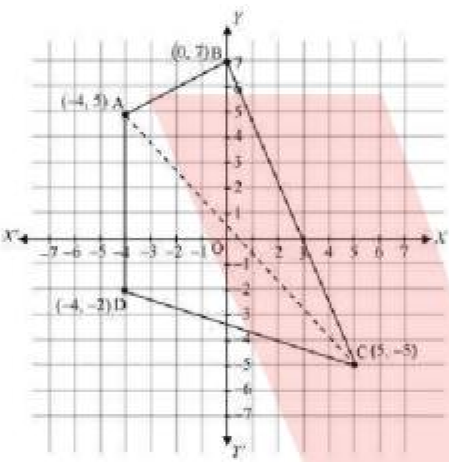
NCERT solutions for class 11 Maths Chapter 10 Straight Lines

Question:1 Draw a quadrilateral in the Cartesian plane, whose vertices are $(-4,5),(0,7),(5,-5)$ and $(-4,-2)$. Also, find its area.

Answer:



Area of ABCD = Area of ABC + Area of ACD

Now, we know that the area of a triangle with vertices $(x_1,y_1),(x_2,y_2)\ and\ (x_3,y_3)$ is given by

$$A=\frac{1}{2}|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$$

Therefore,

Area of triangle ABC $=\frac{1}{2}|-4(7+5)+0(-5-5)+5(5-7)|=\frac{1}{2}|-48-10|=\frac{58}{2}=29$

Similarly,

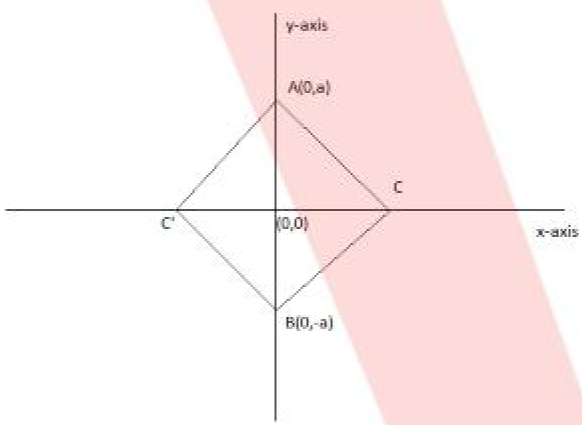
Area of triangle ACD $=\frac{1}{2}|-4(-5+2)+5(-2-5)+-4(5+5)|=\frac{1}{2}|12-35-40|=\frac{63}{2}$

Now,

Area of ABCD = Area of ABC + Area of ACD
$= \frac{121}{2}\ units$

Question:2 The base of an equilateral triangle with side $2a$ lies along the y -axis such that the mid-point of the base is at the origin. Find vertices of the triangle.

Answer:



it is given that it is an equilateral triangle and length of all sides is 2a

The base of the triangle lies on y-axis such origin is the midpoint

Therefore,

Coordinates of point A and B are $(0, a)\ \ and\ \ (0, -a)$ respectively

Now,

Apply Pythagoras theorem in triangle AOC

$AC^2 = OA^2 + OC^2$

$(2a)^2 = a^2 + OC^2$

$OC^2 = 4a^2 - a^2 = 3a^2$

$OC = \pm\sqrt{3}a$

Therefore, coordinates of vertices of the triangle are

$(0, a), (0, -a)\ and\ (\sqrt{3}a, 0)\ \ or\ \ (0, a), (0, -a)\ and\ (-\sqrt{3}a, 0)$

Question:3(i) Find the distance between $P(x_1, y_1)$ and $Q(x_2, y_2)$ when :

PQ is parallel to the y -axis.

Answer:

When PQ is parallel to the y-axis

then, x coordinates are equal i.e. $x_2 = x_1$

Now, we know that the distance between two points is given by
$D = |\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}|$

Now, in this case $x_2 = x_1$

Therefore,
$D = |\sqrt{(x_2 - x_2)^2 + (y_2 - y_1)^2}| = |\sqrt{(y_2 - y_1)^2}| = |(y_2 - y_1)|$

Therefore, the distance between $P(x_1, y_1)$ and $Q(x_2, y_2)$ when PQ is parallel to y-axis is $|(y_2 - y_1)|$

Question:3(ii) Find the distance between $P(x_1, y_1)$ and $Q(x_2, y_2)$ when :

PQ is parallel to the x -axis.

Answer:

When PQ is parallel to the x-axis

then, x coordinates are equal i.e. $y_2 = y_1$

Now, we know that the distance between two points is given by
$D = |\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}|$

Now, in this case $y_2 = y_1$

Therefore,
$D = |\sqrt{(x_2 - x_1)^2 + (y_2 - y_2)^2}| = |\sqrt{(x_2 - x_1)^2}| = |x_2 - x_1|$

Therefore, the distance between $P(x_1, y_1)$ and $Q(x_2, y_2)$ when PQ is parallel to the x-axis is $|x_2 - x_1|$

Question:4 Find a point on the x-axis, which is equidistant from the points $(7, 6)$ and $(3, 4)$.

Answer:

Point is on the x-axis, therefore, y coordinate is 0

Let's assume the point is (x, 0)

Now, it is given that the given point (x, 0) is equidistance from point (7, 6) and (3, 4)

We know that

Distance between two points is given by

$D = |\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}|$

Now,

$D_1 = |\sqrt{(x - 7)^2 + (0 - 6)^2}| = |\sqrt{x^2 + 49 - 14x + 36}| = |\sqrt{x^2 - 14x + 85}|$

and

$D_2 = |\sqrt{(x - 3)^2 + (0 - 4)^2}| = |\sqrt{x^2 + 9 - 6x + 16}| = |\sqrt{x^2 - 6x + 25}|$

Now, according to the given condition

$D_1 = D_2$

$|\sqrt{x^2 - 14x + 85}| = |\sqrt{x^2 - 6x + 25}|$

Squaring both the sides

$x^2 - 14x + 85 = x^2 - 6x + 25$

$8x = 60$

$x = \frac{60}{8} = \frac{15}{2}$

Therefore, the point is $(\frac{15}{2}, 0)$

Question:5 Find the slope of a line, which passes through the origin, and the mid-point of the line segment joining the points $P(0,-4)$ and $B(8,0)$.

Answer:

Mid-point of the line joining the points $P(0,-4)$ and $B(8,0)$. is

$$l=\left(\frac{8}{2},\frac{-4}{2}\right)=(4,-2)$$

It is given that line also passes through origin which means passes through the point (0, 0)

Now, we have two points on the line so we can now find the slope of a line by using formula

$$m=\frac{y_2-y_1}{x_2-x_1}$$

$$m=\frac{-2-0}{4-0}=\frac{-2}{4}=\frac{-1}{2}$$

Therefore, the slope of the line is $\frac{-1}{2}$

Question:6 Without using the Pythagoras theorem, show that the points $(4,4),(3,5)$ and $(-1,-1)$, are the vertices of a right angled triangle.

Answer:

It is given that point A(4,4) , B(3,5) and C(-1,-1) are the vertices of a right-angled triangle

Now,

We know that the distance between two points is given by

$$D=|\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}|$$

Length of AB $=|\sqrt{(4-3)^2+(4-5)^2}|=|\sqrt{1+1}|=\sqrt{2}$

Length of BC $=|\sqrt{(3+1)^2+(5+1)^2}|=|\sqrt{16+36}|=\sqrt{52}$

Length of AC $= |\sqrt{(4+1)^2+(4+1)^2}| = |\sqrt{25+25}| = \sqrt{50}$

Now, we know that Pythagoras theorem is

$H^2 = B^2 + L^2$

Is clear that

$$(\sqrt{52})^2 = (\sqrt{50})^2 + (\sqrt{2})^2$$

$52 = 52$

$i.e$

$BC^2 = AB^2 + AC^2$

Hence proved

Question:7 Find the slope of the line, which makes an angle of 30° with the positive direction of y -axis measured anticlockwise.

Answer:

It is given that the line makes an angle of 30° with the positive direction of y -axis measured anticlockwise

Now, we know that

$m = \tan\theta$

line makes an angle of 30° with the positive direction of y -axis

Therefore, the angle made by line with the positive x-axis is = $90^\circ + 30^\circ = 120^\circ$

Now,

$m = \tan 120^\circ = -\tan 60^\circ = -\sqrt{3}$

Therefore, the slope of the line is $-\sqrt{3}$

Question:8 Find the value of x for which the points $(x, -1), (2, 1)$ and $(4, 5)$ are collinear.

Answer:

Point is collinear which means they lie on the same line by this we can say that their slopes are equal

Given points are A(x,-1) , B(2,1) and C(4,5)

$$Slope = m = \frac{y_2 - y_1}{x_2 - x_1}$$

Now,

The slope of AB = Slope of BC

$$\frac{1+1}{2-x} = \frac{5-1}{4-2}$$

$$\frac{2}{2-x} = \frac{4}{2}$$

$$\frac{2}{2-x} = 2$$

$$2 = 2(2-x)$$
$$2 = 4 - 2x$$
$$-2x = -2$$
$$x = 1$$

Therefore, the value of x is 1

Question:9 Without using the distance formula, show that points $(-2,-1), (4,0), (3,3)$ and $(-3,2)$are the vertices of a parallelogram.

Answer:

Given points are $A(-2,-1), B(4,0), C(3,3)$ and $D(-3,2)$

We know the pair of the opposite side are parallel to each other in a parallelogram

Which means their slopes are also equal

$$Slope = m = \frac{y_2 - y_1}{x_2 - x_1}$$

The slope of AB =

$$\frac{0+1}{4+2}=\frac{1}{6}$$

The slope of BC =

$$\frac{3-0}{3-4}=\frac{3}{-1}=-3$$

The slope of CD =

$$\frac{2-3}{-3-3}=\frac{-1}{-6}=\frac{1}{6}$$

The slope of AD

$$=\frac{2+1}{-3+2}=\frac{3}{-1}=-3$$

We can clearly see that

The slope of AB = Slope of CD (which means they are parallel)

and

The slope of BC = Slope of AD (which means they are parallel)

Hence pair of opposite sides are parallel to each other

Therefore, we can say that points $(-2,-1),(4,0),(3,3)$ and $(-3,2)$ are the vertices of a parallelogram

Question:10 Find the angle between the x-axis and the line joining the points $(3,-1)$ and $(4,-2)$.

Answer:

We know that

$m=\tan\theta$

So, we need to find the slope of line joining points (3,-1) and (4,-2)

Now,

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2+1}{4-3} = -1$$

$$\tan\theta = -1$$
$$\tan\theta = \tan\frac{3\pi}{4} = \tan 135^\circ$$
$$\theta = \frac{3\pi}{4} = 135^\circ$$

Therefore, angle made by line with positive x-axis when measure in anti-clockwise direction is 135°

Question:11 The slope of a line is double of the slope of another line. If tangent of the angle between them is $\frac{1}{3}$, find the slopes of the lines

Answer:

Let $m_1 \ and \ m_2$ are the slopes of lines and θ is the angle between them

Then, we know that

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$$

It is given that $m_2 = 2m_1$ and

$$\tan\theta = \frac{1}{3}$$

Now,

$$\frac{1}{3} = \left|\frac{2m_1 - m_1}{1 + m_1.2m_1}\right|$$
$$\frac{1}{3} = \left|\frac{m_1}{1 + 2m_1^2}\right|$$

Now,

Now,
$$m_1 = \frac{1}{2}\ or\ \frac{-1}{2}\ or\ 1\ or\ -1$$

According to which value of $m_2 = 1\ or\ -1\ or\ 2\ or\ -2$

Therefore, $m_1, m_2 = \frac{1}{2}, 1\ or\ \frac{-1}{2}, -1\ or\ 1, 2\ or\ -1, -2$

Question:12 A line passes through (x_1, y_1) and (h, k). If slope of the line is m, show that $k - y_1 = m(h - x_1)$.

Answer:

Given that A line passes through (x_1, y_1) and (h, k) and slope of the line is m

Now,
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$\Rightarrow m = \frac{k - y_1}{h - x_1}$$
$$\Rightarrow (k - y_1) = m(h - x_1)$$

Hence proved

Question:13 If three points $(h, 0)$, (a, b) and $(0, k)$ lie on a line, show that $\frac{a}{h} + \frac{b}{k} = 1.$

Answer:

Points $A(h, 0)$, $B(a, b)$ and $C(0, k)$ lie on a line so by this we can say that their slopes are also equal

We know that
$$Slope = m = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope of AB = $\frac{b - 0}{a - h} = \frac{b}{a - h}$

Slope of AC = $\frac{k-b}{0-a} = \frac{k-b}{-a}$

Now,

Slope of AB = slope of AC

$\frac{b}{a-h} = \frac{k-b}{-a}$

$-ab = (a-h)(k-b)$

$-ab = ak - ab - hk + hb$

$ak + hb = hk$

Now divide both the sides by hk

$\frac{ak}{hk} + \frac{hb}{hk} = \frac{hk}{hk}$

$\frac{a}{h} + \frac{b}{k} = 1$

Hence proved

Question:14 Consider the following population and year graph, find the slope of the line AB and using it, find what will be the population in the year 2010?

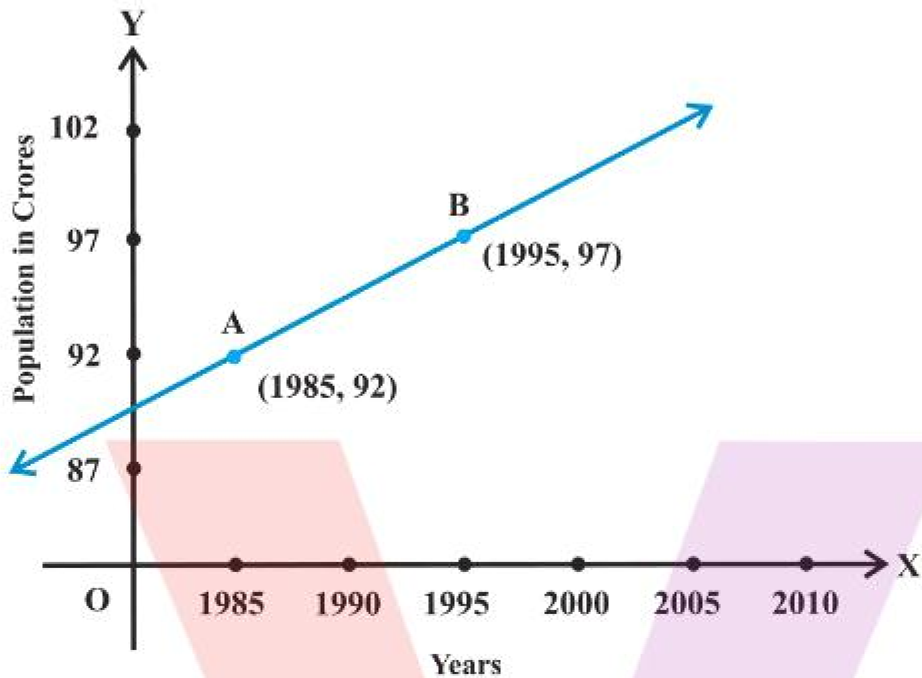


Fig 10.10

Answer:

Given point A(1985,92) and B(1995,97)

Now, we know that

$$Slope = m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{97-92}{1995-1985} = \frac{5}{10} = \frac{1}{2}$$

Therefore, the slope of line AB is $\frac{1}{2}$

Now, the equation of the line passing through the point (1985,92) and with slope = $\frac{1}{2}$ is given by

$$(y-92) = \frac{1}{2}(x-1985)$$

$$2y - 184 = x - 1985$$

$$x - 2y = 1801$$

Now, in the year 2010 the population is

$$2010 - 2y = 1801$$
$$-2y = -209$$
$$y = 104.5$$

Therefore, the population in the year 2010 is 104.5 crore

NCERT solutions for class 11 maths chapter 10 straight lines-Exercise: 10.2

Question:1 Find the equation of the line which satisfy the given conditions:

Write the equations for the x -and y -axes.

Answer:

Equation of x-axis is y = 0

and

Equation of y-axis is x = 0

Question:2 Find the equation of the line which satisfy the given conditions:

Passing through the point $(-4, 3)$ with slope $\frac{1}{2}$.

Answer:

We know that , equation of line passing through point (x_1, y_1) and with slope m is given by

$$(y - y_1) = m(x - x_1)$$

Now, equation of line passing through point (-4,3) and with slope $\frac{1}{2}$ is

$$(y-3)=\frac{1}{2}(x-(-4))$$
$$2y-6=x+4$$
$$x-2y+10=0$$

Therefore, equation of the line is $x-2y+10=0$

Question:3 Find the equation of the line which satisfy the given conditions:

Passing through $(0,0)$ with slope m .

Answer:

We know that the equation of the line passing through the point (x_1,y_1) and with slope m is given by

$$(y-y_1)=m(x-x_1)$$

Now, the equation of the line passing through the point (0,0) and with slope m is

$$(y-0)=m(x-0)$$
$$y=mx$$

Therefore, the equation of the line is $y=mx$

Question:4 Find the equation of the line which satisfy the given conditions:

Passing through $(2,2\sqrt{3})$ and inclined with the x-axis at an angle of 75° .

Answer:

We know that the equation of the line passing through the point (x_1,y_1) and with slope m is given by

$$(y-y_1)=m(x-x_1)$$

we know that

$$m=\tan\theta$$

where θ is angle made by line with positive x-axis measure in the anti-clockwise direction

$m=\tan 75^\circ \qquad (\because \theta=75^\circ\ given)$

$m=\dfrac{\sqrt{3}+1}{\sqrt{3}-1}$

Now, the equation of the line passing through the point $(2,2\sqrt{3})$ and with slope $m=\dfrac{\sqrt{3}+1}{\sqrt{3}-1}$ is

Therefore, the equation of the line is $(\sqrt{3}+1)x-(\sqrt{3}-1)y=4(\sqrt{3}-1)$

Question:5 Find the equation of the line which satisfy the given conditions:

Intersecting the x -axis at a distance of 3 units to the left of origin with slope -2 .

Answer:

We know that the equation of the line passing through the point (x_1,y_1) and with slope m is given by

$(y-y_1)=m(x-x_1)$

Line Intersecting the x -axis at a distance of 3 units to the left of origin which means the point is (-3,0)

Now, the equation of the line passing through the point (-3,0) and with slope -2 is

$(y-0)=-2(x-(-3))$

$y=-2x-6$

$2x+y+6=0$

Therefore, the equation of the line is $2x+y+6=0$

Question:6 Find the equation of the line which satisfy the given conditions:

Intersecting the y -axis at a distance of 2 units above the origin and making an angle of 30° with positive direction of the x-axis.

Answer:

We know that , equation of line passing through point (x_1, y_1) and with slope m is given by

$(y - y_1) = m(x - x_1)$

Line Intersecting the y-axis at a distance of 2 units above the origin which means point is (0,2)

we know that

Now, the equation of the line passing through the point (0,2) and with slope $\frac{1}{\sqrt{3}}$ is

$$(y - 2) = \frac{1}{\sqrt{3}}(x - 0)$$

$$\sqrt{3}(y - 2) = x$$
$$x - \sqrt{3}y + 2\sqrt{3} = 0$$

Therefore, the equation of the line is $x - \sqrt{3}y + 2\sqrt{3} = 0$

Question:7 Find the equation of the line which satisfy the given conditions:

Passing through the points $(-1, 1)$ and $(2, -4)$.

Answer:

We know that , equation of line passing through point (x_1, y_1) and with slope m is given by

$(y - y_1) = m(x - x_1)$

Now, it is given that line passes throught point (-1 ,1) and (2 , -4)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-4 - 1}{2 + 1} = \frac{-5}{3}$$

Now, equation of line passing through point (-1,1) and with slope $\frac{-5}{3}$ is

$$(y-1)=\frac{-5}{3}(x-(-1))$$

$$3(y-1)=-5(x+1)$$
$$3y-3=-5x-5$$
$$5x+3y+2=0$$

Question:8 Find the equation of the line which satisfy the given conditions:

Perpendicular distance from the origin is 5 units and the angle made by the perpendicular with the positive x -axis is 30°.

Answer:

It is given that length of perpendicular is 5 units and angle made by the perpendicular with the positive x -axis is 30°

Therefore, equation of line is

$$x\cos\theta+y\sin\theta=p$$

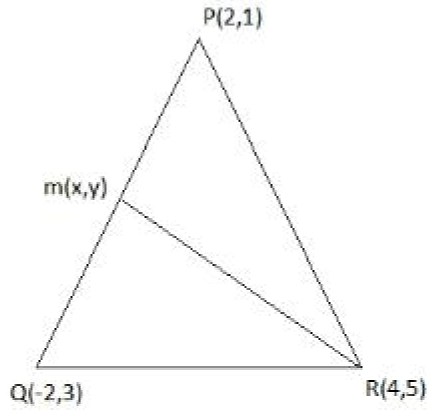
In this case p = 5 and $\theta=30^\circ$

$$x\cos 30^\circ+y\sin 30^\circ=5$$
$$x.\frac{\sqrt{3}}{2}+\frac{y}{2}=5$$
$$\sqrt{3}x+y=10$$

Therefore, equation of the line is $\sqrt{3}x+y=10$

Question:9 The vertices of $\Delta\, PQR$ are $P(2,1), Q(-2,3)$ and $R(4,5)$. Find equation of the median through the vertex R .

Answer:



The vertices of ΔPQR are $P(2,1), Q(-2,3)$ and $R(4,5)$

Let m be RM b the median through vertex R

Coordinates of M (x, y) = $\left(\frac{2-2}{2}, \frac{1+3}{2}\right) = (0,2)$

Now, slope of line RM

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-2}{4-0} = \frac{3}{4}$

Now, equation of line passing through point (x_1, y_1) and with slope m is

$(y - y_1) = m(x - x_1)$

equation of line passing through point (0 , 2) and with slope $\frac{3}{4}$ is

$(y-2) = \frac{3}{4}(x-0)$

$4(y-2) = 3x$
$4y - 8 = 3x$
$3x - 4y + 8 = 0$

Therefore, equation of median is $3x - 4y + 8 = 0$

Question:10 Find the equation of the line passing through $(-3,5)$ and perpendicular to the line through the points $(2,5)$ and $(-3,6)$.

Answer:

It is given that the line passing through $(-3, 5)$ and perpendicular to the line through the points $(2, 5)$and $(-3, 6)$

Let the slope of the line passing through the point (-3,5) is m and

Slope of line passing through points (2,5) and (-3,6)

$$m' = \frac{6-5}{-3-2} = \frac{1}{-5}$$

Now this line is perpendicular to line passing through point (-3,5)

Therefore,

$$m = -\frac{1}{m'} = -\frac{1}{\frac{1}{-5}} = 5$$

Now, equation of line passing through point (x_1, y_1) and with slope m is

$$(y - y_1) = m(x - x_1)$$

equation of line passing through point (-3 , 5) and with slope 5 is

$$(y-5) = 5(x-(-3))$$

$$(y-5) = 5(x+3)$$
$$y - 5 = 5x + 15$$
$$5x - y + 20 = 0$$

Therefore, equation of line is $5x - y + 20 = 0$

Question:11 A line perpendicular to the line segment joining the points $(1, 0)$ and $(2, 3)$ divides it in the ratio $1 : n$. Find the equation of the line.

Answer:

Co-ordinates of point which divide line segment joining the points $(1, 0)$ and $(2, 3)$ in the ratio $1 : n$ is

$$\left(\frac{n(1)+1(2)}{1+n}, \frac{n.(0)+1.(3)}{1+n}\right) = \left(\frac{n+2}{1+n}, \frac{3}{1+n}\right)$$

Let the slope of the perpendicular line is m

And Slope of line segment joining the points $(1, 0)$ and $(2, 3)$ is

$$m' = \frac{3-0}{2-1} = 3$$

Now, slope of perpendicular line is

$$m = -\frac{1}{m'} = -\frac{1}{3}$$

Now, equation of line passing through point (x_1, y_1) and with slope m is

$$(y - y_1) = m(x - x_1)$$

equation of line passing through point $\left(\frac{n+2}{1+n}, \frac{3}{1+n}\right)$ and with slope $-\frac{1}{3}$ is

$$(y - \frac{3}{1+n}) = -\frac{1}{3}(x - (\frac{n+2}{1+n}))$$

$$3y(1+n) - 9 = -x(1+n) + n + 2$$

$$x(1+n) + 3y(1+n) = n + 11$$

Therefore, equation of line is $x(1+n) + 3y(1+n) = n + 11$

Question:12 Find the equation of a line that cuts off equal intercepts on the coordinate axes and passes through the point $(2, 3)$.

Answer:

Let (a, b) are the intercept on x and y-axis respectively

Then, the equation of the line is given by

$$\frac{x}{a} + \frac{y}{b} = 1$$

Intercepts are equal which means a = b

$$\frac{x}{a} + \frac{y}{a} = 1$$

$$x + y = a$$

Now, it is given that line passes through the point (2,3)

Therefore,

$$a = 2 + 3 = 5$$

therefore, equation of the line is $x + y = 5$

Question:13 Find equation of the line passing through the point $(2, 2)$ and cutting off intercepts on the axes whose sum is 9 .

Answer:

Let (a, b) are the intercept on x and y axis respectively

Then, the equation of line is given by

$$\frac{x}{a}+\frac{y}{b}=1$$

It is given that

a + b = 9

b = 9 - a

Now,

$$\frac{x}{a}+\frac{y}{9-a}=1$$

$$x(9-a)+ay=a(9-a)$$
$$9x-ax+ay=9a-a^2$$

It is given that line passes through point (2 ,2)

So,

case (i) a = 6 b = 3

$$\frac{x}{6}+\frac{y}{3}=1$$

$$x+2y=6$$

case (ii) a = 3 , b = 6

$$\frac{x}{3}+\frac{y}{6}=1$$

$$2x+y=6$$

Therefore, equation of line is 2x + y = 6 , x + 2y = 6

Question:14 Find equation of the line through the point $(0, 2)$ making an angle $\frac{2\pi}{3}$ with the positive x -axis. Also, find the equation of line parallel to it and crossing the y -axis at a distance of 2 units below the origin .

Answer:

We know that

$$m = \tan\theta$$
$$m = \tan\frac{2\pi}{3} = -\sqrt{3}$$

Now, equation of line passing through point (0 , 2) and with slope $-\sqrt{3}$ is

$$(y-2) = -\sqrt{3}(x-0)$$
$$\sqrt{3}x + y - 2 = 0$$

Therefore, equation of line is $\sqrt{3}x + y - 2 = 0$ -(i)

Now, It is given that line crossing the y -axis at a distance of 2 units below the origin which means coordinates are (0 ,-2)

This line is parallel to above line which means slope of both the lines are equal

Now, equation of line passing through point (0 , -2) and with slope $-\sqrt{3}$ is

$$(y-(-2)) = -\sqrt{3}(x-0)$$
$$\sqrt{3}x + y + 2 = 0$$

Therefore, equation of line is $\sqrt{3}x + y + 2 = 0$

Question:15 The perpendicular from the origin to a line meets it at the point $(-2, 9)$, find the equation of the line.

Answer:

Let the slope of the line is m

and slope of a perpendicular line is which passes through the origin (0, 0) and (-2, 9) is

$$m' = \frac{9-0}{-2-0} = \frac{9}{-2}$$

Now, the slope of the line is

$$m = -\frac{1}{m'} = \frac{2}{9}$$

Now, the equation of line passes through the point (-2, 9) and with slope $\frac{2}{9}$ is

$$(y-9) = \frac{2}{9}(x-(-2))$$

$$9(y-9) = 2(x+2)$$
$$2x - 9y + 85 = 0$$

Therefore, the equation of the line is $2x - 9y + 85 = 0$

Question:16 The length L (in centimetre) of a copper rod is a linear function of its Celsius temperature C . In an experiment,

if $L = 124.942$ when $C = 20$ and $L = 125.134$ when $C = 110$, express L in terms of C

Answer:

It is given that

If $C = 20$ then $L = 124.942$

and If $C = 110$ then $L = 125.134$

Now, if assume C along x-axis and L along y-axis

Then, we will get coordinates of two points (20 , 124.942) and (110 , 125.134)

Now, the relation between C and L is given by equation

$$(L - 124.942) = \frac{125.134 - 124.942}{110 - 20}(C - 20)$$

$$(L - 124.942) = \frac{0.192}{90}(C - 20)$$

$$L = \frac{0.192}{90}(C - 20) + 124.942$$

Which is the required relation

Question:17 The owner of a milk store finds that, he can sell 980 litres of milk each week at $Rs\ 14/litre$ and 1220 litres of milk each week at $Rs\ 16/litre$. Assuming a linear relationship between selling price and demand, how many litres could he sell weekly at $Rs\ 17/litre$

Answer:

It is given that the owner of a milk store sell

980 litres milk each week at $Rs\ 14/litre$

and 1220 litres of milk each week at $Rs\ 16/litre$

Now, if we assume the rate of milk as x-axis and Litres of milk as y-axis

Then, we will get coordinates of two points i.e. (14, 980) and (16, 1220)

Now, the relation between litres of milk and Rs/litres is given by equation

$$(L-980)=\frac{1220-980}{16-14}(R-14)$$

$$(L-980)=\frac{240}{2}(R-14)$$

$$L-980=120R-1680$$

$$L=120R-700$$

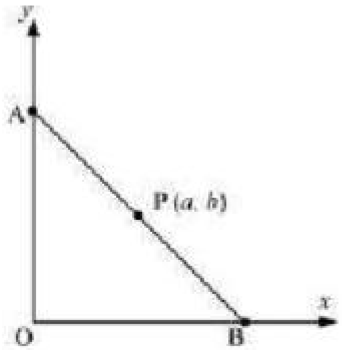
Now, at $Rs\ 17/litre$ he could sell

$$L=120\times 17-700=2040-700=1340$$

He could sell 1340 litres of milk each week at $Rs\ 17/litre$

Question:18 $P(a,b)$ is the mid-point of a line segment between axes. Show that equation of the line is $\frac{x}{a}+\frac{y}{b}=2$.

Answer:



Now, let coordinates of point A is (0 , y) and of point B is (x , 0)

The,

$\frac{x+0}{2} = a \ and \ \frac{0+y}{2} = b$

$x = 2a \ and \ y = 2b$

Therefore, the coordinates of point A is (0 , 2b) and of point B is (2a , 0)

Now, slope of line passing through points (0,2b) and (2a,0) is

$m = \frac{0-2b}{2a-0} = \frac{-2b}{2a} = \frac{-b}{a}$

Now, equation of line passing through point (2a,0) and with slope $\frac{-b}{a}$ is

$(y-0) = \frac{-b}{a}(x-2a)$

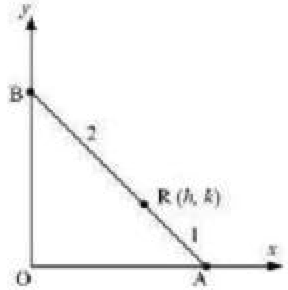
$\frac{y}{b} = -\frac{x}{a} + 2$

$\frac{x}{a} + \frac{y}{b} = 2$

Hence proved

Question:19 Point $R(h,k)$ divides a line segment between the axes in the ratio $1:2$. Find equation of the line.

Answer:



Let the coordinates of Point A is (x,0) and of point B is (0,y)

It is given that point R(h , k) divides the line segment between the axes in the ratio $1:2$

Therefore,

R(h , k)

$$h=\frac{2x}{3} \;\; and \;\; k=\frac{y}{3}$$
$$x=\frac{3h}{2} \;\; and \;\; y=3k$$

Therefore, coordinates of point A is $\left(\frac{3h}{2},0\right)$ and of point B is $(0,3k)$

Now, slope of line passing through points $\left(\frac{3h}{2},0\right)$ and $(0,3k)$ is

$$m=\frac{3k-0}{0-\frac{3h}{2}}=\frac{2k}{-h}$$

Now, equation of line passing through point $(0,3k)$ and with slope $-\frac{2k}{h}$ is

$$(y-3k)=-\frac{2k}{h}(x-0)$$
$$h(y-3k)=-2k(x)$$
$$yh-3kh=-2kx$$
$$2kx+yh=3kh$$

Therefore, the equation of line is $2kx+yh=3kh$

Question:20 By using the concept of equation of a line, prove that the three points $(3,0)$, $(-2,-2)$ and $(8,2)$ are collinear.

Answer:

Points are collinear means they lies on same line

Now, given points are $A(3,0), B(-2,-2)$ and $C(8,2)$

Equation of line passing through point A and B is

$(y-0)=\frac{0+2}{3+2}(x-3)$

$y=\frac{2}{5}(x-3) \Rightarrow 5y=2(x-3)$

$2x-5y=6$

Therefore, the equation of line passing through A and B is $2x-5y=6$

Now, Equation of line passing through point B and C is

$(y-2)=\frac{2+2}{8+2}(x-8)$

$(y-2)=\frac{4}{10}(x-8)$

$(y-2)=\frac{2}{5}(x-8) \Rightarrow 5(y-2)=2(x-8)$

$5y-10=2x-16$

$2x-5y=6$

Therefore, Equation of line passing through point B and C is $2x-5y=6$

When can clearly see that Equation of line passing through point A nd B and through B and C is the same

By this we can say that points $A(3,0), B(-2,-2)$ and $C(8,2)$ are collinear points

NCERT solutions for class 11 maths chapter 10 straight lines-Exercise: 10.3

Question:1(i) Reduce the following equations into slope - intercept form and find their slopes and the y - intercepts.

$x + 7y = 0$

Answer:

Given equation is

$x + 7y = 0$

we can rewrite it as

$y = -\frac{1}{7}x$ -(i)

Now, we know that the Slope-intercept form of the line is

$y = mx + C$ -(ii)

Where m is the slope and C is some constant

On comparing equation (i) with equation (ii)

we will get

$m = -\frac{1}{7}$ and $C = 0$

Therefore, slope and y-intercept are $-\frac{1}{7}\ and\ 0$ respectively

Question:1(ii) Reduce the following equations into slope - intercept form and find their slopes and the y - intercepts.

$6x + 3y - 5 = 0$

Answer:

Given equation is

$6x + 3y - 5 = 0$

we can rewrite it as

$y = -\frac{6}{3}x + \frac{5}{3} \Rightarrow y = -2x + \frac{5}{3}$ -(i)

Now, we know that the Slope-intercept form of line is

$y = mx + C$ -(ii)

Where m is the slope and C is some constant

On comparing equation (i) with equation (ii)

we will get

$m = -2$ and $C = \frac{5}{3}$

Therefore, slope and y-intercept are $-2 \ and \ \frac{5}{3}$ respectively

Question:1(iii) Reduce the following equations into slope - intercept form and find their slopes and the y - intercepts.

$y = 0.$

Answer:

Given equation is

$y = 0$ -(i)

Now, we know that the Slope-intercept form of the line is

$y = mx + C$ -(ii)

Where m is the slope and C is some constant

On comparing equation (i) with equation (ii)

we will get

$m = 0$ and $C = 0$

Therefore, slope and y-intercept are $0 \ and \ 0$ respectively

Question:2(i) Reduce the following equations into intercept form and find their intercepts on the axes.

$3x + 2y - 12 = 0$

Answer:

Given equation is

$3x + 2y - 12 = 0$

we can rewrite it as

$\frac{3x}{12} + \frac{2y}{12} = 1$

$\frac{x}{4} + \frac{y}{6} = 1$ -(i)

Now, we know that the intercept form of line is

$\frac{x}{a} + \frac{y}{b} = 1$ -(ii)

Where a and b are intercepts on x and y axis respectively

On comparing equation (i) and (ii)

we will get

a = 4 and b = 6

Therefore, intercepts on x and y axis are 4 and 6 respectively

Question:2(ii) Reduce the following equations into intercept form and find their intercepts on the axes.

$4x - 3y = 6$

Answer:

Given equation is

$4x - 3y = 6$

we can rewrite it as

$\frac{4x}{6} - \frac{3y}{6} = 1$

$\frac{x}{\frac{3}{2}} - \frac{y}{2} = 1$ -(i)

Now, we know that the intercept form of line is
$\frac{x}{a}+\frac{y}{b}=1$ -(ii)

Where a and b are intercepts on x and y axis respectively

On comparing equation (i) and (ii)

we will get
$a=\frac{3}{2}$ and $b=-2$

Therefore, intercepts on x and y axis are $\frac{3}{2}$ and -2 respectively

Question:2(iii) Reduce the following equations into intercept form and find their intercepts on the axes.

$3y+2=0$

Answer:

Given equation is
$3y+2=0$

we can rewrite it as
$y=\frac{-2}{3}$

Therefore, intercepts on y-axis are $\frac{-2}{3}$

and there is no intercept on x-axis

Question:3(i) Reduce the following equations into normal form. Find their perpendicular distances from the origin and angle between perpendicular and the positive x-axis.

$x-\sqrt{3}y+8=0$

Answer:

Given equation is

$x-\sqrt{3}y+8=0$

we can rewrite it as

$-x+\sqrt{3}y=8$

Coefficient of x is -1 and y is $\sqrt{3}$

Therefore, $\sqrt{(-1)^2+(\sqrt{3})^2}=\sqrt{1+3}=\sqrt{4}=2$

Now, Divide both the sides by 2

we will get

$-\frac{x}{2}+\frac{\sqrt{3}y}{2}=4$

we can rewrite it as

$x\cos 120^\circ+y\sin 120^\circ=4 \qquad -(i)$

Now, we know that the normal form of the line is

$x\cos\theta+y\sin\theta=p \qquad -(ii)$

Where θ is the angle between perpendicular and the positive x-axis and p is the perpendicular distance from the origin

On comparing equation (i) and (ii)

we wiil get

$\theta=120^\circ \ \ and \ \ p=4$

Therefore, the angle between perpendicular and the positive x-axis and perpendicular distance from the origin is $120^\circ \ and \ 4$ respectively

Question:3(ii) Reduce the following equations into normal form. Find their perpendicular distances from the origin and angle between perpendicular and the positive x-axis.

$y-2=0$

Answer:

Given equation is

$y-2=0$

we can rewrite it as

$0.x+y=2$

Coefficient of x is 0 and y is 1

Therefore, $\sqrt{(0)^2+(1)^2}=\sqrt{0+1}=\sqrt{1}=1$

Now, Divide both the sides by 1

we will get

$y=2$

we can rewrite it as

$x\cos 90^\circ + y\sin 90^\circ = 2 \qquad -(i)$

Now, we know that normal form of line is

$x\cos\theta + y\sin\theta = p \qquad -(ii)$

Where θ is the angle between perpendicular and the positive x-axis and p is the perpendicular distance from the origin

On comparing equation (i) and (ii)

we wiil get

$\theta = 90^\circ \ \ and \ \ p=2$

Therefore, the angle between perpendicular and the positive x-axis and perpendicular distance from the origin is $90^\circ \ and \ 2$ respectively

Question:3(iii) Reduce the following equations into normal form. Find their perpendicular distances from the origin and angle between perpendicular and the positive x-axis.

$x-y=4$

Answer:

Given equation is

$x - y = 4$

Coefficient of x is 1 and y is -1

Therefore, $\sqrt{(1)^2 + (-1)^2} = \sqrt{1+1} = \sqrt{2}$

Now, Divide both the sides by $\sqrt{2}$

we wiil get

$\frac{x}{\sqrt{2}} - \frac{y}{\sqrt{2}} = \frac{4}{\sqrt{2}}$

we can rewrite it as

$x \cos 315^\circ + y \sin 315^\circ = 2\sqrt{2} \qquad -(i)$

Now, we know that normal form of line is

$x \cos \theta + y \sin \theta = p \qquad -(ii)$

Where θ is the angle between perpendicular and the positive x-axis and p is the perpendicular distance from the origin

On compairing equation (i) and (ii)

we wiil get

$\theta = 315^\circ \;\; and \;\; p = 2\sqrt{2}$

Therefore, the angle between perpendicular and the positive x-axis and perpendicular distance from the origin is $315^\circ \; and \; 2\sqrt{2}$ respectively

Question:4 Find the distance of the point $(-1, 1)$ from the line $12(x+6) = 5(y-2)$.

Answer:

Given the equation of the line is

$12(x+6) = 5(y-2)$

we can rewrite it as

$12x + 72 = 5y - 10$

$12x - 5y + 82 = 0$

Now, we know that

$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$ where A and B are the coefficients of x and y and C is some constant and (x_1, y_1) is point from which we need to find the distance

In this problem A = 12 , B = -5 , c = 82 and (x_1, y_1) = (-1 , 1)

Therefore,

Therefore, the distance of the point $(-1, 1)$ from the line $12(x+6) = 5(y-2)$ is 5 units

Question:5 Find the points on the x-axis, whose distances from the line $\frac{x}{3} + \frac{y}{4} = 1$ are 4 units.

Answer:

Given equation of line is

$\frac{x}{3} + \frac{y}{4} = 1$

we can rewrite it as

$4x + 3y - 12 = 0$

Now, we know that

$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$

In this problem A = 4 , B = 3 C = -12 and d = 4

point is on x-axis therefore (x_1, y_1) = (x ,0)

Now,

$$4 = \frac{|4.x + 3.0 - 12|}{\sqrt{4^2 + 3^2}} = \frac{|4x - 12|}{\sqrt{16 + 9}} = \frac{|4x - 12|}{\sqrt{25}} = \frac{|4x - 12|}{5}$$
$$20 = |4x - 12|$$
$$4|x - 3| = 20$$
$$|x - 3| = 5$$

Now if x > 3

Then,
$$|x - 3| = x - 3$$
$$x - 3 = 5$$
$$x = 8$$

Therefore, point is (8,0)

and if x < 3

Then,
$$|x - 3| = -(x - 3)$$
$$-x + 3 = 5$$
$$x = -2$$

Therefore, point is (-2,0)

Therefore, the points on the x-axis, whose distances from the line $\frac{x}{3} + \frac{y}{4} = 1$ are 4 units are (8 , 0) and (-2 , 0)

Question:6(i) Find the distance between parallel lines $15x + 8y - 34 = 0$ and $15x + 8y + 31 = 0$.

Answer:

Given equations of lines are

$15x + 8y - 34 = 0$ and $15x + 8y + 31 = 0$

it is given that these lines are parallel

Therefore,
$$d = \frac{|C_2 - C_1|}{\sqrt{A^2 + B^2}}$$
$$A = 15, B = 8, C_1 = -34 \ and \ C_2 = 31$$

Now,

Therefore, the distance between two lines is $\frac{65}{17}\ units$

Question:6(ii) Find the distance between parallel lines $l(x+y)+p=0$ and $l(x+y)-r=0$

Answer:

Given equations of lines are

$l(x+y)+p=0$ and $l(x+y)-r=0$

it is given that these lines are parallel

Therefore,

$$d=\frac{|C_2-C_1|}{\sqrt{A^2+B^2}}$$

$$A=l, B=l, C_1=-r\ and\ C_2=p$$

Now,

$$d=\frac{|p-(-r)|}{\sqrt{l^2+l^2}}=\frac{|p+r|}{\sqrt{2l^2}}=\frac{|p+r|}{\sqrt{2}|l|}$$

Therefore, the distance between two lines is $\frac{1}{\sqrt{2}}\left|\frac{p+r}{l}\right|$

Question:7 Find equation of the line parallel to the line $3x-4y+2=0$ and passing through the point $(-2,3)$.

Answer:

It is given that line is parallel to line $3x-4y+2=0$ which implies that the slopes of both the lines are equal

we can rewrite it as

$y = \frac{3x}{4} + \frac{1}{2}$

The slope of line $3x - 4y + 2 = 0$ = $\frac{3}{4}$

Now, the equation of the line passing through the point $(-2, 3)$ and with slope $\frac{3}{4}$ is

$(y - 3) = \frac{3}{4}(x - (-2))$

$4(y - 3) = 3(x + 2)$

$4y - 12 = 3x + 6$

$3x - 4y + 18 = 0$

Therefore, the equation of the line is $3x - 4y + 18 = 0$

Question:8 Find equation of the line perpendicular to the line $x - 7y + 5 = 0$ and having x intercept 3 .

Answer:

It is given that line is perpendicular to the line $x - 7y + 5 = 0$

we can rewrite it as

$y = \frac{x}{7} + \frac{5}{7}$

Slope of line $x - 7y + 5 = 0$ (m') = $\frac{1}{7}$

Now,

The slope of the line is $m = \frac{-1}{m'} = -7 \qquad (\because \textit{lines are perpendicular})$

Now, the equation of the line with x intercept 3 i.e. (3, 0) and with slope -7 is

$(y - 0) = -7(x - 3)$

$y = -7x + 21$

$7x + y - 21 = 0$

Question:9 Find angles between the lines $\sqrt{3}x + y = 1$ and $x + \sqrt{3}y = 1$.

Answer:

Given equation of lines are
$\sqrt{3}x + y = 1$ and $x + \sqrt{3}y = 1$

Slope of line $\sqrt{3}x + y = 1$ is, $m_1 = -\sqrt{3}$

And

Slope of line $x + \sqrt{3}y = 1$ is , $m_2 = -\frac{1}{\sqrt{3}}$

Now, if θ is the angle between the lines

Then,

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$$

$$\tan\theta = \frac{1}{\sqrt{3}} \qquad or \qquad \tan\theta = -\frac{1}{\sqrt{3}}$$

$$\theta = \frac{\pi}{6} = 30^\circ \qquad or \qquad \theta = \frac{5\pi}{6} = 150^\circ$$

Therefore, the angle between the lines is $30^\circ\ and\ 150^\circ$

Question:10 The line through the points $(h, 3)$ and $(4, 1)$ intersects the line $7x - 9y - 19 = 0$ at right angle. Find the value of h .

Answer:

Line passing through points (h ,3) and (4 ,1)

Therefore,Slope of the line is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{3-1}{h-4}$$

This line intersects the line $7x - 9y - 19 = 0$ at right angle

Therefore, the Slope of both the lines are negative times inverse of each other

Slope of line $7x - 9y - 19 = 0$, $m' = \frac{7}{9}$

Now,

$$m = -\frac{1}{m'}$$

$$\frac{2}{h-4} = -\frac{9}{7}$$

$$14 = -9(h-4)$$

$$14 = -9h + 36$$

$$-9h = -22$$

$$h = \frac{22}{9}$$

Therefore, the value of h is $\frac{22}{9}$

Question:11 Prove that the line through the point (x_1, y_1) and parallel to the line $Ax + By + C = 0$ is $A(x - x_1) + B(y - y_1) = 0.$

Answer:

It is given that line is parallel to the line $Ax + By + C = 0$

Therefore, their slopes are equal

The slope of line $Ax + By + C = 0$, $m' = \frac{-A}{B}$

Let the slope of other line be m

Then,

$$m = m' = \frac{-A}{B}$$

Now, the equation of the line passing through the point (x_1, y_1) and with slope $-\frac{A}{B}$ is

$$(y - y_1) = -\frac{A}{B}(x - x_1)$$

$$B(y - y_1) = -A(x - x_1)$$

$$A(x - x_1) + B(y - y_1) = 0$$

Hence proved

Question:12 Two lines passing through the point $(2, 3)$ intersects each other at an angle of 60° . If slope of one line is 2 , find equation of the other line .

Answer:

Let the slope of two lines are $m_1 \ and \ m_2$ respectively

It is given the lines intersects each other at an angle of 60° and slope of the line is 2

Now,

$$m_1 = m \ and \ m_2 = 2 \ and \ \theta = 60^\circ$$

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$$

$$\tan 60^\circ = \left|\frac{2 - m}{1 + 2m}\right|$$

$$\sqrt{3} = \left|\frac{2 - m}{1 + 2m}\right|$$

Now, the equation of line passing through point (2 ,3) and with slope $\frac{2-\sqrt{3}}{2\sqrt{3}+1}$ is

$(y-3)=\frac{2-\sqrt{3}}{2\sqrt{3}+1}(x-2)$

$(2\sqrt{3}+1)(y-3)=(2-\sqrt{3})(x-2)$

$x(\sqrt{3}-2)+y(2\sqrt{3}+1)=-1+8\sqrt{3}$ -(i)

Similarly,

Now , equation of line passing through point (2 ,3) and with slope $\frac{-(2+\sqrt{3})}{2\sqrt{3}-1}$ is

$(y-3)=\frac{-(2+\sqrt{3})}{2\sqrt{3}-1}(x-2)$

$(2\sqrt{3}-1)(y-3)=-(2+\sqrt{3})(x-2)$

$x(2+\sqrt{3})+y(2\sqrt{3}-1)=1+8\sqrt{3}$ -(ii)

Therefore, equation of line is $x(\sqrt{3}-2)+y(2\sqrt{3}+1)=-1+8\sqrt{3}$ or $x(2+\sqrt{3})+y(2\sqrt{3}-1)=1+8\sqrt{3}$

Question:13 Find the equation of the right bisector of the line segment joining the points $(3,4)$ and $(-1,2)$.

Answer:

Right bisector means perpendicular line which divides the line segment into two equal parts

Now, lines are perpendicular which means their slopes are negative times inverse of each other

Slope of line passing through points $(3,4)$ and $(-1,2)$ is

$m' = \frac{4-2}{3+1} = \frac{2}{4} = \frac{1}{2}$

Therefore, Slope of bisector line is

$m = -\frac{1}{m'} = -2$

Now, let (h , k) be the point of intersection of two lines

It is given that point (h,k) divides the line segment joining point $(3, 4)$ and $(-1, 2)$ into two equal part which means it is the mid point

Therefore,

$h = \frac{3-1}{2} = 1 \quad and \quad k = \frac{4+2}{2} = 3$

$(h, k) = (1, 3)$

Now, equation of line passing through point (1,3) and with slope -2 is

$(y-3) = -2(x-1)$

$y - 3 = -2x + 2$

$2x + y = 5$

Therefore, equation of line is $2x + y = 5$

Question:14 Find the coordinates of the foot of perpendicular from the point $(-1, 3)$ to the line $3x - 4y - 16 = 0$.

Answer:

Let suppose the foot of perpendicular is (x_1, y_1)

We can say that line passing through the point (x_1, y_1) and $(-1, 3)$ is perpendicular to the line $3x - 4y - 16 = 0$

Now,

The slope of the line $3x - 4y - 16 = 0$ is , $m' = \frac{3}{4}$

And

The slope of the line passing through the point (x_1, y_1) and $(-1, 3)$ is, $m = \frac{y-3}{x+1}$

lines are perpendicular

Therefore,

Now, the point (x_1, y_1) also lies on the line $3x - 4y - 16 = 0$

Therefore,

$3x_1 - 4y_1 = 16 \qquad - (ii)$

On solving equation (i) and (ii)

we will get

$x_1 = \frac{68}{25}$ and $y_1 = -\frac{49}{25}$

Therefore, $(x_1, y_1) = \left(\frac{68}{25}, -\frac{49}{25}\right)$

Question:15 The perpendicular from the origin to the line $y = mx + c$ meets it at the point $(-1, 2)$. Find the values of m and c .

Answer:

We can say that line passing through point $(0, 0)$ and $(-1, 2)$ is perpendicular to line $y = mx + c$

Now,

The slope of the line passing through the point $(0, 0)$ and $(-1, 2)$ is

, $m = \frac{2-0}{-1-0} = -2$

lines are perpendicular

Therefore,

$m = -\frac{1}{m'} = \frac{1}{2}$ - (i)

Now, the point $(-1, 2)$ also lies on the line $y = mx + c$

Therefore,

$$2 = \frac{1}{2}.(-1) + C$$

$$C = \frac{5}{2} \qquad -(ii)$$

Therefore, the value of m and C is $\frac{1}{2}$ and $\frac{5}{2}$ respectively

Question:16 If p and q are the lengths of perpendiculars from the origin to the lines $x\cos\theta - y\sin\theta = k\cos 2\theta$ and $x\sec\theta + y\ cosec\,\theta = k$, respectively, prove that $p^2 + 4q^2 = k^2$.

Answer:

Given equations of lines are $x\cos\theta - y\sin\theta = k\cos 2\theta$ and $x\sec\theta + y\ cosec\,\theta = k$

We can rewrite the equation $x\sec\theta + y\ cosec\,\theta = k$ as

$$x\sin\theta + y\cos\theta = k\sin\theta\cos\theta$$

Now, we know that

$$d = \left|\frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}\right|$$

In equation $x\cos\theta - y\sin\theta = k\cos 2\theta$

$$A = \cos\theta, B = -\sin\theta, C = -k\cos 2\theta\ and\ (x_1, y_1) = (0, 0)$$

$$p = \left|\frac{\cos\theta.0 - \sin\theta.0 - k\cos 2\theta}{\sqrt{\cos^2\theta + (-\sin\theta)^2}}\right| = |-k\cos 2\theta|$$

Similarly,

in the equation $x\sin\theta + y\cos\theta = k\sin\theta\cos\theta$

$A = \sin\theta, B = \cos\theta, C = -k\sin\theta\cos\theta \; and \; (x_1, y_1) = (0,0)$

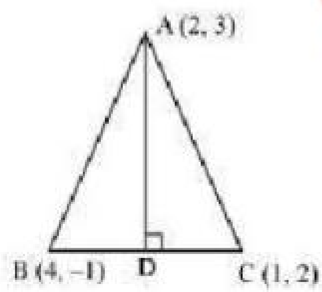
Now,

$$p^2 + 4q^2 = (|-k\cos 2\theta|)^2 + 4.(|-\frac{k\sin 2\theta}{2})^2 = k^2\cos^2 2\theta + 4.\frac{k^2\sin^2 2\theta}{4}$$
$$= k^2(\cos^2 2\theta + \sin^2 2\theta)$$
$$= k^2$$

Hence proved

Question:17 In the triangle ABC with vertices $A(2,3)$, $B(4,-1)$ and $C(1,2)$, find the equation and length of altitude from the vertex A .

Answer:



Let suppose foot of perpendicular is (x_1, y_1)

We can say that line passing through point $(x_1, y_1) \; and \; A(2,3)$ is perpendicular to line passing through point $B(4,-1) \; and \; C(1,2)$

Now,

Slope of line passing through point $B(4,-1) \; and \; C(1,2)$ is

, $m' = \frac{2+1}{1-4} = \frac{3}{-3} = -1$

And

Slope of line passing through point $(x_1, y_1)\ and\ (2,3)$ is , m

lines are perpendicular

Therefore,
$$m = -\frac{1}{m'} = 1$$

Now, equation of line passing through point (2 ,3) and slope with 1

$$(y-3) = 1(x-2)$$

$x - y + 1 = 0$ -(i)

Now, equation line passing through point $B(4,-1)\ and\ C(1,2)$ is

$$(y-2) = -1(x-1)$$

$$x + y - 3 = 0$$

Now, perpendicular distance of (2,3) from the $x + y - 3 = 0$ is

-(ii)

Therefore, equation and length of the line is $x - y + 1 = 0$ and $\sqrt{2}$ respectively

Question:18 If p is the length of perpendicular from the origin to the line whose intercepts on the axes are a and b , then show that $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$.

Answer:

we know that intercept form of line is
$$\frac{x}{a} + \frac{y}{b} = 1$$

we know that
$$d = \left|\frac{Ax_1 + bx_2 + C}{\sqrt{A^2 + B^2}}\right|$$

In this problem

$$A=\frac{1}{a},B=\frac{1}{b},C=-1\ and\ (x_1,y_1)=(0,0)$$

On squaring both the sides

we will get
$$\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$$

Hence proved

NCERT solutions for class 11 maths chapter 10 straight lines-Miscellaneous Exercise

Question:1(a) Find the values of k for which the line $(k-3)x-(4-k^2)y+k^2-7k+6=0$ is

Parallel to the x-axis.

Answer:

Given equation of line is
$(k-3)x-(4-k^2)y+k^2-7k+6=0$

and equation of x-axis is $y=0$

it is given that these two lines are parallel to each other

Therefore, their slopes are equal

Slope of $y=0$ is , $m'=0$

and

Slope of line $(k-3)x-(4-k^2)y+k^2-7k+6=0$ is , $m=\frac{k-3}{4-k^2}$

Now,

$m = m'$
$\dfrac{k-3}{4-k^2} = 0$

$k - 3 = 0$

$k = 3$

Therefore, value of k is 3

Question:1(b) Find the values of k for which the line $(k-3)x - (4-k^2)y + k^2 - 7k + 6 = 0$ is

Parallel to the y-axis.

Answer:

Given equation of line is
$(k-3)x - (4-k^2)y + k^2 - 7k + 6 = 0$

and equation of y-axis is $x = 0$

it is given that these two lines are parallel to each other

Therefore, their slopes are equal

Slope of $y = 0$ is , $m' = \infty = \dfrac{1}{0}$

and

Slope of line $(k-3)x - (4-k^2)y + k^2 - 7k + 6 = 0$ is , $m = \dfrac{k-3}{4-k^2}$

Now,

$m = m'$
$\dfrac{k-3}{4-k^2} = \dfrac{1}{0}$

$4 - k^2 = 0$

$k = \pm 2$

Therefore, value of k is ± 2

Question:1(c) Find the values of k for which the line $(k-3)x-(4-k^2)y+k^2-7k+6=0$ is Passing through the origin.

Answer:

Given equation of line is

$(k-3)x-(4-k^2)y+k^2-7k+6=0$

It is given that it passes through origin (0,0)

Therefore,

$(k-3).0-(4-k^2).0+k^2-7k+6=0$

$k^2-7k+6=0$

$k^2-6k-k+6=0$

$(k-6)(k-1)=0$

$k=6\ or\ 1$

Therefore, value of k is $6\ or\ 1$

Question:2 Find the values of θ and p , if the equation $x\cos\theta+y\sin\theta=p$ is the normal form of the line $\sqrt{3}x+y+2=0$.

Answer:

The normal form of the line is $x\cos\theta+y\sin\theta=p$

Given the equation of lines is

$\sqrt{3}x+y+2=0$

First, we need to convert it into normal form. So, divide both the sides by $\sqrt{(\sqrt{3})^2+1^2}=\sqrt{3+1}=\sqrt{4}=2$

$-\frac{\sqrt{3}\cos\theta}{2}-\frac{y}{2}=1$

On comparing both

we will get
$\cos\theta = -\frac{\sqrt{3}}{2}, \sin\theta = -\frac{1}{2}\ and\ p = 1$
$\theta = \frac{7\pi}{6}\ and\ p = 1$
Therefore, the answer is $\theta = \frac{7\pi}{6}\ and\ p = 1$

Question:3 Find the equations of the lines, which cut-off intercepts on the axes whose sum and product are 1 and -6 , respectively.

Answer:

Let the intercepts on x and y-axis are a and b respectively

It is given that

$a + b = 1\ \ and\ \ a.b = -6$

$a = 1 - b$

$\Rightarrow b.(1 - b) = -6$

$\Rightarrow b - b^2 = -6$

$\Rightarrow b^2 - b - 6 = 0$

$\Rightarrow b^2 - 3b + 2b - 6 = 0$

$\Rightarrow (b + 2)(b - 3) = 0$

$\Rightarrow b = -2\ and\ 3$

Now, when $b = -2 \Rightarrow a = 3$

and when $b = 3 \Rightarrow a = -2$

We know that the intercept form of the line is
$\frac{x}{a} + \frac{y}{b} = 1$

Case (i) when a = 3 and b = -2
$\frac{x}{3} + \frac{y}{-2} = 1$

$\Rightarrow 2x - 3y = 6$

Case (ii) when a = -2 and b = 3

$$\frac{x}{-2} + \frac{y}{3} = 1$$

$\Rightarrow -3x + 2y = 6$

Therefore, equations of lines are $2x - 3y = 6 \; and \; -3x + 2y = 6$

Question:4 What are the points on the y -axis whose distance from the line $\frac{x}{3} + \frac{y}{4} = 1$ is 4 units.

Answer:

Given the equation of the line is

$$\frac{x}{3} + \frac{y}{4} = 1$$

we can rewrite it as

$4x + 3y = 12$

Let's take point on y-axis is $(0, y)$

It is given that the distance of the point $(0, y)$ from line $4x + 3y = 12$ is 4 units

Now,

$$d = \left| \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}} \right|$$

In this problem $A = 4, B = 3, C = -12, d = 4 \;\; and \;\; (x_1, y_1) = (0, y)$

Case (i)

$$4 = \frac{3y - 12}{5}$$

$20 = 3y - 12$

$y = \frac{32}{3}$

Therefore, the point is $\left(0, \frac{32}{3}\right)$ -(i)

Case (ii)

$4 = -\left(\frac{3y - 12}{5}\right)$

$20 = -3y + 12$

$y = -\frac{8}{3}$

Therefore, the point is $\left(0, -\frac{8}{3}\right)$ -(ii)

Therefore, points on the y -axis whose distance from the line $\frac{x}{3} + \frac{y}{4} = 1$ is 4 units are $\left(0, \frac{32}{3}\right)$ and $\left(0, -\frac{8}{3}\right)$

Question:5 Find perpendicular distance from the origin to the line joining the points $(\cos\theta, \sin\theta)$ and $(\cos\phi, \sin\phi)$. .

Answer:

Equation of line passing through the points $(\cos\theta, \sin\theta)$ and $(\cos\phi, \sin\phi)$ is

$(y - \sin\theta) = \frac{\sin\phi - \sin\theta}{\cos\phi - \cos\theta}(x - \cos\theta)$

$\Rightarrow (\cos\phi - \cos\theta)(y - \sin\theta) = (\sin\phi - \sin\theta)(x - \cos\theta)$

$\Rightarrow x(\sin\phi - \sin\theta) - y(\cos\phi - \cos\theta) = \sin(\theta - \phi)$

$(\because \cos a \sin b - \sin a \cos b = \sin(a - b))$

Now, distance from origin(0,0) is

$$d=\left|\frac{-\sin(\theta-\phi)}{1+1-2\cos(\theta-\phi)}\right|$$

$(\because \cos a\cos b+\sin a\sin b=\cos(a-b)\quad and\quad \sin^2 a+\cos^2 a=1)$

$$d=\left|\frac{-\sin(\theta-\phi)}{2(1-\cos(\theta-\phi))}\right|$$

$$d=\left|\frac{-2\sin\frac{\theta-\phi}{2}\cos\frac{\theta-\phi}{2}}{2\sin\frac{\theta-\phi}{2}}\right|$$

$$d=\left|\cos\frac{\theta-\phi}{2}\right|$$

Question:6 Find the equation of the line parallel to y-axis and drawn through the point of intersection of the lines $x-7y+5=0$ and $3x+y=0$.

Answer:

Point of intersection of the lines $x-7y+5=0$ and $3x+y=0$

$$\left(-\frac{5}{22},\frac{15}{22}\right)$$

It is given that this line is parallel to y - axis i.e. $x=0$ which means their slopes are equal

Slope of $x=0$ is , $m'=\infty=\frac{1}{0}$

Let the Slope of line passing through point $\left(-\frac{5}{22},\frac{15}{22}\right)$ is m

Then,

$$m=m'=\frac{1}{0}$$

Now, equation of line passing through point $\left(-\frac{5}{22}, \frac{15}{22}\right)$ and with slope $\frac{1}{0}$ is
$(y - \frac{15}{22}) = \frac{1}{0}(x + \frac{5}{22})$
$x = -\frac{5}{22}$

Therefore, equation of line is $x = -\frac{5}{22}$

Question:7 Find the equation of a line drawn perpendicular to the line $\frac{x}{4} + \frac{y}{6} = 1$ through the point, where it meets the y -axis.

Answer:

given equation of line is
$\frac{x}{4} + \frac{y}{6} = 1$

we can rewrite it as

$3x + 2y = 12$

Slope of line $3x + 2y = 12$, $m' = -\frac{3}{2}$

Let the Slope of perpendicular line is m
$m = -\frac{1}{m'} = \frac{2}{3}$

Now, the ponit of intersection of $3x + 2y = 12$ and $x = 0$ is $(0, 6)$

Equation of line passing through point $(0, 6)$ and with slope $\frac{2}{3}$ is
$(y - 6) = \frac{2}{3}(x - 0)$

$3(y - 6) = 2x$

$2x - 3y + 18 = 0$

Therefore, equation of line is $2x - 3y + 18 = 0$

Question:8 Find the area of the triangle formed by the lines $y - x = 0, x + y = 0$ and $x - k = 0$.

Answer:

Given equations of lines are

$y - x = 0 \qquad - (i)$

$x + y = 0 \qquad - (ii)$

$x - k = 0 \qquad - (iii)$

The point if intersection of (i) and (ii) is (0,0)

The point if intersection of (ii) and (iii) is (k,-k)

The point if intersection of (i) and (iii) is (k,k)

Therefore, the vertices of triangle formed by three lines are $(0, 0), (k, -k)$ *and* (k, k)

Now, we know that area of triangle whose vertices are $(x_1, y_1), (x_2, y_2)$ *and* (x_3, y_3) is

$$A = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

$$A = \frac{1}{2}|0(-k - k) + k(k - 0) + k(0 + k)|$$

$$A = \frac{1}{2}|k^2 + k^2|$$

$$A = \frac{1}{2}|2k^2|$$

$$A = k^2$$

Therefore, area of triangle is k^2 *square units*

Question:9 Find the value of p so that the three lines $3x + y - 2 = 0, px + 2y - 3 = 0$ and $2x - y - 3 = 0$ may intersect at one point.

Answer:

Point of intersection of lines $3x + y - 2 = 0$ and $2x - y - 3 = 0$ is $(1, -1)$

Now, $(1, -1)$ must satisfy equation $px + 2y - 3 = 0$

Therefore,

$p(1) + 2(-1) - 3 = 0$

$p - 2 - 3 = 0$

$p = 5$

Therefore, the value of p is 5

Question:10 If three lines whose equations are $y = m_1x + c_1, y = m_2x + c_2$ and $y = m_3x + c_3$ are concurrent, then show that $m_1(c_2 - c_3) + m_2(c_3 - c_1) + m_3(c_1 - c_2) = 0$.

Answer:

Concurrent lines means they all intersect at the same point

Now, given equation of lines are

$y = m_1x + c_1 \qquad -(i)$

$y = m_2x + c_2 \qquad -(ii)$

$y = m_3x + c_3 \qquad -(iii)$

Point of intersection of equation (i) and (ii) $\left(\frac{c_2 - c_1}{m_1 - m_2}, \frac{m_1c_2 - m_2c_1}{m_1 - m_2}\right)$

Now, lines are concurrent which means point $\left(\frac{c_2 - c_1}{m_1 - m_2}, \frac{m_1c_2 - m_2c_1}{m_1 - m_2}\right)$ also satisfy equation (iii)

Therefore,

$$\frac{m_1c_2 - m_2c_1}{m_1 - m_2} = m_3.\left(\frac{c_2 - c_1}{m_1 - m_2}\right) + c_3$$

$$m_1c_2 - m_2c_1 = m_3(c_2 - c_1) + c_3(m_1 - m_2)$$

$$m_1(c_2 - c_3) + m_2(c_3 - c_1) + m_3(c_1 - c_2) = 0$$

Hence proved

Question:11 Find the equation of the lines through the point $(3, 2)$ which make an angle of 45° with the line $x - 2y = 3$.

Answer:

Given the equation of the line is

$x - 2y = 3$

The slope of line $x - 2y = 3$, $m_2 = \dfrac{1}{2}$

Let the slope of the other line is, $m_1 = m$

Now, it is given that both the lines make an angle 45° with each other

Therefore,

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1m_2}\right|$$

$$\tan 45^\circ = \left|\frac{\frac{1}{2} - m}{1 + \frac{m}{2}}\right|$$

$$1 = \left|\frac{1 - 2m}{2 + m}\right|$$

Now,

Case (i)

$$1 = \frac{1 - 2m}{2 + m}$$

$2+m=1-2m$
$m=-\frac{1}{3}$

Equation of line passing through the point $(3,2)$ and with slope $-\frac{1}{3}$
$(y-2)=-\frac{1}{3}(x-3)$

$3(y-2)=-1(x-3)$

$x+3y=9 \qquad -(i)$

Case (ii)
$1=-\left(\frac{1-2m}{2+m}\right)$

$2+m=-(1-2m)$

$m=3$

Equation of line passing through the point $(3,2)$ and with slope 3 is
$(y-2)=3(x-3)$

$3x-y=7 \qquad -(ii)$

Therefore, equations of lines are $3x-y=7$ and $x+3y=9$

Question:12 Find the equation of the line passing through the point of intersection of the lines $4x+7y-3=0$ and $2x-3y+1=0$ that has equal intercepts on the axes.

Answer:

Point of intersection of the lines $4x+7y-3=0$ and $2x-3y+1=0$ is $\left(\frac{1}{13},\frac{5}{13}\right)$

We know that the intercept form of the line is
$\frac{x}{a}+\frac{y}{b}=1$

It is given that line make equal intercepts on x and y axis

Therefore,

a = b

Now, the equation reduces to

$x + y = a$ -(i)

It passes through point $\left(\frac{1}{13}, \frac{5}{13}\right)$

Therefore,
$a = \frac{1}{13} + \frac{5}{13} = \frac{6}{13}$

Put the value of a in equation (i)

we will get

$13x + 13y = 6$

Therefore, equation of line is $13x + 13y = 6$

Question:13 Show that the equation of the line passing through the origin and making an angle $\theta$with the line $y = mx + c$ is $\frac{y}{x} = \frac{m \pm \tan\theta}{1 \mp m\tan\theta}$.

Answer:

Slope of line $y = mx + c$ is m

Let the slope of other line is m'

It is given that both the line makes an angle θ with each other

Therefore,
$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$

$\tan\theta = \left|\frac{m - m'}{1 + mm'}\right|$

$\mp(1 + mm')\tan\theta = (m - m')$

$\mp\tan\theta + m'(\mp m\tan\theta + 1) = m$

$m' = \frac{m \pm \tan\theta}{1 \mp m\tan\theta}$

Now, equation of line passing through origin (0,0) and with slope $\frac{m \pm \tan\theta}{1 \mp m\tan\theta}$ is

$(y-0) = \frac{m \pm \tan\theta}{1 \mp m\tan\theta}(x-0)$

$\frac{y}{x} = \frac{m \pm \tan\theta}{1 \mp m\tan\theta}$

Hence proved

Question:14 In what ratio, the line joining $(-1, 1)$ and $(5, 7)$ is divided by the line $x + y = 4$?

Answer:

Equation of line joining $(-1, 1)$ and $(5, 7)$ is

$(y-1) = \frac{7-1}{5+1}(x+1)$

$\Rightarrow (y-1) = \frac{6}{6}(x+1)$

$\Rightarrow (y-1) = 1(x+1)$

$\Rightarrow x - y + 2 = 0$

Now, point of intersection of lines $x + y = 4$ and $x - y + 2 = 0$ is $(1, 3)$

Now, let's suppose point $(1, 3)$ divides the line segment joining $(-1, 1)$ and $(5, 7)$ in $1 : k$

Then,

$(1,3) = \left(\frac{k(-1)+1(5)}{k+1}, \frac{k(1)+1(7)}{k+1}\right)$

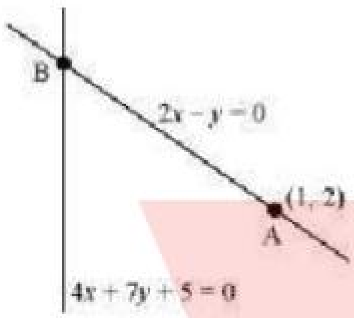
$1 = \frac{-k+5}{k+1} \;\; and \;\; 3 = \frac{k+7}{k+1}$

$\Rightarrow k = 2$

Therefore, the line joining $(-1, 1)$ and $(5, 7)$ is divided by the line $x + y = 4$ in ratio $1 : 2$

Question:15 Find the distance of the line $4x + 7y + 5 = 0$ from the point $(1, 2)$ along the line $2x - y = 0$.

Answer:



point $(1, 2)$ lies on line $2x - y = 0$

Now, point of intersection of lines $2x - y = 0$ and $4x + 7y + 5 = 0$ is $\left(-\frac{5}{18}, -\frac{5}{9}\right)$

Now, we know that the distance between two point is given by

$$d = |\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}|$$

$$d = |\sqrt{(1 + \frac{5}{18})^2 + (2 + \frac{5}{9})^2}|$$

$$d = |\sqrt{(\frac{23}{18})^2 + (\frac{23}{9})^2}|$$

$$d = \left|\sqrt{\frac{529}{324} + \frac{529}{81}}\right|$$

$$d = \left|\sqrt{\frac{529 + 2116}{324}}\right| = \left|\sqrt{\frac{2645}{324}}\right| = \frac{23\sqrt{5}}{18}$$

Therefore, the distance of the line $4x + 7y + 5 = 0$ from the point $(1, 2)$ along the line $2x - y = 0$ is $\frac{23\sqrt{5}}{18}\ units$

Question:16 Find the direction in which a straight line must be drawn through the point $(-1, 2)$ so that its point of intersection with the line $x + y = 4$ may be at a distance of 3 units from this point.

Answer:

Let (x_1, y_1) be the point of intersection

it lies on line $x + y = 4$

Therefore,

$$x_1 + y_1 = 4$$
$$x_1 = 4 - y_1 \qquad -(i)$$

Distance of point (x_1, y_1) from $(-1, 2)$ is 3

Therefore,

$$3 = |\sqrt{(x_1+1)^2 + (y_1-2)^2}|$$

Square both the sides and put value from equation (i)

When $y_1 = 2 \Rightarrow x_1 = 2$ point is $(2, 2)$

and

When $y_1 = 5 \Rightarrow x_1 = -1$ point is $(-1, 5)$

Now, slope of line joining point $(2, 2)$ and $(-1, 2)$ is

$$m = \frac{2-2}{-1-2} = 0$$

Therefore, line is parallel to x-axis -(i)

or

slope of line joining point $(-1, 5)$ and $(-1, 2)$

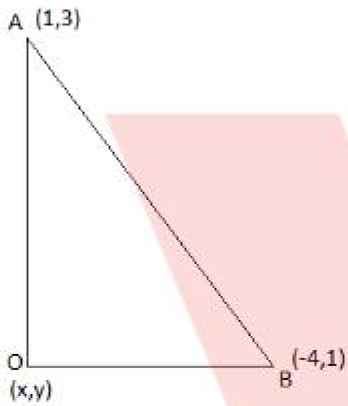
$$m = \frac{5-2}{-1+2} = \infty$$

Therefore, line is parallel to y-axis -(ii)

Therefore, line is parallel to x -axis or parallel to y-axis

Question:17 The hypotenuse of a right angled triangle has its ends at the points $(1, 3)$ and $(-4, 1)$ Find an equation of the legs (perpendicular sides) of the triangle.

Answer:



Slope of line OA and OB are negative times inverse of each other

Slope of line OA is , $m = \frac{3-y}{1-x} \Rightarrow (3-y) = m(1-x)$

Slope of line OB is , $-\frac{1}{m} = \frac{1-y}{-4-x} \Rightarrow (x+4) = m(1-y)$

Now,

Now, for a given value of m we get these equations

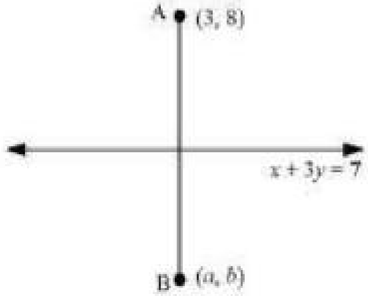
If $m = \infty$

$1 - x = 0 \quad and \quad 1 - y = 0$

$x = 1 \quad and \quad y = 1$

Question:18 Find the image of the point $(3, 8)$ with respect to the line $x + 3y = 7$ assuming the line to be a plane mirror.

Answer:



Let point (a, b) is the image of point $(3, 8)$ w.r.t. to line $x + 3y = 7$

line $x + 3y = 7$ is perpendicular bisector of line joining points $(3, 8)$ and (a, b)

Slope of line $x + 3y = 7$, $m' = -\frac{1}{3}$

Slope of line joining points $(3, 8)$ and (a, b) is , $m = \frac{8-b}{3-a}$

Now,

$$m = -\frac{1}{m'} \qquad (\because lines\ are\ perpendicular)$$

$$\frac{8-b}{3-a} = 3$$

$$8 - b = 9 - 3a$$

$$3a - b = 1 \qquad -(i)$$

Point of intersection is the midpoint of line joining points $(3, 8)$ and (a, b)

Therefore,

Point of intersection is $\left(\frac{3+a}{2}, \frac{b+8}{2}\right)$

Point $\left(\frac{3+a}{2}, \frac{b+8}{2}\right)$ also satisfy the line $x + 3y = 7$

Therefore,

$$\frac{3+a}{2} + 3.\frac{b+8}{2} = 7$$

$$a + 3b = -13 \qquad -(ii)$$

On solving equation (i) and (ii) we will get

$(a, b) = (-1, -4)$

Therefore, the image of the point $(3, 8)$ with respect to the line $x + 3y = 7$ is $(-1, -4)$

Question:19 If the lines $y = 3x + 1$ and $2y = x + 3$ are equally inclined to the line $y = mx + 4$, find the value of m .

Answer:

Given equation of lines are

$y = 3x + 1 \qquad -(i)$

$2y = x + 3 \qquad -(ii)$

$y = mx + 4 \qquad -(iii)$

Now, it is given that line (i) and (ii) are equally inclined to the line (iii)

Slope of line $y = 3x + 1$ is , $m_1 = 3$

Slope of line $2y = x + 3$ is , $m_2 = \frac{1}{2}$

Slope of line $y = mx + 4$ is , $m_3 = m$

Now, we know that

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

Now,

$$\tan\theta_1 = \left|\frac{3 - m}{1 + 3m}\right| \text{ and } \tan\theta_2 = \left|\frac{\frac{1}{2} - m}{1 + \frac{m}{2}}\right|$$

It is given that $\tan\theta_1 = \tan\theta_2$

Therefore,

$$\left|\frac{3 - m}{1 + 3m}\right| = \left|\frac{1 - 2m}{2 + m}\right|$$

$$\frac{3 - m}{1 + 3m} = \pm\left(\frac{1 - 2m}{2 + m}\right)$$

Now, if $\frac{3-m}{1+3m} = \left(\frac{1-2m}{2+m}\right)$

Then,

$(2+m)(3-m) = (1-2m)(1+3m)$

$6+m-m^2 = 1+m-6m^2$

$5m^2 = -5$

$m = \sqrt{-1}$

Which is not possible

Now, if $\frac{3-m}{1+3m} = -\left(\frac{1-2m}{2+m}\right)$

Then,

$(2+m)(3-m) = -(1-2m)(1+3m)$

$6+m-m^2 = -1-m+6m^2$

$7m^2 - 2m - 7 = 0$

Therefore, the value of m is $\frac{1 \pm 5\sqrt{2}}{7}$

Question:20 If the sum of the perpendicular distances of a variable point $P(x, y)$ from the lines $x + y - 5 = 0$ and $3x - 2y + 7 = 0$ is always 10 . Show that P must move on a line.

Answer:

Given the equation of line are

$x + y - 5 = 0 \qquad -(i)$

$3x - 2y + 7 = 0 \qquad -(ii)$

Now, perpendicular distances of a variable point $P(x, y)$ from the lines are

$$d_1 = \left|\frac{1.x + 1.y - 5}{\sqrt{1^2 + 1^2}}\right| d_2 = \left|\frac{3.x - 2.y + 7}{\sqrt{3^2 + 2^2}}\right|$$
$$d_1 = \left|\frac{x + y - 5}{\sqrt{2}}\right| d_2 = \left|\frac{3x - 2y + 7}{\sqrt{13}}\right|$$

Now, it is given that

$d_1 + d_2 = 10$

Therefore,
$$\frac{x + y - 5}{\sqrt{2}} + \frac{3x - 2y + 7}{\sqrt{13}} = 10$$
$$(assuming\ x + y - 5 > 0\ and\ 3x - 2y + 7 > 0)$$
$$(x + y - 5)\sqrt{13} + (3x - 2y + 7)\sqrt{2} = 10\sqrt{26}$$

$$x(\sqrt{13} + 3\sqrt{2}) + y(\sqrt{13} - 2\sqrt{2}) = 10\sqrt{26} + 5\sqrt{13} - 7\sqrt{2}$$

Which is the equation of the line

Hence proved

Question:21 Find equation of the line which is equidistant from parallel lines $9x + 6y - 7 = 0$ and $3x + 2y + 6 = 0$.

Answer:

Let's take the point $p(a, b)$ which is equidistance from parallel lines $9x + 6y - 7 = 0$ and $3x + 2y + 6 = 0$

Therefore,
$$d_1 = \left|\frac{9.a + 6.b - 7}{\sqrt{9^2 + 6^2}}\right| d_2 = \left|\frac{3.a + 2.b + 6}{\sqrt{3^2 + 2^2}}\right|$$
$$d_1 = \left|\frac{9a + 6b - 7}{\sqrt{117}}\right| d_2 = \left|\frac{3a + 2b + 6}{\sqrt{13}}\right|$$

It is that $d_1 = d_2$

Therefore,

$$\left|\frac{9a+6b-7}{3\sqrt{13}}\right| = \left|\frac{3a+2b+6}{\sqrt{13}}\right|$$

$$(9a+6b-7) = \pm 3(3a+2b+6)$$

Now, case (i)

$$(9a+6b-7) = 3(3a+2b+6)$$

$$25 = 0$$

Therefore, this case is not possible

Case (ii)

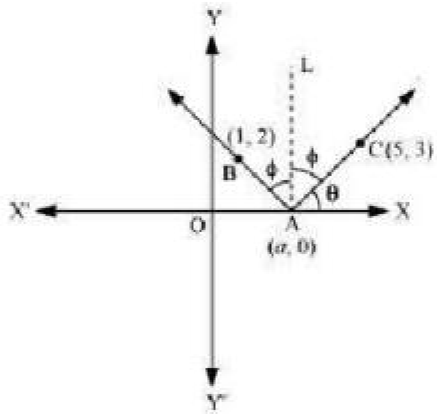
$$(9a+6b-7) = -3(3a+2b+6)$$

$$18a+12b+11 = 0$$

Therefore, the required equation of the line is $18a+12b+11=0$

Question:22 A ray of light passing through the point $(1, 2)$ reflects on the x -axis at point A and the reflected ray passes through the point $(5, 3)$. Find the coordinates of A .

Answer:



From the figure above we can say that

The slope of line AC $(m) = \tan\theta$

Therefore,
$$\tan\theta = \frac{3-0}{5-a} = \frac{3}{5-a} \qquad (i)$$

Similarly,

The slope of line AB $(m') = \tan(180^\circ - \theta)$

Therefore,
$$\tan(180^\circ - \theta) = \frac{2-0}{1-a}$$
$$-\tan\theta = \frac{2}{1-a}$$
$$\tan\theta = \frac{2}{a-1} \qquad -(ii)$$

Now, from equation (i) and (ii) we will get
$$\frac{3}{5-a} = \frac{2}{a-1}$$
$$\Rightarrow 3(a-1) = 2(5-a)$$
$$\Rightarrow 3a - 3 = 10 - 2a$$
$$\Rightarrow 5a = 13$$
$$\Rightarrow a = \frac{13}{5}$$

Therefore, the coordinates of A . is $\left(\frac{13}{5}, 0\right)$

Question:23 Prove that the product of the lengths of the perpendiculars drawn from the points $(\sqrt{a^2-b^2},0)$ and $(-\sqrt{a^2-b^2},0)$ to the line $\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$ is b^2.

Answer:

Given equation id line is

$$\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$$

We can rewrite it as

$$xb\cos\theta+ya\sin\theta=ab$$

Now, the distance of the line $xb\cos\theta+ya\sin\theta=ab$ from the point $(\sqrt{a^2-b^2},0)$ is given by

Similarly,

The distance of the line $xb\cos\theta+ya\sin\theta=ab$ from the point $(-\sqrt{a^2-b^2},0)$ is given by

$$=\left|\frac{-((b\cos\theta.\sqrt{a^2-b^2})^2-(ab)^2)}{(b\cos\theta)^2+(a\sin\theta)^2}\right|$$

$$=\left|\frac{-b^2\cos^2\theta.(a^2-b^2)+a^2b^2)}{(b\cos\theta)^2+(a\sin\theta)^2}\right|$$

$$=\left|\frac{-a^2b^2\cos^2\theta+b^4\cos^2\theta+a^2b^2)}{b^2\cos^2\theta+a^2\sin^2\theta}\right|$$

$$=\left|\frac{-b^2(a^2\cos^2\theta-b^2\cos^2\theta-a^2)}{b^2\cos^2\theta+a^2\sin^2\theta}\right|$$

$$=\left|\frac{+b^2(b^2\cos^2\theta+a^2\sin^2\theta)}{b^2\cos^2\theta+a^2\sin^2\theta}\right|$$

$= b^2$

Hence proved

Question:24 A person standing at the junction (crossing) of two straight paths represented by the equations $2x - 3y + 4 = 0$ and $3x + 4y - 5 = 0$ wants to reach the path whose equation is $6x - 7y + 8 = 0$ in the least time. Find equation of the path that he should follow.

Answer:

point of intersection of lines $2x - 3y + 4 = 0$ and $3x + 4y - 5 = 0$ (junction) is $\left(-\frac{1}{17}, \frac{22}{17}\right)$

Now, person reaches to path $6x - 7y + 8 = 0$ in least time when it follow the path perpendicular to it

Now,

Slope of line $6x - 7y + 8 = 0$ is , $m' = \frac{6}{7}$

let the slope of line perpendicular to it is , m

Then,

$m = -\frac{1}{m} = -\frac{7}{6}$

Now, equation of line passing through point $\left(-\frac{1}{17}, \frac{22}{17}\right)$ and with slope $-\frac{7}{6}$ is

$\left(y - \frac{22}{17}\right) = -\frac{7}{6}\left(x - (-\frac{1}{17})\right)$

$\Rightarrow 6(17y - 22) = -7(17x + 1)$

$\Rightarrow 102y - 132 = -119x - 7$

$\Rightarrow 119x + 102y = 125$

Therefore, the required equation of line is $119x + 102y = 125$