NCERT solutions for class 11 maths chapter 5 Complex Numbers and Quadratic Equations-Exercise: 5.1

Question:1 Express each of the complex number in the form $a + ib$.

$$(5i)\left(-\frac{3}{5}i\right)$$

Answer:

On solving

$$(5i)\left(-\frac{3}{5}i\right)$$

we will get

$$(5i)\left(-\frac{3}{5}i\right) = 5 \times (-\frac{3}{5}) \times i \times i$$

$$= -3 \times i^2 \, (\because i^2 = -1)$$

$$= -3 \times -1$$

$$= 3$$

Now, in the form of $a + ib$ we can write it as

$$= 3 + 0i$$

Question:2 Express each of the complex number in the form $a + ib$.

$$i^9 + i^{19}$$

Answer:

We know that $i^4 = 1$

Now, we will reduce $i^9 + i^{19}$ into

$$i^9 + i^{19} = (i^4)^2.i + (i^4)^3.i^3$$
$$= (1)^2.i + (1)^3.(-i)\,(\because i^4 = 1, i^3 = -i \; and \; i^2 = -1)$$
$$= i - i = 0$$

Now, in the form of $a + ib$ we can write it as

$o + io$

Therefore, the answer is $o + io$

Question:3 Express each of the complex number in the form a+ib.

i^{-39}

Answer:

We know that $i^4 = 1$

Now, we will reduce i^{-39} into

$$i^{-39} = (i^4)^{-9}.i^{-3}$$
$$= (1)^{-9}.(-i)^{-1}\,(\because i^4 = 1, i^3 = -i)$$
$$= \frac{1}{-i}$$
$$= \frac{1}{-i} \times \frac{i}{i}$$
$$= \frac{i}{-i^2}\,(\because i^2 = -1)$$
$$= \frac{i}{-(-1)}$$
$$= i$$

Now, in the form of $a + ib$ we can write it as

$o + i1$

Therefore, the answer is $o + i1$

Question:4 Express each of the complex number in the form a+ib.

$3(7+7i)+i(7+7i)$

Answer:

Given problem is

$3(7+7i)+i(7+7i)$

Now, we will reduce it into

$$3(7+7i)+i(7+7i) = 21+21i+7i+7i^2$$
$$= 21+21i+7i+7(-1)\,(\because i^2=-1)$$
$$= 21+21i+7i-7$$
$$= 14+28i$$

Therefore, the answer is $14 + i28$

Question:5 Express each of the complex number in the form $a + ib$.

$(1-i)-(-1+6i)$

Answer:

Given problem is

$(1-i)-(-1+6i)$

Now, we will reduce it into

$(1-i)-(-1+6i) = 1-i+1-6i$

$= 2-7i$

Therefore, the answer is $2-7i$

Question:6 Express each of the complex number in the form $a+ib$.

$\left(\frac{1}{5}+i\frac{2}{5}\right)-\left(4+i\frac{5}{2}\right)$

Answer:

Given problem is
$\left(\frac{1}{5}+i\frac{2}{5}\right)-\left(4+i\frac{5}{2}\right)$

Now, we will reduce it into

$= \frac{1-20}{5}+i\frac{(4-25)}{10}$
$= -\frac{19}{5}-i\frac{21}{10}$

Therefore, the answer is $-\frac{19}{5}-i\frac{21}{10}$

Question:7 Express each of the complex number in the form $a+ib$.

Answer:

Given problem is

Now, we will reduce it into

$$= \frac{1+4+12}{3} + i\frac{(7+1-3)}{3}$$
$$= \frac{17}{3} + i\frac{5}{3}$$

Therefore, the answer is $\frac{17}{3} + i\frac{5}{3}$

Question:8 Express each of the complex number in the form $a + ib$.

$(1-i)^4$

Answer:

The given problem is

$(1-i)^4$

Now, we will reduce it into

$$(1-i)^4 = ((1-i)^2)^2$$
$$= (1^2 + i^2 - 2.1.i)^2 \,(using\ (a-b)^2 = a^2 + b^2 - 2ab)$$
$$= (1-1-2i)^2 \,(\because i^2 = -1)$$
$$= (-2i)^2$$
$$= 4i^2$$
$$= -4$$

Therefore, the answer is $-4 + i0$

Question:9 Express each of the complex number in the form $a + ib$.

$$\left(\frac{1}{3}+3i\right)^3$$

Answer:

Given problem is

$$\left(\frac{1}{3}+3i\right)^3$$

Now, we will reduce it into

$$(using\ (a+b)^3=a^3+b^3+3a^2b+3ab^2)$$
$$=\frac{1}{27}+27i^3+i+9i^2$$
$$=\frac{1}{27}+27(-i)+i+9(-1)(\because i^3=-i\ and\ i^2=-1)$$
$$=\frac{1}{27}-27i+i-9$$
$$=\frac{1-243}{27}-26i$$
$$=-\frac{242}{27}-26i$$

Therefore, the answer is

$$-\frac{242}{27}-26i$$

Question:10 Express each of the complex number in the form $a+ib$.

$$\left(-2-\frac{1}{3}i\right)^3$$

Answer:

Given problem is

$$\left(-2-\frac{1}{3}i\right)^3$$

Now, we will reduce it into

$$(using\ (a+b)^3 = a^3+b^3+3a^2b+3ab^2)$$
$$= -\left(8+\frac{1}{27}i^3+3.4.\frac{1}{3}i+3.\frac{1}{9}i^2.2\right)$$
$$= -\left(8+\frac{1}{27}(-i)+4i+\frac{2}{3}(-1)\right)(\because i^3=-i\ and\ i^2=-1)$$
$$= -\left(8-\frac{1}{27}i+4i-\frac{2}{3}\right)$$
$$= -\left(\frac{(-1+108)}{27}i+\frac{24-2}{3}\right)$$
$$= -\frac{22}{3}-i\frac{107}{27}$$

Therefore, the answer is $-\frac{22}{3}-i\frac{107}{27}$

Question:11 Find the multiplicative inverse of each of the complex numbers.

$4-3i$

Answer:

Let $z=4-3i$

Then,

$\bar{z}=4+3i$

And

$|z|^2=4^2+(-3)^2=16+9=25$

Now, the multiplicative inverse is given by

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{4+3i}{25} = \frac{4}{25} + i\frac{3}{25}$$

Therefore, the multiplicative inverse is

$$\frac{4}{25} + i\frac{3}{25}$$

Question:12 Find the multiplicative inverse of each of the complex numbers.

$\sqrt{5} + 3i$

Answer:

Let $z = \sqrt{5} + 3i$

Then,

$\bar{z} = \sqrt{5} - 3i$

And

$|z|^2 = (\sqrt{5})^2 + (3)^2 = 5 + 9 = 14$

Now, the multiplicative inverse is given by

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{\sqrt{5} - 3i}{14} = \frac{\sqrt{5}}{14} - i\frac{3}{14}$$

Therefore, the multiplicative inverse is $\frac{\sqrt{5}}{14} - i\frac{3}{14}$

Question:13 Find the multiplicative inverse of each of the complex numbers.

$-i$

Answer:

Let $z = -i$

Then,

$\bar{z} = i$

And
$|z|^2 = (0)^2 + (1)^2 = 0 + 1 = 1$

Now, the multiplicative inverse is given by
$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{i}{1} = 0 + i$

Therefore, the multiplicative inverse is $0 + i1$

Question:14 Express the following expression in the form of $a + ib$:

$$\frac{(3+i\sqrt{5})(3-i\sqrt{5})}{(\sqrt{3}+\sqrt{2}i)-(\sqrt{3}-i\sqrt{2})}$$

Answer:

Given problem is
$$\frac{(3+i\sqrt{5})(3-i\sqrt{5})}{(\sqrt{3}+\sqrt{2}i)-(\sqrt{3}-i\sqrt{2})}$$

Now, we will reduce it into

$$(using\ (a-b)(a+b) = a^2 - b^2)$$
$$= \frac{9-5i^2}{\sqrt{3}+\sqrt{2}i-\sqrt{3}+\sqrt{2}i}$$
$$= \frac{9-5(-1)}{2\sqrt{2}i} \quad (\because i^2 = -1)$$
$$= \frac{14}{2\sqrt{2}i} \times \frac{\sqrt{2}i}{\sqrt{2}i}$$
$$= \frac{7\sqrt{2}i}{2i^2}$$

$= -\frac{7\sqrt{2}i}{2}$

Therefore, answer is $0 - i\frac{7\sqrt{2}}{2}$

NCERT solutions for class 11 maths chapter 5 Complex Numbers and Quadratic Equations-Exercise: 5.2

Question:1 Find the modulus and the arguments of each of the complex numbers.

$z = -1 - i\sqrt{3}$

Answer:

Given the problem is

$z = -1 - i\sqrt{3}$

Now, let

$r\cos\theta = -1 \quad and \quad r\sin\theta = -\sqrt{3}$

Square and add both the sides

$r^2(\cos^2\theta + \sin^2\theta) = (-1)^2 + (-\sqrt{3})^2 \; (\because \cos^2\theta + \sin^2\theta = 1)$

$r^2 = 1 + 3$

$r^2 = 4$

$r = 2 \; (\because r > 0)$

Therefore, the modulus is 2

Now,

$2\cos\theta = -1 \quad and \quad 2\sin\theta = -\sqrt{3}$

$\cos\theta = -\frac{1}{2} \quad and \quad \sin\theta = -\frac{\sqrt{3}}{2}$

Since, both the values of $\cos\theta \; and \; \sin\theta$ is negative and we know that they are negative in III quadrant

Therefore,

Argument = $-\left(\pi-\frac{\pi}{3}\right)=-\frac{2\pi}{3}$

Therefore, the argument is

$-\frac{2\pi}{3}$

Question:2 Find the modulus and the arguments of each of the complex numbers.

$z=-\sqrt{3}+i$

Answer:

Given the problem is

$z=-\sqrt{3}+i$

Now, let

$r\cos\theta=-\sqrt{3} \quad and \quad r\sin\theta=1$

Square and add both the sides

$r^2(\cos^2\theta+\sin^2\theta)=(-\sqrt{3})^2+(1)^2\,(\because \cos^2\theta+\sin^2\theta=1)$

$r^2=1+3$

$r^2=4$

$r=2\,(\because r>0)$

Therefore, the modulus is 2

Now,

$2\cos\theta=-\sqrt{3} \quad and \quad 2\sin\theta=1$

$\cos\theta=-\frac{\sqrt{3}}{2} \quad and \quad \sin\theta=\frac{1}{2}$

Since values of $\cos\theta$ is negative and value $\sin\theta$ is positive and we know that this is the case in II quadrant

Therefore,

Argument = $\left(\pi - \frac{\pi}{6}\right) = \frac{5\pi}{6}$

Therefore, the argument is

$\frac{5\pi}{6}$

Question:3 Convert each of the complex numbers in the polar form:

$1 - i$

Answer:

Given problem is

$z = 1 - i$

Now, let

$r\cos\theta = 1 \quad and \quad r\sin\theta = -1$

Square and add both the sides

$r^2(\cos^2\theta + \sin^2\theta) = (1)^2 + (-1)^2 \, (\because \cos^2\theta + \sin^2\theta = 1)$

$r^2 = 1 + 1$

$r^2 = 2$

$r = \sqrt{2} \, (\because r > 0)$

Therefore, the modulus is $\sqrt{2}$

Now,

$\sqrt{2}\cos\theta = 1 \quad and \quad \sqrt{2}\sin\theta = -1$

$\cos\theta = \frac{1}{\sqrt{2}} \quad and \quad \sin\theta = -\frac{1}{\sqrt{2}}$

Since values of $\sin\theta$ is negative and value $\cos\theta$ is positive and we know that this is the case in the IV quadrant

Therefore,

$\theta = -\frac{\pi}{4} \qquad (lies\ in\ IV\ quadrant)$

Therefore,

$$1 - i = r\cos\theta + ir\sin\theta$$
$$= \sqrt{2}\cos\left(-\frac{\pi}{4}\right) + i\sqrt{2}\sin\left(-\frac{\pi}{4}\right)$$
$$= \sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)$$

Therefore, the required polar form is $\sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)$

Question:4 Convert each of the complex numbers in the polar form:

$-1 + i$

Answer:

Given the problem is

$z = -1 + i$

Now, let

$r\cos\theta = -1 \quad and \quad r\sin\theta = 1$

Square and add both the sides

$r^2(\cos^2\theta + \sin^2\theta) = (1)^2 + (-1)^2\, (\because \cos^2\theta + \sin^2\theta = 1)$

$r^2 = 1 + 1$

$r^2 = 2$

$r = \sqrt{2}\, (\because r > 0)$

Therefore, the modulus is $\sqrt{2}$

Now,

$$\sqrt{2}\cos\theta = -1 \quad and \quad \sqrt{2}\sin\theta = 1$$
$$\cos\theta = -\frac{1}{\sqrt{2}} \quad and \quad \sin\theta = \frac{1}{\sqrt{2}}$$

Since values of $\cos\theta$ is negative and value $\sin\theta$ is positive and we know that this is the case in II quadrant

Therefore,

Therefore,

$$-1+i = r\cos\theta + ir\sin\theta$$
$$= \sqrt{2}\cos\left(\frac{3\pi}{4}\right) + i\sqrt{2}\sin\left(\frac{3\pi}{4}\right)$$
$$= \sqrt{2}\left(\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)\right)$$

Therefore, the required polar form is $\sqrt{2}\left(\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)\right)$

Question:5 Convert each of the complex numbers in the polar form:

$-1-i$

Answer:

Given problem is

$z = -1-i$

Now, let

$r\cos\theta = -1 \quad and \quad r\sin\theta = -1$

Square and add both the sides

$r^2(\cos^2\theta + \sin^2\theta) = (-1)^2 + (-1)^2 \; (\because \cos^2\theta + \sin^2\theta = 1)$

$r^2 = 1+1$

$r^2 = 2$

$r = \sqrt{2}$ &nbsnbsp; $(\because r > 0)$

Therefore, the modulus is $\sqrt{2}$

Now,

$$\sqrt{2}\cos\theta = -1 \quad and \quad \sqrt{2}\sin\theta = -1$$
$$\cos\theta = -\frac{1}{\sqrt{2}} \quad and \quad \sin\theta = -\frac{1}{\sqrt{2}}$$

Since values of both $\cos\theta$ and $\sin\theta$ is negative and we know that this is the case in III quadrant

Therefore,

Therefore,

$$-1 - i = r\cos\theta + ir\sin\theta$$
$$= \sqrt{2}\cos\left(-\frac{3\pi}{4}\right) + i\sqrt{2}\sin\left(-\frac{3\pi}{4}\right)$$
$$= \sqrt{2}\left(\cos\left(-\frac{3\pi}{4}\right) + i\sin\left(-\frac{3\pi}{4}\right)\right)$$

Therefore, the required polar form is $\sqrt{2}\left(\cos\left(-\frac{3\pi}{4}\right) + i\sin\left(-\frac{3\pi}{4}\right)\right)$

Question:6 Convert each of the complex numbers in the polar form:

-3

Answer:

Given problem is

$z = -3$

Now, let

$$r\cos\theta = -3 \quad and \quad r\sin\theta = 0$$

Square and add both the sides

$$r^2(\cos^2\theta + \sin^2\theta) = (-3)^2 + (0)^2 \, (\because \cos^2\theta + \sin^2\theta = 1)$$

$r^2 = 9 + 0$

$r^2 = 9$

$r = 3\,(\because r > 0)$

Therefore, the modulus is **3**

Now,

$3\cos\theta = -3 \quad and \quad 3\sin\theta = 0$

$\cos\theta = -1 \quad and \quad \sin\theta = 0$

Since values of $\cos\theta$ is negative and $\sin\theta$ is Positive and we know that this is the case in II quadrant

Therefore,

$\theta = \pi \qquad\qquad (lies\ in\ II\ quadrant)$

Therefore,

$-3 = r\cos\theta + ir\sin\theta$

$= 3\cos(\pi) + i3\sin(\pi)$

$= 3(\cos\pi + i\sin\pi)$

Therefore, the required polar form is $3(\cos\pi + i\sin\pi)$

Question:7 Convert each of the complex numbers in the polar form:

$\sqrt{3} + i$

Answer:

Given problem is

$z = \sqrt{3} + i$

Now, let

$r\cos\theta = \sqrt{3} \quad and \quad r\sin\theta = 1$

Square and add both the sides

$r^2(\cos^2\theta + \sin^2\theta) = (\sqrt{3})^2 + (1)^2\ (\because \cos^2\theta + \sin^2\theta = 1)$

$r^2 = 3 + 1$

$r^2 = 4$

$r = 2\ (\because r > 0)$

Therefore, the modulus is **2**

Now,

$2\cos\theta = \sqrt{3} \quad and \quad 2\sin\theta = 1$

$\cos\theta = \dfrac{\sqrt{3}}{2} \quad and \quad \sin\theta = \dfrac{1}{2}$

Since values of Both $\cos\theta$ and $\sin\theta$ is Positive and we know that this is the case in I quadrant

Therefore,

$\theta = \dfrac{\pi}{6} \qquad (lies\ in\ I\ quadrant)$

Therefore,

$\sqrt{3} + i = r\cos\theta + ir\sin\theta$

$= 2\cos\left(\dfrac{\pi}{6}\right) + i2\sin\left(\dfrac{\pi}{6}\right)$

$= 2\left(\cos\dfrac{\pi}{6} + i\sin\dfrac{\pi}{6}\right)$

Therefore, the required polar form is $2\left(\cos\dfrac{\pi}{6} + i\sin\dfrac{\pi}{6}\right)$

Question:8 Convert each of the complex numbers in the polar form:

i

Answer:

Given problem is

$z = i$

Now, let

$r\cos\theta = 0 \quad and \quad r\sin\theta = 1$

Square and add both the sides

$r^2(\cos^2\theta + \sin^2\theta) = (0)^2 + (1)^2\ (\because \cos^2\theta + \sin^2\theta = 1)$

$r^2 = 0 + 1$

$r^2 = 1$

$r = 1\ (\because r > 0)$

Therefore, the modulus is **1**

Now,

$1\cos\theta = 0 \quad and \quad 1\sin\theta = 1$

$\cos\theta = 0 \quad and \quad \sin\theta = 1$

Since values of Both $\cos\theta$ and $\sin\theta$ is Positive and we know that this is the case in I quadrant

Therefore,

$\theta = \frac{\pi}{2} \qquad (lies\ in\ I\ quadrant)$

Therefore,

$$\begin{aligned} i &= r\cos\theta + ir\sin\theta \\ &= 1\cos\left(\frac{\pi}{2}\right) + i1\sin\left(\frac{\pi}{2}\right) \\ &= \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} \end{aligned}$$

Therefore, the required polar form is $\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$

NCERT solutions for class 11 maths chapter 5 Complex Numbers and Quadratic Equations-Exercise: 5.3

Question:1 Solve each of the following equations: $x^2 + 3 = 0$

Answer:

Given equation is

$x^2 + 3 = 0$

Now, we know that the roots of the quadratic equation is given by the formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In this case value of **a = 1 , b = 0 and c = 3**

Therefore,

$$\frac{-0 \pm \sqrt{0^2 - 4.1.(3)}}{2.1} = \frac{\pm\sqrt{-12}}{2} = \frac{\pm 2\sqrt{3}i}{2} = \pm\sqrt{3}i$$

Therefore, the solutions of requires equation are $\pm\sqrt{3}i$

Question:2 Solve each of the following equations: $2x^2 + x + 1 = 0$

Answer:

Given equation is

$2x^2 + x + 1 = 0$

Now, we know that the roots of the quadratic equation are given by the formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In this case value of **a = 2 , b = 1 and c = 1**

Therefore,

$$\frac{-1\pm\sqrt{1^2-4.2.1}}{2.2}=\frac{-1\pm\sqrt{1-8}}{4}=\frac{-1\pm\sqrt{-7}}{4}=\frac{-1\pm\sqrt{7}i}{4}$$

Therefore, the solutions of requires equation are

$$\frac{-1\pm\sqrt{7}i}{4}$$

Question:3 Solve each of the following equations: $x^2+3x+9=0$

Answer:

Given equation is

$x^2+3x+9=0$

Now, we know that the roots of the quadratic equation are given by the formula

$$\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

In this case value of **a = 1 , b = 3 and c = 9**

Therefore,

$$\frac{-3\pm\sqrt{3^2-4.1.9}}{2.1}=\frac{-3\pm\sqrt{9-36}}{2}=\frac{-3\pm\sqrt{-27}}{2}=\frac{-3\pm3\sqrt{3}i}{2}$$

Therefore, the solutions of requires equation are

$$\frac{-3\pm3\sqrt{3}i}{2}$$

Question:4 Solve each of the following equations: $-x^2+x-2=0$

Answer:

Given equation is

$-x^2+x-2=0$

Now, we know that the roots of the quadratic equation is given by the formula

$$\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

In this case value of **a = -1 , b = 1 and c = -2**

Therefore,

Therefore, the solutions of equation are

$$\frac{-1 \pm \sqrt{7}i}{-2}$$

Question:5 Solve each of the following equations: $x^2 + 3x + 5 = 0$

Answer:

Given equation is

$x^2 + 3x + 5 = 0$

Now, we know that the roots of the quadratic equation are given by the formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In this case value of **a = 1 , b = 3 and c = 5**

Therefore,

$$\frac{-3 \pm \sqrt{3^2 - 4.1.5}}{2.1} = \frac{-3 \pm \sqrt{9 - 20}}{2} = \frac{-3 \pm \sqrt{-11}}{2} = \frac{-3 \pm \sqrt{11}i}{2}$$

Therefore, the solutions of the equation are $\frac{-3 \pm \sqrt{11}i}{2}$

Question:6 Solve each of the following equations: $x^2 - x + 2 = 0$

Answer:

Given equation is

$x^2 - x + 2 = 0$

Now, we know that the roots of the quadratic equation are given by the formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In this case value of **a = 1 , b = -1 and c = 2**

Therefore,

Therefore, the solutions of equation are $\frac{1 \pm \sqrt{7}i}{2}$

Question:7 Solve each of the following equations: $\sqrt{2}x^2 + x + \sqrt{2} = 0$

Answer:

Given equation is

$\sqrt{2}x^2 + x + \sqrt{2} = 0$

Now, we know that the roots of the quadratic equation is given by the formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In this case the value of $a = \sqrt{2}, b = 1 \ and \ c = \sqrt{2}$

Therefore,

Therefore, the solutions of the equation are $\frac{-1 \pm \sqrt{7}i}{2\sqrt{2}}$

Question:8 Solve each of the following equations: $\sqrt{3}x^2 - \sqrt{2}x + 3\sqrt{3} = 0$

Answer:

Given equation is

$\sqrt{3}x^2 - \sqrt{2}x + 3\sqrt{3} = 0$

Now, we know that the roots of the quadratic equation are given by the formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In this case the value of $a = \sqrt{3}, b = -\sqrt{2}\ and\ c = 3\sqrt{3}$

Therefore,
$$= \frac{\sqrt{2} \pm \sqrt{34}i}{2\sqrt{3}}$$

Therefore, the solutions of the equation are $\frac{\sqrt{2} \pm \sqrt{34}i}{2\sqrt{3}}$

Question:9 Solve each of the following equations: $x^2 + x + \frac{1}{\sqrt{2}} = 0$

Answer:

Given equation is
$$x^2 + x + \frac{1}{\sqrt{2}} = 0$$

Now, we know that the roots of the quadratic equation is given by the formula
$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In this case the value of $a = 1, b = 1\ and\ c = \frac{1}{\sqrt{2}}$

Therefore,
$$\frac{-1 \pm \sqrt{1^2 - 4.1.\frac{1}{\sqrt{2}}}}{2.1} = \frac{-1 \pm \sqrt{1 - 2\sqrt{2}}}{2} = \frac{-1 \pm \sqrt{-(2\sqrt{2} - 1)}}{2}$$
$$= \frac{-1 \pm \sqrt{(2\sqrt{2} - 1)}i}{2}$$

Therefore, the solutions of the equation are

$$\frac{-1 \pm \sqrt{(2\sqrt{2} - 1)}i}{2}$$

Question:10 Solve each of the following equations:

$$x^2 + \frac{x}{\sqrt{2}} + 1 = 0$$

Answer:

Given equation is
$$x^2 + \frac{x}{\sqrt{2}} + 1 = 0$$

Now, we know that the roots of the quadratic equation are given by the formula
$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In this case the value of $a = 1, b = \frac{1}{\sqrt{2}} \text{ and } c = 1$

Therefore,
$$= \frac{-1 \pm \sqrt{7}i}{2\sqrt{2}}$$

Therefore, the solutions of the equation are

$$\frac{-1 \pm \sqrt{7}i}{2\sqrt{2}}$$

NCERT solutions for class 11 maths chapter 5 Complex Numbers and Quadratic Equations-Miscellaneous Exercise

Question:1 Evaluate $\left[i^{18} + \left(\frac{1}{i}\right)^{25}\right]^3$.

Answer:

The given problem is
$$\left[i^{18} + \left(\frac{1}{i}\right)^{25}\right]^3$$

Now, we will reduce it into

$= \left[1^4.(-1) + \frac{1}{1^6.i}\right]^3 \ (\because i^4 = 1, i^2 = -1)$

$= \left[-1 + \frac{1}{i}\right]^3$

$= \left[-1 + \frac{1}{i} \times \frac{i}{i}\right]^3$

$= \left[-1 + \frac{i}{i^2}\right]^3$

$= \left[-1 + \frac{i}{-1}\right]^3 = [-1 - i]^3$

Now,

$-(1+i)^3 = -(1^3 + i^3 + 3.1^2.i + 3.1.i^2)$

$(using\ (a+b)^3 = a^3 + b^3 + 3.a^2.b + 3.a.b^2)$

$= -(1 - i + 3i + 3(-1)) \ (\because i^3 = -i, i^2 = -1)$

$= -(1 - i + 3i - 3) = -(-2 + 2i)$

$= 2 - 2i$

Therefore, answer is $2 - 2i$

Question:2 For any two complex numbers z_1 and z_2, prove that $Re(z_1 z_2) = Re\ z_1\ Re\ z_2 - Imz_1\ Imz_2$

Answer:

Let two complex numbers are

$z_1 = x_1 + iy_1$

$z_2 = x_2 + iy_2$

Now,

$z_1.z_2 = (x_1 + iy_1).(x_2 + iy_2)$

$= x_1 x_2 + ix_1 y_2 + iy_1 x_2 + i^2 y_1 y_2$

$= x_1x_2 + ix_1y_2 + iy_1x_2 - y_1y_2 (\because i^2 = -1)$

$= x_1x_2 - y_1y_2 + i(x_1y_2 + y_1x_2)$

$Re(z_1z_2) = x_1x_2 - y_1y_2$

$= Re(z_1z_2) - Im(z_1z_2)$

Hence proved

Question:3 Reduce $\left(\frac{1}{1-4i} - \frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)$ to the standard form.

Answer:

Given problem is
$\left(\frac{1}{1-4i} - \frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)$

Now, we will reduce it into

$$= \left(\frac{1+i-2+8i}{1-4i+i-4i^2}\right)\left(\frac{3-4i}{5+i}\right)$$
$$= \left(\frac{-1+9i}{1-3i-4(-1)}\right)\left(\frac{3-4i}{5+i}\right)$$
$$= \left(\frac{-1+9i}{5-3i}\right)\left(\frac{3-4i}{5+i}\right)$$

Now, multiply numerator an denominator by $(14+5i)$

$$\Rightarrow \frac{33+31i}{2(14-5i)} \times \frac{14+5i}{14+5i}$$
$$\Rightarrow \frac{462+165i+434i+155i^2}{2(14^2-(5i)^2)} (using\ (a-b)(a+b) = a^2 - b^2)$$

$$\Rightarrow \frac{462+599i-155}{2(196-25i^2)}$$
$$\Rightarrow \frac{307+599i}{2(196+25)} = \frac{307+599i}{2\times 221} = \frac{307+599i}{442} = \frac{307}{442} + i\frac{599}{442}$$

Therefore, answer is $\frac{307}{442} + i\frac{599}{442}$

Question:4 If $x - iy = \sqrt{\frac{a-ib}{c-id}}$, prove that $(x^2+y^2)^2 = \frac{a^2+b^2}{c^2+d^2}$.

Answer:

the given problem is

$$x - iy = \sqrt{\frac{a-ib}{c-id}}$$

Now, multiply the numerator and denominator by

$$\sqrt{c+id}$$
$$x - iy = \sqrt{\frac{a-ib}{c-id} \times \frac{c+id}{c+id}}$$
$$= \sqrt{\frac{(ac+bd)+i(ad-bc)}{c^2-i^2d^2}} = \sqrt{\frac{(ac+bd)+i(ad-bc)}{c^2+d^2}}$$

Now, square both the sides

$$(x-iy)^2 = \left(\sqrt{\frac{(ac+bd)+i(ad-bc)}{c^2+d^2}}\right)^2$$
$$= \frac{(ac+bd)+i(ad-bc)}{c^2+d^2}$$
$$x^2 - y^2 - 2ixy = \frac{(ac+bd)+i(ad-bc)}{c^2+d^2}$$

On comparing the real and imaginary part, we obtain

$$x^2 - y^2 = \frac{ac + bd}{c^2 + d^2} \;\; and \;\; -2xy = \frac{ad - bc}{c^2 + d^2} \quad -(i)$$

Now,

$$(x^2 + y^2)^2 = (x^2 - y^2)^2 + 4x^2y^2$$
$$= \left(\frac{ac + bd}{c^2 + d^2}\right)^2 + \left(\frac{ad - bc}{c^2 + d^2}\right)^2 \quad (using\ (i))$$
$$= \frac{a^2c^2 + b^2d^2 + 2acbd + a^2d^2 + b^2c^2 - 2adbc}{(c^2 + d^2)^2}$$
$$= \frac{a^2c^2 + b^2d^2 + a^2d^2 + b^2c^2}{(c^2 + d^2)^2}$$
$$= \frac{a^2(c^2 + d^2) + b^2(c^2 + d^2)}{(c^2 + d^2)^2}$$
$$= \frac{(a^2 + b^2)(c^2 + d^2)}{(c^2 + d^2)^2}$$
$$= \frac{(a^2 + b^2)}{(c^2 + d^2)}$$

Hence proved

Question:5(i) Convert the following in the polar form:

$$\frac{1 + 7i}{(2 - i)^2}$$

Answer:

Let

$$z = \frac{1 + 7i}{(2 - i)^2} = \frac{1 + 7i}{4 + i^2 - 4i} = \frac{1 + 7i}{4 - 1 - 4i} = \frac{1 + 7i}{3 - 4i}$$

Now, multiply the numerator and denominator by $3 + 4i$

$$\Rightarrow z = \frac{1+7i}{3-4i} \times \frac{3+4i}{3+4i} = \frac{3+4i+21i+28i^2}{3^2+4^2} = \frac{-25+25i}{25} = -1+i$$

Now,

let

$r\cos\theta = -1 \quad and \quad r\sin\theta = 1$

On squaring both and then add

$r^2(\cos^2\theta + \sin^2\theta) = (-1)^2 + 1^2$

$$r^2 = 2$$
$$r = \sqrt{2} \qquad\qquad (\because r > 0)$$

Now,

$$\sqrt{2}\cos\theta = -1 \quad and \quad \sqrt{2}\sin\theta = 1$$
$$\cos\theta = -\frac{1}{\sqrt{2}} \quad and \quad \sin\theta = \frac{1}{\sqrt{2}}$$

Since the value of $\cos\theta$ is negative and $\sin\theta$ is positive this is the case in II quadrant

Therefore,

$$z = r\cos\theta + ir\sin\theta$$
$$= \sqrt{2}\cos\frac{3\pi}{4} + i\sqrt{2}\sin\frac{3\pi}{4}$$
$$= \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$$

Therefore, the required polar form is

$$\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$$

Question:5(ii) Convert the following in the polar form:

$$\frac{1+3i}{1-2i}$$

Answer:

Let

$z = \frac{1+3i}{1-2i}$

Now, multiply the numerator and denominator by $1+2i$

$\Rightarrow z = \frac{1+3i}{1-2i} \times \frac{1+2i}{i+2i} = \frac{1+2i+3i-6}{1+4} = \frac{-5+5i}{5} = -1+i$

Now,

let

$r\cos\theta = -1 \quad and \quad r\sin\theta = 1$

On squaring both and then add

$r^2(\cos^2\theta + \sin^2\theta) = (-1)^2 + 1^2$

$r^2 = 2$

$r = \sqrt{2} \qquad\qquad (\because r > 0)$

Now,

$\sqrt{2}\cos\theta = -1 \quad and \quad \sqrt{2}\sin\theta = 1$

$\cos\theta = -\frac{1}{\sqrt{2}} \quad and \quad \sin\theta = \frac{1}{\sqrt{2}}$

Since the value of $\cos\theta$ is negative and $\sin\theta$ is positive this is the case in II quadrant

Therefore,

$$z = r\cos\theta + ir\sin\theta$$
$$= \sqrt{2}\cos\frac{3\pi}{4} + i\sqrt{2}\sin\frac{3\pi}{4}$$
$$= \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$$

Therefore, the required polar form is

$$\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$$

Question:6 Solve each of the equation: $3x^2 - 4x + \frac{20}{3} = 0$

Answer:

Given equation is
$3x^2 - 4x + \frac{20}{3} = 0$

Now, we know that the roots of the quadratic equation are given by the formula
$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

In this case the value of

$a = 3, b = -4 \ and \ c = \frac{20}{3}$

Therefore,
$\frac{-(-4) \pm \sqrt{(-4)^2 - 4.3.\frac{20}{3}}}{2.3} = \frac{4 \pm \sqrt{16 - 80}}{6} = \frac{4 \pm \sqrt{-64}}{6} = \frac{4 \pm 8i}{6} = \frac{2}{3} \pm i\frac{4}{3}$

Therefore, the solutions of requires equation are

$\frac{2}{3} \pm i\frac{4}{3}$

Question:7 Solve each of the equation: $x^2 - 2x + \frac{3}{2} = 0$

Answer:

Given equation is
$x^2 - 2x + \frac{3}{2} = 0$

Now, we know that the roots of the quadratic equation are given by the formula
$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

In this case the value of $a = 1, b = -2 \ and \ c = \frac{3}{2}$

Therefore,

$$\frac{-(-2) \pm \sqrt{(-2)^2 - 4.1.\frac{3}{2}}}{2.1} = \frac{2 \pm \sqrt{4-6}}{2} = \frac{2 \pm \sqrt{-2}}{2} = \frac{2 \pm i\sqrt{2}}{2} = 1 \pm i\frac{\sqrt{2}}{2}$$

Therefore, the solutions of requires equation are

$$1 \pm i\frac{\sqrt{2}}{2}$$

Question:8 Solve each of the equation: $27x^2 - 10x + 1 = 0$.

Answer:

Given equation is

$$27x^2 - 10x + 1 = 0$$

Now, we know that the roots of the quadratic equation are given by the formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In this case the value of $a = 27, b = -10 \ and \ c = 1$

Therefore,

$$\frac{-(-10) \pm \sqrt{(-10)^2 - 4.27.1}}{2.27} = \frac{10 \pm \sqrt{100 - 108}}{54} = \frac{10 \pm \sqrt{-8}}{54}$$

$$= \frac{10 \pm i2\sqrt{2}}{54} = \frac{5}{27} \pm i\frac{\sqrt{2}}{27}$$

Therefore, the solutions of requires equation are $\frac{5}{27} \pm i\frac{\sqrt{2}}{27}$

Question:9 Solve each of the equation: $21x^2 - 28x + 10 = 0$

Answer:

Given equation is

$$21x^2 - 28x + 10 = 0$$

Now, we know that the roots of the quadratic equation are given by the formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In this case the value of $a = 21, b = -28 \; and \; c = 10$

Therefore,
$$\frac{-(-28) \pm \sqrt{(-28)^2 - 4.21.10}}{2.21} = \frac{28 \pm \sqrt{784 - 840}}{42} = \frac{28 \pm \sqrt{-56}}{42}$$
$$= \frac{28 \pm i2\sqrt{14}}{42} = \frac{2}{3} \pm i\frac{\sqrt{14}}{21}$$

Therefore, the solutions of requires equation are

$$\frac{2}{3} \pm i\frac{\sqrt{14}}{21}$$

Question:10 If $z_1 = 2 - i, z_2 = 1 + i$, find $\left|\frac{z_1 + z_2 + 1}{z_1 - z_2 + 1}\right|$.

Answer:

It is given that
$z_1 = 2 - i, z_2 = 1 + i$

Then,

Now, multiply the numerator and denominator by $1 + i$

Now,
$|1 + i| = \sqrt{1^2 + 1^2} = \sqrt{1 + 1} = \sqrt{2}$

Therefore, the value of

$\left|\frac{z_1 + z_2 + 1}{z_1 - z_2 + 1}\right|$ is $\sqrt{2}$

Question:11 If $a + ib = \frac{(x+i)^2}{2x^2 + 1}$, prove that $a^2 + b^2 = \frac{(x^2+1)^2}{(2x^2+1)^2}$.

Answer:

It is given that
$a + ib = \frac{(x+i)^2}{2x^2+1}$

Now, we will reduce it into

On comparing real and imaginary part. we will get
$a = \frac{x^2-1}{2x^2+1}\ and\ b = \frac{2x}{2x^2+1}$

Now,
$a^2 + b^2 = \left(\frac{x^2-1}{2x^2+1}\right)^2 + \left(\frac{2x}{2x^2+1}\right)^2$
$= \frac{x^4+1-2x^2+4x^2}{(2x^2+1)^2}$
$= \frac{x^4+1+2x^2}{(2x^2+1)^2}$
$= \frac{(x^2+1)^2}{(2x^2+1)^2}$

Hence proved

Question:12(i) Let $z_1 = 2 - i, z_2 = -2 + i.$ Find

$Re\left(\frac{z_1 z_2}{\bar{z_1}}\right)$

Answer:

It is given that
$z_1 = 2 - i\ and\ z_2 = -2 + i$

Now,

$z_1 z_2 = (2-i)(-2+i) = -4+2i+2i-i^2 = -4+4i+1 = -3+4i$

And

$\bar{z}_1 = 2+i$

Now,

$= \frac{-2+11i}{5} = -\frac{2}{5} + i\frac{11}{5}$

Now,

$Re\left(\frac{z_1 z_2}{z_1}\right) = -\frac{2}{5}$

Therefore, the answer is

$-\frac{2}{5}$

Question:12(ii) Let $z_1 = 2-i, z_2 = -2+i$. Find

$Im\left(\frac{1}{z_1 \bar{z}_1}\right)$

Answer:

It is given that

$z_1 = 2-i$

Therefore,

$\bar{z}_1 = 2+i$

NOw,

$z_1 \bar{z}_1 = (2-i)(2+i) = 2^2 - i^2 = 4+1 = 5\,(using\ (a-b)(a+b) = a^2 - b^2)$

Now,

$\frac{1}{z_1 \bar{z}_1} = \frac{1}{5}$

Therefore,

$$Im\left(\frac{1}{z_1\bar{z}_1}\right) = 0$$

Therefore, the answer is **0**

Question:13 Find the modulus and argument of the complex number $\frac{1+2i}{1-3i}$.

Answer:

Let
$$z = \frac{1+2i}{1-3i}$$

Now, multiply the numerator and denominator by $(1+3i)$

$$= -\frac{1}{2} + i\frac{1}{2}$$

Therefore,
$$r\cos\theta = -\frac{1}{2} \quad and \quad r\sin\theta = \frac{1}{2}$$

Square and add both the sides
$$r^2\cos^2\theta + r^2\sin^2\theta = \left(-\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2$$
$$r^2(\cos^2\theta + \sin^2\theta) = \left(\frac{1}{4}\right) + \left(\frac{1}{4}\right)$$

$$r = \frac{1}{\sqrt{2}} \qquad\qquad (\because r > 0)$$

Therefore, the modulus is $\frac{1}{\sqrt{2}}$

Now,
$$\frac{1}{\sqrt{2}}\cos\theta = -\frac{1}{2} \quad and \quad \frac{1}{\sqrt{2}}\sin\theta = \frac{1}{2}$$
$$\cos\theta = -\frac{1}{\sqrt{2}} \quad and \quad \sin\theta = \frac{1}{\sqrt{2}}$$

Since the value of $\cos\theta$ is negative and the value of $\sin\theta$ is positive and we know that it is the case in II quadrant

Therefore,

Argument $= \left(\pi - \frac{\pi}{4}\right) = \frac{3\pi}{4}$

Therefore, Argument and modulus are $\frac{3\pi}{4} \quad and \quad \frac{1}{\sqrt{2}}$ respectively

Question:14 Find the real numbers x andy if $(x - iy)(3 + 5i)$ is the conjugate of $-6 - 24i$.

Answer:

Let

$z = (x - iy)(3 + 5i) = 3x + 5xi - 3yi - 5yi^2 = 3x + 5y + i(5x - 3y)$

Therefore,

$\bar{z} = (3x + 5y) - i(5x - 3y) \qquad\qquad - (i)$

Now, it is given that

$\bar{z} = -6 - 24i \qquad\qquad - (ii)$

Compare (i) and (ii) we will get

$(3x + 5y) - i(5x - 3y) = -6 - 24i$

On comparing real and imaginary part. we will get

$3x + 5y = -6 \quad and \quad 5x - 3y = 24$

On solving these we will get

$x = 3 \quad and \quad y = -3$

Therefore, the value of x and y are 3 and -3 respectively

Question:15 Find the modulus of $\frac{1+i}{1-i}-\frac{1-i}{1+i}$.

Answer:

Let

$$z=\frac{1+i}{1-i}-\frac{1-i}{1+i}$$

Now, we will reduce it into

$$z=\frac{1+i}{1-i}-\frac{1-i}{1+i}=\frac{(1+i)^2-(1-i)^2}{(1+i)(1-i)}=\frac{1^2+i^2+2i-1^2-i^2+2i}{1^2-i^2}$$

$$=\frac{4i}{1+1}=\frac{4i}{2}=2i$$

Now,

$$r\cos\theta=0 \quad and \quad r\sin\theta=2$$

square and add both the sides. we will get,

$$r^2\cos^2\theta+r^2\sin^2\theta=0^2+2^2$$

$$r^2(\cos^2\theta+\sin^2\theta)=4$$

$$r^2=4 \qquad (\because \cos^2\theta+\sin^2\theta=1)$$

$$r=2 \qquad (\because r>0)$$

Therefore, modulus of

$\frac{1+i}{1-i}-\frac{1-i}{1+i}$ is **2**

Question:16 If $(x+iy)^3=u+iv$, then show that $\frac{u}{x}+\frac{v}{y}=4(x^2-y^2)$.

Answer:

it is given that

$$(x+iy)^3=u+iv$$

Now, expand the Left-hand side

$x^3 + (iy)^3 + 3.(x)^2.iy + 3.x.(iy)^2 = u + iv$

$x^3 + i^3y^3 + 3x^2iy + 3xi^2y^2 = u + iv$

$x^3 - iy^3 + 3x^2iy - 3xy^2 = u + iv\,(\because i^3 = -i \;\; and \;\; i^2 = -1)$

$x^3 - 3xy^2 + i(3x^2y - y^3) = u + iv$

On comparing real and imaginary part. we will get,

$u = x^3 - 3xy^2 \quad and \quad v = 3x^2y - y^3$

Now,

$\frac{u}{x} + \frac{v}{y} = \frac{x(x^2 - 3y^2)}{x} + \frac{y(3x^2 - y^2)}{y}$

$= x^2 - 3y^2 + 3x^2 - y^2$

$= 4x^2 - 4y^2$

$= 4(x^2 - y^2)$

Hence proved

Question:17 If α and β are different complex numbers with $|\beta| = 1$, then find $\left|\frac{\beta - \alpha}{1 - \bar{\alpha}\beta}\right|$.

Answer:

Let

$\alpha = a + ib$ and $\beta = x + iy$

It is given that

$|\beta| = 1 \Rightarrow \sqrt{x^2 + y^2} = 1 \Rightarrow x^2 + y^2 = 1$

and

$\bar{\alpha} = a - ib$

Now,

$$= \left|\frac{(x-a)+i(y-b)}{(1-ax-yb)-i(bx-ay)}\right|$$
$$= \frac{\sqrt{(x-a)^2+(y-b)^2}}{\sqrt{(1-ax-yb)^2+(bx-ay)^2}}$$
$$= \frac{\sqrt{x^2+a^2-2xa+y^2+b^2-yb}}{\sqrt{1+a^2x^2+b^2y^2-2ax+2abxy-by+b^2x^2+a^2y^2-2abxy}}$$
$$= \frac{\sqrt{(x^2+y^2)+a^2-2xa+b^2-yb}}{\sqrt{1+a^2(x^2+y^2)+b^2(x^2+y^2)-2ax+2abxy-by-2abxy}}$$
$$= \frac{\sqrt{1+a^2-2xa+b^2-yb}}{\sqrt{1+a^2+b^2-2ax-by}} \left(\because x^2+y^2=1 \ given\right)$$
$$= 1$$

Therefore, value of $\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|$ is **1**

Question:18 Find the number of non-zero integral solutions of the equation $|1-i|^x = 2^x$.

Answer:

Given problem is

$|1-i|^x = 2^x$

Now,

$(\sqrt{1^2+(-1)^2})^x = 2^x$

$(\sqrt{1+1})^x = 2^x$

$\left(\sqrt{2}\right)^x = 2^x$

$2^{\frac{x}{2}} = 2^x$

$\frac{x}{2} = x$

$\frac{x}{2} = 0$

x = 0 is the only possible solution to the given problem

Therefore, there are **0** number of non-zero integral solutions of the equation $|1-i|^x = 2^x$

Question:19 If $(a+ib)(c+id)(e+if)(g+ih) = A+iB$, then show that $(a^2+b^2)(c^2+d^2)(e^2+f^2)(g^2+h^2) = A^2+B^2$

Answer:

It is given that
$(a+ib)(c+id)(e+if)(g+ih) = A+iB$,

Now, take mod on both sides
$|(a+ib)(c+id)(e+if)(g+ih)| = |A+iB|$
$|(a+ib)||(c+id)||(e+if)||(g+ih)| = |A+iB| (\because |z_1 z_2| = |z_1||z_2|)$
$(\sqrt{a^2+b^2})(\sqrt{c^2+d^2})(\sqrt{e^2+f^2})(\sqrt{g^2+h^2}) = (\sqrt{A^2+B^2})$

Square both the sides. we will get

$$(a^2+b^2)(c^2+d^2)(e^2+f^2)(g^2+h^2) = (A^2+B^2)$$

Hence proved

Question:20 If $\left(\frac{1+i}{1-i}\right)^m = 1,$ then find the least positive integral value of m .

Answer:

Let
$z = \left(\frac{1+i}{1-i}\right)^m$

Now, multiply both numerator and denominator by $(1+i)$

We will get,

$$z = \left(\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right)^m$$

$$= \left(\frac{(1+i)^2}{1^2 - i^2}\right)^m$$

$$= \left(\frac{1^2 + i^2 + 2i}{1+1}\right)^m$$

$$= \left(\frac{1-1+2i}{2}\right)^m \quad (\because i^2 = -1)$$

$$= \left(\frac{2i}{2}\right)^m$$

$$= i^m$$

We know that $i^4 = 1$

Therefore, the least positive integral value of m is **4**