 = (\sqrt{a^2 - b^2})^2$

$x^2 + 2ax + a^2 + y^2 + 2by + b^2 = a^2 - b^2$

$x^2 + y^2 + 2ax + 2by + 2b^2 = 0$

Question:6 Find the centre and radius of the circles.

$(x + 5)^2 + (y - 3)^2 = 36$

Answer:

As we know,

The equation of the circle with centre (h, k) and radius r is given by ;

$(x - h)^2 + (y - k)^2 = r^2$

Given here

$(x + 5)^2 + (y - 3)^2 = 36$

Can also be written in the form

$(x - (-5))^2 + (y - 3)^2 = 6^2$

So, from comparing, we can see that

$r = 6$

Hence the Radius of the circle is 6.

Question:7 Find the centre and radius of the circles.

$x^2 + y^2 - 4x - 8y - 45 = 0$

Answer:

As we know,

The equation of the circle with centre (h, k) and radius r is given by ;

$$(x-h)^2+(y-k)^2=r^2$$

Given here

$$x^2+y^2-4x-8y-45=0$$

Can also be written in the form

$$(x-2)^2+(y-4)^2=(\sqrt{65})^2$$

So, from comparing, we can see that

$$r=\sqrt{65}$$

Hence the Radius of the circle is $\sqrt{65}$.

Question:8 Find the centre and radius of the circles.

$$x^2+y^2-8x+10y-12=0$$

Answer:

As we know,

The equation of the circle with centre (h, k) and radius r is given by ;

$$(x-h)^2+(y-k)^2=r^2$$

Given here

$$x^2 + y^2 - 8x + 10y - 12 = 0$$

Can also be written in the form

$$(x-4)^2 + (y-(-5))^2 = (\sqrt{53})^2$$

So, from comparing, we can see that

$$r = \sqrt{53}$$

Hence the radius of the circle is $\sqrt{53}$.

Question:9 Find the centre and radius of the circles.

$$2x^2 + 2y^2 - x = 0$$

Answer:

As we know,

The equation of the circle with centre (h, k) and radius r is given by ;

$$(x-h)^2 + (y-k)^2 = r^2$$

Given here

$$2x^2 + 2y^2 - x = 0$$

Can also be written in the form

$$\left(x - \frac{1}{4}\right)^2 + (y-0)^2 = \left(\frac{1}{4}\right)^2$$

So, from comparing, we can see that

$$r = \frac{1}{4}$$

Hence Center of the circle is the $\left(\frac{1}{4}, 0\right)$ Radius of the circle is $\frac{1}{4}$.

Question:10 Find the equation of the circle passing through the points (4,1) and (6,5) and whose centre is on the line $4x + y = 16$.

Answer:

As we know,

The equation of the circle with centre (h, k) and radius r is given by ;

$$(x-h)^2 + (y-k)^2 = r^2$$

Given Here,

Condition 1: the circle passes through points (4,1) and (6,5)

$$(4-h)^2 + (1-k)^2 = r^2$$

$$(6-h)^2 + (5-k)^2 = r^2$$

Here,

$$(4-h)^2 + (1-k)^2 = (6-h)^2 + (5-k)^2$$

$$(4-h)^2 - (6-h)^2 + (1-k)^2 - (5-k)^2 = 0$$

$$(-2)(10-2h) + (-4)(6-2k) = 0$$

$-20+4h-24+8k=0$

$4h+8k=44$

Now, Condition 2:centre is on the line $4x+y=16$.

$4h+k=16$

From condition 1 and condition 2

$h=3,\ k=4$

Now lets substitute this value of h and k in condition 1 to find out r

$(4-3)^2+(1-4)^2=r^2$

$1+9=r^2$

$r=\sqrt{10}$

So now, the Final Equation of the circle is

$(x-3)^2+(y-4)^2=(\sqrt{10})^2$

$x^2-6x+9+y^2-8y+16=10$

$x^2+y^2-6x-8y+15=0$

Question:11 Find the equation of the circle passing through the points (2,3) and (–1,1) and hose centre is on the line $x-3y-11=0$.

Answer:

As we know,

The equation of the circle with centre (h, k) and radius r is given by ;

$(x-h)^2+(y-k)^2=r^2$

Given Here,

Condition 1: the circle passes through points (2,3) and (–1,1)

$(2-h)^2+(3-k)^2=r^2$

$(-1-h)^2+(1-k)^2=r^2$

Here,

$(2-h)^2+(3-k)^2=(-1-h)^2+(1-k)^2$

$(2-h)^2-(-1-h)^2+(3-k)^2-(1-k)^2=0$

$(3)(1-2h)+(2)(4-2k)=0$

$3-6h+8-4k=0$

$6h+4k=11$

Now, Condition 2: centre is on the line. $x-3y-11=0$

$h-3k=11$

From condition 1 and condition 2

$h=\frac{7}{2},\ k=\frac{-5}{2}$

Now let's substitute this value of h and k in condition 1 to find out r

$$\left(2-\frac{7}{2}\right)^2+\left(3+\frac{5}{2}\right)^2=r^2$$

$$\frac{9}{4}+\frac{121}{4}=r^2$$

$$r^2=\frac{130}{4}$$

So now, the Final Equation of the circle is

$$\left(x-\frac{7}{2}\right)^2+\left(y+\frac{5}{2}\right)^2=\frac{130}{4}$$

$$x^2-7x+\frac{49}{4}+y^2+5y+\frac{25}{4}=\frac{130}{4}$$

$$x^2+y^2-7x+5y-\frac{56}{4}=0$$

$$x^2+y^2-7x+5y-14=0$$

Question:12 Find the equation of the circle with radius 5 whose centre lies on *x* -axis and passes through the point (2,3).

Answer:

As we know,

The equation of the circle with centre (h, k) and radius r is given by ;

$$(x-h)^2+(y-k)^2=r^2$$

So let the circle be,

$(x-h)^2+(y-k)^2=r^2$

Since it's radius is 5 and its centre lies on x-axis,

$(x-h)^2+(y-0)^2=5^2$

And Since it passes through the point (2,3).

$(2-h)^2+(3-0)^2=5^2$

$(2-h)^2=25-9$

$(2-h)^2=16$

$(2-h)=4 \; or \; (2-h)=-4$

$h=-2 \; or \; 6$

When $h=-2$,The equation of the circle is :

$(x-(-2))^2+(y-0)^2=5^2$

$x^2+4x+4+y^2=25$

$x^2+y^2+4x-21=0$

When $h=6$ The equation of the circle is :

$(x-6)^2+(y-0)^2=5^2$

$x^2-12x+36+y^2=25$

$x^2+y^2-12x+11=0$

Question:13 Find the equation of the circle passing through (0,0) and making intercepts *a* and *b* on the coordinate axes.

Answer:

Let the equation of circle be,

$$(x-h)^2+(y-k)^2=r^2$$

Now since this circle passes through (0,0)

$$(0-h)^2+(0-k)^2=r^2$$

$$h^2+k^2=r^2$$

Now, this circle makes an intercept of *a* and *b* on the coordinate axes.it means circle passes through the point (a,0) and (0,b)

So,

$$(a-h)^2+(0-k)^2=r^2$$

$$a^2-2ah+h^2+k^2=r^2$$

$$a^2-2ah=0$$

$$a(a-2h)=0$$

$$a=0 \; or \; a-2h=0$$

Since $a \neq 0 \; so \; a-2h=0$

$$h=\frac{a}{2}$$

Similarly,

$$(0-h)^2+(b-k)^2=r^2$$

$$h^2+b^2-2bk+k^2=r^2$$

$$b^2-2bk=0$$

$$b(b-2k)=0$$

Since b is not equal to zero.

$$k=\frac{b}{2}$$

So Final equation of the Circle ;

$$x^2-ax+\frac{a^2}{4}+y^2-bx+\frac{b^2}{4}=\frac{a^2}{4}+\frac{b^2}{4}$$

$$x^2+y^2-ax-bx=0$$

Question:14 Find the equation of a circle with centre (2,2) and passes through the point (4,5).

Answer:

Let the equation of circle be :

$$(x-h)^2+(y-k)^2=r^2$$

Now, since the centre of the circle is (2,2), our equation becomes

$(x-2)^2+(y-2)^2=r^2$

Now, Since this passes through the point (4,5)

$(4-2)^2+(5-2)^2=r^2$

$4+9=r^2$

$r^2=13$

Hence The Final equation of the circle becomes

$(x-2)^2+(y-2)^2=13$

$x^2-4x+4+y^2-4y+4=13$

$x^2+y^2-4x-4y-5=0$

Question:15 Does the point (–2.5, 3.5) lie inside, outside or on the circle $x^2+y^2=25$?

Answer:

Given, a circle

$x^2+y^2=25$

As we can see center of the circle is (0,0)

Now the distance between (0,0) and (–2.5, 3.5) is

$d=\sqrt{(-2.5-0)^2+(3.5-0)^2}$

$d = \sqrt{6.25 + 12.25}$

$d = \sqrt{18.5} \approx 4.3$ $d = \sqrt{18.5} \approx 4.3 < 5$

Since distance between the given point and center of the circle is less than radius of the circle, the point lie inside the circle.

NCERT solutions for class 11 maths chapter 11 conic sections-Exercise: 11.2

Question:1 Find the coordinates of the focus, axis of the parabola, the equation of the directrix and the length of the latus rectum.

$y^2 = 12x$

Answer:

Given, a parabola with equation

$y^2 = 12x$

This is parabola of the form $y^2 = 4ax$ which opens towards the right.

So,

By comparing the given parabola equation with the standard equation, we get,

$4a = 12$

$a = 3$

Hence,

Coordinates of the focus :

$(a, 0) = (3, 0)$

Axis of the parabola:

It can be seen that the axis of this parabola is X-Axis.

The equation of the directrix

$x = -a, \Rightarrow x = -3 \Rightarrow x + 3 = 0$

The length of the latus rectum:

$4a = 4(3) = 12$.

Question:2 Find the coordinates of the focus, axis of the parabola, the equation of the directrix and the length of the latus rectum.

$x^2 = 6y$

Answer:

Given, a parabola with equation

$x^2 = 6y$

This is parabola of the form $x^2 = 4ay$ which opens upward.

So,

By comparing the given parabola equation with the standard equation, we get,

$$4a = 6$$
$$a = \frac{3}{2}$$

Hence,

Coordinates of the focus :

$$(0, a) = \left(0, \frac{3}{2}\right)$$

Axis of the parabola:

It can be seen that the axis of this parabola is Y-Axis.

The equation of the directrix

$$y = -a, \Rightarrow y = -\frac{3}{2} \Rightarrow y + \frac{3}{2} = 0$$

The length of the latus rectum:

$$4a = 4(\frac{3}{2}) = 6$$.

Question:3 Find the coordinates of the focus, axis of the parabola, the equation of the directrix and the length of the latus rectum.

$$y^2 = -8x$$

Answer:

Given, a parabola with equation

$y^2 = -8x$

This is parabola of the form $y^2 = -4ax$ which opens towards left.

So,

By comparing the given parabola equation with the standard equation, we get,

$-4a = -8$

$a = 2$

Hence,

Coordinates of the focus :

$(-a, 0) = (-2, 0)$

Axis of the parabola:

It can be seen that the axis of this parabola is X-Axis.

The equation of the directrix

$x = a, \Rightarrow x = 2 \Rightarrow x - 2 = 0$

The length of the latus rectum:

$4a = 4(2) = 8$.

Question:4 Find the coordinates of the focus, axis of the parabola, the equation of the directrix and the length of the latus rectum.

$x^2 = -16y$

Answer:

Given, a parabola with equation

$x^2 = -16y$

This is parabola of the form $x^2 = -4ay$ which opens downwards.

So,

By comparing the given parabola equation with the standard equation, we get,

$-4a = -16$

$a = 4$

Hence,

Coordinates of the focus :

$(0, -a) = (0, -4)$

Axis of the parabola:

It can be seen that the axis of this parabola is Y-Axis.

The equation of the directrix

$y = a, \Rightarrow y = 4 \Rightarrow y - 4 = 0$

The length of the latus rectum:

$4a = 4(4) = 16$.

Question:5 Find the coordinates of the focus, axis of the parabola, the equation of the directrix and the length of the latus rectum.

$y^2 = 10x$

Answer:

Given, a parabola with equation

$y^2 = 10x$

This is parabola of the form $y^2 = 4ax$ which opens towards the right.

So,

By comparing the given parabola equation with the standard equation, we get,

$$4a = 10$$
$$a = \frac{10}{4} = \frac{5}{2}$$

Hence,

Coordinates of the focus :

$$(a, 0) = \left(\frac{5}{2}, 0\right)$$

Axis of the parabola:

It can be seen that the axis of this parabola is X-Axis.

The equation of the directrix

$$x = -a, \Rightarrow x = -\frac{5}{2} \Rightarrow x + \frac{5}{2} = 0 \Rightarrow 2x + 5 = 0$$

The length of the latus rectum:

$$4a = 4(\frac{5}{2}) = 10$$

Question:6 Find the coordinates of the focus, axis of the parabola, the equation of the directrix and the length of the latus rectum.

$$x^2 = -9y$$

Answer:

Given, a parabola with equation

$$x^2 = -9y$$

This is parabola of the form $x^2 = -4ay$ which opens downwards.

So

By comparing the given parabola equation with the standard equation, we get,

$$-4a = -9$$

$$a = \frac{9}{4}$$

Hence,

Coordinates of the focus :

$$(0, -a) = \left(0, -\frac{9}{4}\right)$$

Axis of the parabola:

It can be seen that the axis of this parabola is Y-Axis.

The equation of the directrix

$$y = a, \Rightarrow y = \frac{9}{4} \Rightarrow y - \frac{9}{4} = 0$$

The length of the latus rectum:

$$4a = 4\left(\frac{9}{4}\right) = 9$$

Question:7 Find the equation of the parabola that satisfies the given conditions:

Focus (6,0); directrix $x = -6$

Answer:

Given, in a parabola,

Focus : (6,0) And Directrix : $x = -6$

Here,

Focus is of the form (a, 0), which means it lies on the X-axis. And Directrix is of the form $x = -a$ which means it lies left to the Y-Axis.

These are the condition when the standard equation of a parabola is. $y^2 = 4ax$

Hence the Equation of Parabola is

$y^2 = 4ax$

Here, it can be seen that:

$a = 6$

Hence the Equation of the Parabola is:

$\Rightarrow y^2 = 4ax \Rightarrow y^2 = 4(6)x$

$\Rightarrow y^2 = 24x$.

Question:8 Find the equation of the parabola that satisfies the given conditions:

Focus (0,–3); directrix $y = 3$

Answer:

Given,in a parabola,

Focus : Focus (0,–3); directrix $y = 3$

Here,

Focus is of the form (0,-a), which means it lies on the Y-axis. And Directrix is of the form $y = a$ which means it lies above X-Axis.

These are the conditions when the standard equation of a parabola is $x^2 = -4ay$.

Hence the Equation of Parabola is

$x^2 = -4ay$

Here, it can be seen that:

$a = 3$

Hence the Equation of the Parabola is:

$\Rightarrow x^2 = -4ay \Rightarrow x^2 = -4(3)y$

$\Rightarrow x^2 = -12y$.

Question:9 Find the equation of the parabola that satisfies the given conditions:

Vertex (0,0); focus (3,0)

Answer:

Given,

Vertex (0,0) And focus (3,0)

As vertex of the parabola is (0,0) and focus lies in the positive X-axis, The parabola will open towards the right, And the standard equation of such parabola is

$y^2 = 4ax$

Here it can be seen that $a = 3$

So, the equation of a parabola is

$\Rightarrow y^2 = 4ax \Rightarrow y^2 = 4(3)x$

$\Rightarrow y^2 = 12x$.

Question:10 Find the equation of the parabola that satisfies the given conditions:

Vertex (0,0); focus (-2,0)

Answer:

Given,

Vertex (0,0) And focus (-2,0)

As vertex of the parabola is (0,0) and focus lies in the negative X-axis, The parabola will open towards left, And the standard equation of such parabola is

$y^2 = -4ax$

Here it can be seen that $a = 2$

So, the equation of a parabola is

$\Rightarrow y^2 = -4ax \Rightarrow y^2 = -4(2)x$

$\Rightarrow y^2 = -8x$.

Question:11 Find the equation of the parabola that satisfies the given conditions:

Vertex (0,0) passing through (2,3) and axis is along *x* -axis.

Answer:

Given

The Vertex of the parabola is (0,0).

The parabola is passing through (2,3) and axis is along *the* x -axis, it will open towards right. and the standard equation of such parabola is

$$y^2 = 4ax$$

Now since it passes through (2,3)

$$3^2 = 4a(2)$$

$$9 = 8a$$

$$a = \frac{8}{9}$$

So the Equation of Parabola is ;

$$\Rightarrow y^2 = 4\left(\frac{9}{8}\right)x$$

$$\Rightarrow y^2 = \left(\frac{9}{2}\right)x$$

$$\Rightarrow 2y^2 = 9x$$

Question:12 Find the equation of the parabola that satisfies the given conditions:

Vertex (0,0), passing through (5,2) and symmetric with respect to y -axis.

Answer:

Given a parabola,

with Vertex (0,0), passing through (5,2) and symmetric with respect to *the y* -axis.

Since the parabola is symmetric with respect to Y=axis, it's axis will ve Y-axis. and since it passes through the point (5,2), it must go through the first quadrant.

So the standard equation of such parabola is

$x^2 = 4ay$

Now since this parabola is passing through (5,2)

$5^2 = 4a(2)$

$25 = 8a$

$a = \frac{25}{8}$

Hence the equation of the parabola is

$\Rightarrow x^2 = 4\left(\frac{25}{8}\right)y$

$\Rightarrow x^2 = \left(\frac{25}{2}\right)y$

$\Rightarrow 2x^2 = 25y$

NCERT solutions for class 11 maths chapter 11 conic sections-Exercise: 11.3

Question:1 Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse.

$$\frac{x^2}{36} + \frac{y^2}{16} = 1$$

Answer:

Given

The equation of the ellipse

$$\frac{x^2}{36} + \frac{y^2}{16} = 1$$

As we can see from the equation, the major axis is along X-axis and the minor axis is along Y-axis.

On comparing the given equation with the standard equation of an ellipse, which is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

We get

$a = 6$ and $b = 4$.

So,

$$c = \sqrt{a^2 - b^2} = \sqrt{6^2 - 4^2}$$

$$c = \sqrt{20} = 2\sqrt{5}$$

Hence,

Coordinates of the foci:

$(c, 0)\ and\ (-c, 0) = (2\sqrt{5}, 0)\ and\ (-2\sqrt{5}, 0)$

The vertices:

$(a, 0)\ and\ (-a, 0) = (6, 0)\ and\ (-6, 0)$

The length of the major axis:

$2a = 2(6) = 12$

The length of minor axis:

$2b = 2(4) = 8$

The eccentricity :

$$e = \frac{c}{a} = \frac{2\sqrt{5}}{6} = \frac{\sqrt{5}}{3}$$

The length of the latus rectum:

$$\frac{2b^2}{a} = \frac{2(4)^2}{6} = \frac{32}{6} = \frac{16}{3}$$

Question:2 Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse.

$$\frac{x^2}{4} + \frac{y^2}{25} = 1$$

Answer:

Given

The equation of the ellipse

$$\frac{x^2}{4} + \frac{y^2}{25} = 1$$

As we can see from the equation, the major axis is along Y-axis and the minor axis is along X-axis.

On comparing the given equation with the standard equation of such ellipse, which is

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

We get

$a = 5$ and $b = 2$.

So,

$$c = \sqrt{a^2 - b^2} = \sqrt{5^2 - 2^2}$$

$$c = \sqrt{21}$$

Hence,

Coordinates of the foci:

$$(0, c)\ and\ (0, -c) = (0, \sqrt{21})\ and\ (0, -\sqrt{21})$$

The vertices:

$$(0, a)\ and\ (0, -a) = (0, 5)\ and\ (0, -5)$$

The length of the major axis:

$$2a = 2(5) = 10$$

The length of minor axis:

$$2b = 2(2) = 4$$

The eccentricity :

$$e = \frac{c}{a} = \frac{\sqrt{21}}{6}$$

The length of the latus rectum:

$$\frac{2b^2}{a} = \frac{2(2)^2}{5} = \frac{8}{5}$$

Question:3 Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse.

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

Answer:

Given

The equation of the ellipse

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

As we can see from the equation, the major axis is along X-axis and the minor axis is along Y-axis.

On comparing the given equation with the standard equation of an ellipse, which is

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

We get

$a = 4$ and $b = 3$.

So,

$c = \sqrt{a^2 - b^2} = \sqrt{4^2 - 3^2}$

$c = \sqrt{7}$

Hence,

Coordinates of the foci:

$(c, 0)\ and\ (-c, 0) = (\sqrt{7}, 0)\ and\ (-\sqrt{7}, 0)$

The vertices:

$(a, 0)\ and\ (-a, 0) = (4, 0)\ and\ (-4, 0)$

The length of the major axis:

$2a = 2(4) = 8$

The length of minor axis:

$2b = 2(3) = 6$

The eccentricity :

$e = \frac{c}{a} = \frac{\sqrt{7}}{4}$

The length of the latus rectum:

$$\frac{2b^2}{a} = \frac{2(3)^2}{4} = \frac{18}{4} = \frac{9}{2}$$

Question:4 Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse.

$$\frac{x^2}{25} + \frac{y^2}{100} = 1$$

Answer:

Given

The equation of the ellipse

$$\frac{x^2}{25} + \frac{y^2}{100} = 1$$

As we can see from the equation, the major axis is along Y-axis and the minor axis is along X-axis.

On comparing the given equation with the standard equation of such ellipse, which is

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

We get

$a = 10$ and $b = 5$.

So,

$$c = \sqrt{a^2 - b^2} = \sqrt{10^2 - 5^2}$$

$c = \sqrt{75} = 5\sqrt{3}$

Hence,

Coordinates of the foci:

$(0, c)\ and\ (0, -c) = (0, 5\sqrt{3})\ and\ (0, -5\sqrt{3})$

The vertices:

$(0, a)\ and\ (0, -a) = (0, 10)\ and\ (0, -10)$

The length of the major axis:

$2a = 2(10) = 20$

The length of minor axis:

$2b = 2(5) = 10$

The eccentricity :

$e = \frac{c}{a} = \frac{5\sqrt{3}}{10} = \frac{\sqrt{3}}{2}$

The length of the latus rectum:

$\frac{2b^2}{a} = \frac{2(5)^2}{10} = \frac{50}{10} = 5$

Question:5 Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse.

$\frac{x^2}{49} + \frac{y^2}{36} = 1$

Answer:

Given

The equation of ellipse

$$\frac{x^2}{49} + \frac{y^2}{36} = 1 \Rightarrow \frac{x^2}{7^2} + \frac{y^2}{6^2} = 1$$

As we can see from the equation, the major axis is along X-axis and the minor axis is along Y-axis.

On comparing the given equation with standard equation of ellipse, which is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

We get

$a = 7$ and $b = 6$.

So,

$$c = \sqrt{a^2 - b^2} = \sqrt{7^2 - 6^2}$$

$$c = \sqrt{13}$$

Hence,

Coordinates of the foci:

$$(c, 0)\ and\ (-c, 0) = (\sqrt{13}, 0)\ and\ (-\sqrt{13}, 0)$$

The vertices:

$(a, 0)\ and\ (-a, 0) = (7, 0)\ and\ (-7, 0)$

The length of major axis:

$2a = 2(7) = 14$

The length of minor axis:

$2b = 2(6) = 12$

The eccentricity :

$$e = \frac{c}{a} = \frac{\sqrt{13}}{7}$$

The length of the latus rectum:

$$\frac{2b^2}{a} = \frac{2(6)^2}{7} = \frac{72}{7}$$

Question:6 Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse.

$$\frac{x^2}{100} + \frac{y^2}{400} = 1$$

Answer:

Given

The equation of the ellipse

$$\frac{x^2}{100} + \frac{y^2}{400} = 1 \Rightarrow \frac{x^2}{10^2} + \frac{y^2}{20^2} = 1$$

As we can see from the equation, the major axis is along Y-axis and the minor axis is along X-axis.

On comparing the given equation with the standard equation of such ellipse, which is

$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

We get

$a = 20$ and $b = 10$.

So,

$c = \sqrt{a^2 - b^2} = \sqrt{20^2 - 10^2}$

$c = \sqrt{300} = 10\sqrt{3}$

Hence,

Coordinates of the foci:

$(0, c)\ and\ (0, -c) = (0, 10\sqrt{3})\ and\ (0, -10\sqrt{3})$

The vertices:

$(0, a)\ and\ (0, -a) = (0, 20)\ and\ (0, -20)$

The length of the major axis:

$2a = 2(20) = 40$

The length of minor axis:

$2b = 2(10) = 20$

The eccentricity :

$$e = \frac{c}{a} = \frac{10\sqrt{3}}{20} = \frac{\sqrt{3}}{2}$$

The length of the latus rectum:

$$\frac{2b^2}{a} = \frac{2(10)^2}{20} = \frac{200}{20} = 10$$

Question:7 Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse.

$36x^2 + 4y^2 = 144$

Answer:

Given

The equation of the ellipse

$$36x^2 + 4y^2 = 144$$

$$\Rightarrow \frac{36}{144}x^2 + \frac{4}{144}y^2 = 1$$

$$\Rightarrow \frac{1}{4}x^2 + \frac{1}{36}y^2 = 1$$

$$\frac{x^2}{2^2} + \frac{y^2}{6^2} = 1$$

As we can see from the equation, the major axis is along Y-axis and the minor axis is along X-axis.

On comparing the given equation with the standard equation of such ellipse, which is

$$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$$

We get

$a=6$ and $b=2$.

So,

$$c=\sqrt{a^2-b^2}=\sqrt{6^2-2^2}$$

$$c=\sqrt{32}=4\sqrt{2}$$

Hence,

Coordinates of the foci:

$$(0,c)\ and\ (0,-c)=(0,4\sqrt{2})\ and\ (0,-4\sqrt{2})$$

The vertices:

$$(0,a)\ and\ (0,-a)=(0,6)\ and\ (0,-6)$$

The length of the major axis:

$$2a=2(6)=12$$

The length of minor axis:

$$2b=2(2)=4$$

The eccentricity :

$$e = \frac{c}{a} = \frac{4\sqrt{2}}{6} = \frac{2\sqrt{2}}{3}$$

The length of the latus rectum:

$$\frac{2b^2}{a} = \frac{2(2)^2}{6} = \frac{8}{6} = \frac{4}{3}$$

Question:8 Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse.

$$16x^2 + y^2 = 16$$

Answer:

Given

The equation of the ellipse

$$16x^2 + y^2 = 16$$

$$\frac{16x^2}{16} + \frac{y^2}{16} = 1$$

$$\frac{x^2}{1^2} + \frac{y^2}{4^2} = 1$$

As we can see from the equation, the major axis is along Y-axis and the minor axis is along X-axis.

On comparing the given equation with the standard equation of such ellipse, which is

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

We get

$a = 4$ and $b = 1$.

So,

$c = \sqrt{a^2 - b^2} = \sqrt{4^2 - 1^2}$

$c = \sqrt{15}$

Hence,

Coordinates of the foci:

$(0, c)\ and\ (0, -c) = (0, \sqrt{15})\ and\ (0, -\sqrt{15})$

The vertices:

$(0, a)\ and\ (0, -a) = (0, 4)\ and\ (0, -4)$

The length of the major axis:

$2a = 2(4) = 8$

The length of minor axis:

$2b = 2(1) = 2$

The eccentricity :

$e = \frac{c}{a} = \frac{\sqrt{15}}{4}$

The length of the latus rectum:

$\frac{2b^2}{a} = \frac{2(1)^2}{4} = \frac{2}{4} = \frac{1}{2}$

Question:9 Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse.

$4x^2 + 9y^2 = 36$

Answer:

Given

The equation of the ellipse

$4x^2 + 9y^2 = 36$

$\frac{4x^2}{36} + \frac{9y^2}{36} = 1$

$\frac{x^2}{9} + \frac{y^2}{4} = 1$

$\frac{x^2}{3^2} + \frac{y^2}{2^2} = 1$

As we can see from the equation, the major axis is along X-axis and the minor axis is along Y-axis.

On comparing the given equation with the standard equation of an ellipse, which is

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

We get

$a = 3$ and $b = 2$.

So,

$c = \sqrt{a^2 - b^2} = \sqrt{3^2 - 2^2}$

$c = \sqrt{5}$

Hence,

Coordinates of the foci:

$(c, 0)\ and\ (-c, 0) = (\sqrt{5}, 0)\ and\ (-\sqrt{5}, 0)$

The vertices:

$(a, 0)\ and\ (-a, 0) = (3, 0)\ and\ (-3, 0)$

The length of the major axis:

$2a = 2(3) = 6$

The length of minor axis:

$2b = 2(2) = 4$

The eccentricity :

$e = \frac{c}{a} = \frac{\sqrt{5}}{3}$

The length of the latus rectum:

$\frac{2b^2}{a} = \frac{2(2)^2}{3} = \frac{8}{3}$

Question:10 Find the equation for the ellipse that satisfies the given conditions:

Vertices (± 5, 0), foci (± 4, 0)

Answer:

Given, In an ellipse,

Vertices (± 5, 0), foci (± 4, 0)

Here Vertices and focus of the ellipse are in X-axis so the major axis of this ellipse will be X-axis.

Therefore, the equation of the ellipse will be of the form:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

Where a and b are the length of the semimajor axis and semiminor axis respectively.

So on comparing standard parameters(vertices and foci) with the given one, we get

$a=5$ and $c=4$

Now, As we know the relation,

$$a^2=b^2+c^2$$

$$b^2=a^2-c^2$$

$$b=\sqrt{a^2-c^2}$$

$$b=\sqrt{5^2-4^2}$$

$$b=\sqrt{9}$$

$$b=3$$

Hence, The Equation of the ellipse will be :

$$\frac{x^2}{5^2}+\frac{y^2}{3^2}=1$$

Which is

$$\frac{x^2}{25}+\frac{y^2}{9}=1.$$

Question:11 Find the equation for the ellipse that satisfies the given conditions:

Vertices (0, ± 13), foci (0, ± 5)

Answer:

Given, In an ellipse,

Vertices (0, ± 13), foci (0, ± 5)

Here Vertices and focus of the ellipse are in Y-axis so the major axis of this ellipse will be Y-axis.

Therefore, the equation of the ellipse will be of the form:

$$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$$

Where a and b are the length of the semimajor axis and semiminor axis respectively.

So on comparing standard parameters(vertices and foci) with the given one, we get

$a=13$ and $c=5$

Now, As we know the relation,

$a^2 = b^2 + c^2$

$b^2 = a^2 - c^2$

$b = \sqrt{a^2 - c^2}$

$b = \sqrt{13^2 - 5^2}$

$b = \sqrt{169 - 25}$

$b = \sqrt{144}$

$b = 12$

Hence, The Equation of the ellipse will be :

$\frac{x^2}{12^2} + \frac{y^2}{13^3} = 1$

Which is

$\frac{x^2}{144} + \frac{y^2}{169} = 1$.

Question:12 Find the equation for the ellipse that satisfies the given conditions:

Vertices (± 6, 0), foci (± 4, 0)

Answer:

Given, In an ellipse,

Vertices (± 6, 0), foci (± 4, 0)

Here Vertices and focus of the ellipse are in X-axis so the major axis of this ellipse will be X-axis.

Therefore, the equation of the ellipse will be of the form:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

Where a and b are the length of the semimajor axis and semiminor axis respectively.

So on comparing standard parameters(vertices and foci) with the given one, we get

$a=6$ and $c=4$

Now, As we know the relation,

$$a^2=b^2+c^2$$

$$b^2=a^2-c^2$$

$$b=\sqrt{a^2-c^2}$$

$$b=\sqrt{6^2-4^2}$$

$$b=\sqrt{36-16}$$

$$b=\sqrt{20}$$

Hence, The Equation of the ellipse will be :

$$\frac{x^2}{6^2}+\frac{y^2}{(\sqrt{20})^2}=1$$

Which is

$\frac{x^2}{36}+\frac{y^2}{20}=1$.

Question:13 Find the equation for the ellipse that satisfies the given conditions:

Ends of major axis (± 3, 0), ends of minor axis (0, ± 2)

Answer:

Given, In an ellipse,

Ends of the major axis (± 3, 0), ends of minor axis (0, ± 2)

Here, the major axis of this ellipse will be X-axis.

Therefore, the equation of the ellipse will be of the form:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

Where a and b are the length of the semimajor axis and semiminor axis respectively.

So on comparing standard parameters(ends of the major and minor axis) with the given one, we get

$a=3$ and $b=2$

Hence, The Equation of the ellipse will be :

$$\frac{x^2}{3^2}+\frac{y^2}{2^2}=1$$

Which is

$$\frac{x^2}{9}+\frac{y^2}{4}=1$$.

Question:14 Find the equation for the ellipse that satisfies the given conditions:

Ends of major axis (0, ± $\sqrt{5}$), ends of minor axis (± 1, 0)

Answer:

Given, In an ellipse,

Ends of the major axis (0, ± $\sqrt{5}$), ends of minor axis (± 1, 0)

Here, the major axis of this ellipse will be Y-axis.

Therefore, the equation of the ellipse will be of the form:

$$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$$

Where a and b are the length of the semimajor axis and semiminor axis respectively.

So on comparing standard parameters(ends of the major and minor axis) with the given one, we get

$a=\sqrt{5}$ and $b=1$

Hence, The Equation of the ellipse will be :

$$\frac{x^2}{1^2}+\frac{y^2}{(\sqrt{5})^2}=1$$

Which is

$\frac{x^2}{1}+\frac{y^2}{5}=1$.

Question:15 Find the equation for the ellipse that satisfies the given conditions:

Length of major axis 26, foci (± 5, 0)

Answer:

Given, In an ellipse,

Length of major axis 26, foci (± 5, 0)

Here, the focus of the ellipse is in X-axis so the major axis of this ellipse will be X-axis.

Therefore, the equation of the ellipse will be of the form:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

Where a and b are the length of the semimajor axis and semiminor axis respectively.

So on comparing standard parameters(Length of semimajor axis and foci) with the given one, we get

$2a=26 \Rightarrow a=13$ and $c=5$

Now, As we know the relation,

$$a^2=b^2+c^2$$

$$b^2=a^2-c^2$$

$$b=\sqrt{a^2-c^2}$$

$b = \sqrt{13^2 - 5^2}$

$b = \sqrt{144}$

$b = 12$

Hence, The Equation of the ellipse will be :

$$\frac{x^2}{13^2} + \frac{y^2}{12^2} = 1$$

Which is

$$\frac{x^2}{169} + \frac{y^2}{144} = 1$$.

Question:16 Find the equation for the ellipse that satisfies the given conditions:

Length of minor axis 16, foci (0, ± 6).

Answer:

Given, In an ellipse,

Length of minor axis 16, foci (0, ± 6).

Here, the focus of the ellipse is on the Y-axis so the major axis of this ellipse will be Y-axis.

Therefore, the equation of the ellipse will be of the form:

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

Where a and b are the length of the semimajor axis and semiminor axis respectively.

So on comparing standard parameters(length of semi-minor axis and foci) with the given one, we get

$2b = 16 \Rightarrow b = 8$ and $c = 6$

Now, As we know the relation,

$a^2 = b^2 + c^2$

$a = \sqrt{b^2 + c^2}$

$a = \sqrt{8^2 + 6^2}$

$a = \sqrt{64 + 36}$

$a = \sqrt{100}$

$a = 10$

Hence, The Equation of the ellipse will be :

$\frac{x^2}{8^2} + \frac{y^2}{10^3} = 1$

Which is

$\frac{x^2}{64} + \frac{y^2}{100} = 1$.

Question:17 Find the equation for the ellipse that satisfies the given conditions:

Foci (± 3, 0), *a* = 4

Answer:

Given, In an ellipse,

V Foci (± 3, 0), *a* = 4

Here foci of the ellipse are in X-axis so the major axis of this ellipse will be X-axis.

Therefore, the equation of the ellipse will be of the form:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Where a and b are the length of the semimajor axis and semiminor axis respectively.

So on comparing standard parameters(vertices and foci) with the given one, we get

$a = 4$ and $c = 3$

Now, As we know the relation,

$$a^2 = b^2 + c^2$$

$$b^2 = a^2 - c^2$$

$$b = \sqrt{a^2 - c^2}$$

$$b = \sqrt{4^2 - 3^2}$$

$$b = \sqrt{7}$$

Hence, The Equation of the ellipse will be :

$$\frac{x^2}{4^2} + \frac{y^2}{(\sqrt{7})^2} = 1$$

Which is

$$\frac{x^2}{16}+\frac{y^2}{7}=1.$$

Question:18 Find the equation for the ellipse that satisfies the given conditions:

b = 3, c = 4, centre at the origin; foci on the x axis.

Answer:

Given,In an ellipse,

b = 3, c = 4, centre at the origin; foci on the x axis.

Here foci of the ellipse are in X-axis so the major axis of this ellipse will be X-axis.

Therefore, the equation of the ellipse will be of the form:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

Where a and b are the length of the semimajor axis and semiminor axis respectively.

Also Given,

$b=3$ and $c=4$

Now, As we know the relation,

$$a^2=b^2+c^2$$

$$a^2=3^2+4^2$$

$a^2 = 25$

$a = 5$

Hence, The Equation of the ellipse will be :

$$\frac{x^2}{5^2} + \frac{y^2}{3^2} = 1$$

Which is

$$\frac{x^2}{25} + \frac{y^2}{9} = 1.$$

Question:19 Find the equation for the ellipse that satisfies the given conditions:

Centre at (0,0), major axis on the y-axis and passes through the points (3, 2) and (1,6).

Answer:

Given,in an ellipse

Centre at (0,0), major axis on the y-axis and passes through the points (3, 2) and (1,6).

Since, The major axis of this ellipse is on the Y-axis, the equation of the ellipse will be of the form:

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

Where a and b are the length of the semimajor axis and semiminor axis respectively.

Now since the ellipse passes through points,(3, 2)

$$\frac{3^2}{b^2} + \frac{2^2}{a^2} = 1$$

$$9a^2 + 4b^2 = a^2b^2$$

since the ellipse also passes through points,(1, 6).

$$\frac{1^2}{b^2} + \frac{6^2}{a^2} = 1$$

$$a^2 + 36b^2 = a^2b^2$$

On solving these two equation we get

$a^2 = 40$ and $b^2 = 10$

Thus, The equation of the ellipse will be

$$\frac{x^2}{10} + \frac{y^2}{40} = 1$$.

Question:20 Find the equation for the ellipse that satisfies the given conditions:

Major axis on the x-axis and passes through the points (4,3) and (6,2) .

Answer:

Given, in an ellipse

Major axis on the x-axis and passes through the points (4,3) and (6,2).

Since The major axis of this ellipse is on the X-axis, the equation of the ellipse will be of the form:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

Where a and b are the length of the semimajor axis and semiminor axis respectively.

Now since the ellipse passes through the point,(4,3)

$$\frac{4^2}{a^2}+\frac{3^2}{b^2}=1$$

$$16b^2+9a^2=a^2b^2$$

since the ellipse also passes through the point (6,2).

$$\frac{6^2}{a^2}+\frac{2^2}{b^2}=1$$

$$4a^2+36b^2=a^2b^2$$

On solving this two equation we get

$a^2=52$ and $b^2=13$

Thus, The equation of the ellipse will be

$$\frac{x^2}{52}+\frac{y^2}{13}=1$$

NCERT solutions for class 11 maths chapter 11 conic sections-Exercise: 11.4

Question:1 Find the coordinates of the foci and the vertices, the eccentricity and the length of the latus rectum of the hyperbolas.

$$\frac{x^2}{16}-\frac{y^2}{9}=1$$

Answer:

Given a Hyperbola equation,

$$\frac{x^2}{16}-\frac{y^2}{9}=1$$

Can also be written as

$$\frac{x^2}{4^2}-\frac{y^2}{3^2}=1$$

Comparing this equation with the standard equation of the hyperbola:

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

We get,

$a=4$ and $b=3$

Now, As we know the relation in a hyperbola,

$$c^2=a^2+b^2$$

$$c=\sqrt{a^2+b^2}$$

$$c=\sqrt{4^2+3^2}$$

$$c=5$$

Here as we can see from the equation that the axis of the hyperbola is X -axis. So,

Coordinates of the foci:

$$(c,0)\ and\ (-c,0)=(5,0)\ and\ (-5,0)$$

The Coordinates of vertices:

$$(a,0)\ and\ (-a,0)=(4,0)\ and\ (-4,0)$$

The Eccentricity:

$$e=\frac{c}{a}=\frac{5}{4}$$

The Length of the latus rectum :

$$\frac{2b^2}{a}=\frac{2(3)^2}{4}=\frac{18}{4}=\frac{9}{2}$$

Question:2 Find the coordinates of the foci and the vertices, the eccentricity and the length of the latus rectum of the hyperbolas.

$$\frac{y^2}{9}-\frac{x^2}{27}=1$$

Answer:

Given a Hyperbola equation,

$$\frac{y^2}{9}-\frac{x^2}{27}=1$$

Can also be written as

$$\frac{y^2}{3^2}-\frac{x^2}{(\sqrt{27})^2}=1$$

Comparing this equation with the standard equation of the hyperbola:

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

We get,

$a = 3$ and $b = \sqrt{27}$

Now, As we know the relation in a hyperbola,

$$c^2 = a^2 + b^2$$

$$c = \sqrt{a^2 + b^2}$$

$$c = \sqrt{3^2 + (\sqrt{27})^2}$$

$$c = \sqrt{36}$$

$$c = 6$$

Here as we can see from the equation that the axis of the hyperbola is Y-axis. So,

Coordinates of the foci:

$$(0, c)\ and\ (0, -c) = (0, 6)\ and\ (0, -6)$$

The Coordinates of vertices:

$$(0, a)\ and\ (0, -a) = (0, 3)\ and\ (0, -3)$$

The Eccentricity:

$$e = \frac{c}{a} = \frac{6}{3} = 2$$

The Length of the latus rectum :

$$\frac{2b^2}{a} = \frac{2(27)}{3} = \frac{54}{3} = 18$$

Question:3 Find the coordinates of the foci and the vertices, the eccentricity and the length of the latus rectum of the hyperbolas.

$$9y^2 - 4x^2 = 36$$

Answer:

Given a Hyperbola equation,

$$9y^2 - 4x^2 = 36$$

Can also be written as

$$\frac{9y^2}{36} - \frac{4x^2}{36} = 1$$

$$\frac{y^2}{2^2} - \frac{x^2}{3^2} = 1$$

Comparing this equation with the standard equation of the hyperbola:

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

We get,

$a = 2$ and $b = 3$

Now, As we know the relation in a hyperbola,

$$c^2 = a^2 + b^2$$

$c=\sqrt{a^2+b^2}$

$c=\sqrt{2^2+3^2}$

$c=\sqrt{13}$

Hence,

Coordinates of the foci:

$(0,c)\ and\ (0,-c)=(0,\sqrt{13})\ and\ (0,-\sqrt{13})$

The Coordinates of vertices:

$(0,a)\ and\ (0,-a)=(0,2)\ and\ (0,-2)$

The Eccentricity:

$e=\frac{c}{a}=\frac{\sqrt{13}}{2}$

The Length of the latus rectum :

$\frac{2b^2}{a}=\frac{2(9)}{2}=\frac{18}{2}=9$

Question:4 Find the coordinates of the foci and the vertices, the eccentricity and the length of the latus rectum of the hyperbolas.

$16x^2-9y^2=576$

Answer:

Given a Hyperbola equation,

$16x^2 - 9y^2 = 576$

Can also be written as

$\frac{16x^2}{576} - \frac{9y^2}{576} = 1$

$\frac{x^2}{36} - \frac{y^2}{64} = 1$

$\frac{x^2}{6^2} - \frac{y^2}{8^2} = 1$

Comparing this equation with the standard equation of the hyperbola:

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

We get,

$a = 6$ and $b = 8$

Now, As we know the relation in a hyperbola,

$c^2 = a^2 + b^2$

$c = \sqrt{a^2 + b^2}$

$c = \sqrt{6^2 + 8^2}$

$c = 10$

Therefore,

Coordinates of the foci:

$(c, 0)\ and\ (-c, 0) = (10, 0)\ and\ (-10, 0)$

The Coordinates of vertices:

$(a, 0)\ and\ (-a, 0) = (6, 0)\ and\ (-6, 0)$

The Eccentricity:

$$e = \frac{c}{a} = \frac{10}{6} = \frac{5}{3}$$

The Length of the latus rectum :

$$\frac{2b^2}{a} = \frac{2(8)^2}{6} = \frac{128}{6} = \frac{64}{3}$$

Question:5 Find the coordinates of the foci and the vertices, the eccentricity and the length of the latus rectum of the hyperbolas.

$5y^2 - 9x^2 = 36$

Answer:

Given a Hyperbola equation,

$5y^2 - 9x^2 = 36$

Can also be written as

$$\frac{5y^2}{36} - \frac{9x^2}{36} = 1$$

$$\frac{y^2}{\frac{36}{5}} - \frac{x^2}{4} = 1$$

$$\frac{y^2}{(\frac{6}{\sqrt{5}})^2} - \frac{x^2}{2^2} = 1$$

Comparing this equation with the standard equation of the hyperbola:

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

We get,

$$a = \frac{6}{\sqrt{5}}$$

and $b = 2$

Now, As we know the relation in a hyperbola,

$$c^2 = a^2 + b^2$$

$$c = \sqrt{a^2 + b^2}$$

$$c = \sqrt{(\frac{6}{\sqrt{5}})^2 + 2^2}$$

$$c = \sqrt{\frac{56}{5}}$$

$$c = 2\sqrt{\frac{14}{5}}$$

Here as we can see from the equation that the axis of the hyperbola is Y-axis. So,

Coordinates of the foci:

$$(0, c)\ and\ (0, -c) = \left(0, 2\sqrt{\frac{14}{5}}\right)\ and\ \left(0, -2\sqrt{\frac{14}{5}}\right)$$

The Coordinates of vertices:

$$(0, a)\ and\ (0, -a) = \left(0, \frac{6}{\sqrt{5}}\right)\ and\ \left(0, -\frac{6}{\sqrt{5}}\right)$$

The Eccentricity:

$$e = \frac{c}{a} = \frac{2\sqrt{\frac{14}{5}}}{\frac{6}{\sqrt{5}}} = \frac{\sqrt{14}}{3}$$

The Length of the latus rectum :

$$\frac{2b^2}{a} = \frac{2(4)}{\frac{6}{\sqrt{5}}} = \frac{4\sqrt{5}}{3}$$

Question:6 Find the coordinates of the foci and the vertices, the eccentricity and the length of the latus rectum of the hyperbolas.

$$49y^2 - 16x^2 = 784$$

Answer:

Given a Hyperbola equation,

$$49y^2 - 16x^2 = 784$$

Can also be written as

$$\frac{49y^2}{784} - \frac{16x^2}{784} = 1$$

$$\frac{y^2}{16} - \frac{x^2}{49} = 1$$

$$\frac{y^2}{4^2} - \frac{x^2}{7^2} = 1$$

Comparing this equation with the standard equation of the hyperbola:

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

We get,

$a = 4$ and $b = 7$

Now, As we know the relation in a hyperbola,

$$c^2 = a^2 + b^2$$

$$c = \sqrt{a^2 + b^2}$$

$$c = \sqrt{4^2 + 7^2}$$

$$c = \sqrt{65}$$

Therefore,

Coordinates of the foci:

$$(0, c)\ and\ (0, -c) = (0, \sqrt{65})\ and\ (0, -\sqrt{65})$$

The Coordinates of vertices:

$$(0, a)\ and\ (0, -a) = (0, 4)\ and\ (0, -4)$$

The Eccentricity:

$e = \frac{c}{a} = \frac{\sqrt{65}}{4}$

The Length of the latus rectum :

$\frac{2b^2}{a} = \frac{2(49)}{4} = \frac{98}{4} = \frac{49}{2}$

Question:7 Find the equations of the hyperbola satisfying the given conditions.

Vertices (± 2, 0), foci (± 3, 0)

Answer:

Given, in a hyperbola

Vertices (± 2, 0), foci (± 3, 0)

Here, Vertices and focii are on the X-axis so, the standard equation of the Hyperbola will be ;

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

By comparing the standard parameter (Vertices and foci) with the given one, we get

$a = 2$ and $c = 3$

Now, As we know the relation in a hyperbola

$c^2 = a^2 + b^2$

$b^2 = c^2 - a^2$

$b^2 = 3^2 - 2^2$

$b^2 = 9 - 4 = 5$

Hence,The Equation of the hyperbola is ;

$$\frac{x^2}{4} - \frac{y^2}{5} = 1$$

Question:8 Find the equations of the hyperbola satisfying the given conditions.

Vertices (0, ± 5), foci (0, ± 8)

Answer:

Given, in a hyperbola

Vertices (0, ± 5), foci (0, ± 8)

Here, Vertices and focii are on the Y-axis so, the standard equation of the Hyperbola will be ;

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

By comparing the standard parameter (Vertices and foci) with the given one, we get

$a = 5$ and $c = 8$

Now, As we know the relation in a hyperbola

$c^2 = a^2 + b^2$

$b^2 = c^2 - a^2$

$b^2 = 8^2 - 5^2$

$b^2 = 64 - 25 = 39$

Hence, The Equation of the hyperbola is ;

$$\frac{y^2}{25} - \frac{x^2}{39} = 1$$.

Question:9 Find the equations of the hyperbola satisfying the given conditions.

Vertices (0, ± 3), foci (0, ± 5)

Answer:

Given, in a hyperbola

Vertices (0, ± 3), foci (0, ± 5)

Here, Vertices and focii are on the Y-axis so, the standard equation of the Hyperbola will be ;

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

By comparing the standard parameter (Vertices and foci) with the given one, we get

$a = 3$ and $c = 5$

Now, As we know the relation in a hyperbola

$c^2 = a^2 + b^2$

$b^2 = c^2 - a^2$

$b^2 = 5^2 - 3^2$

$b^2 = 25 - 9 = 16$

Hence, The Equation of the hyperbola is ;

$$\frac{y^2}{9} - \frac{x^2}{16} = 1.$$

Question:10 Find the equations of the hyperbola satisfying the given conditions.

Foci (± 5, 0), the transverse axis is of length 8.

Answer:

Given, in a hyperbola

Foci (± 5, 0), the transverse axis is of length 8.

Here, focii are on the X-axis so, the standard equation of the Hyperbola will be ;

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

By comparing the standard parameter (transverse axis length and foci) with the given one, we get

$2a = 8 \Rightarrow a = 4$ and $c = 5$

Now, As we know the relation in a hyperbola

$c^2 = a^2 + b^2$

$b^2 = c^2 - a^2$

$b^2 = 5^2 - 4^2$

$b^2 = 25 - 16 = 9$

Hence, The Equation of the hyperbola is ;

$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$